



PHD

Thai Business Cycles: Theory and Practice

Wongpunya, Nipit

Award date:
2009

Awarding institution:
University of Bath

[Link to publication](#)

Alternative formats

If you require this document in an alternative format, please contact:
openaccess@bath.ac.uk

Copyright of this thesis rests with the author. Access is subject to the above licence, if given. If no licence is specified above, original content in this thesis is licensed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International (CC BY-NC-ND 4.0) Licence (<https://creativecommons.org/licenses/by-nc-nd/4.0/>). Any third-party copyright material present remains the property of its respective owner(s) and is licensed under its existing terms.

Take down policy

If you consider content within Bath's Research Portal to be in breach of UK law, please contact: openaccess@bath.ac.uk with the details. Your claim will be investigated and, where appropriate, the item will be removed from public view as soon as possible.

THAI BUSINESS CYCLES: THEORY AND PRACTICE

by

Nipit Wongpunya

A thesis submitted for the degree of

Doctor of Philosophy

University of Bath

Department of Economics and International Development

October 2009

COPYRIGHT

Attention is drawn to the fact that copyright of this thesis rests with its author. This copy of this thesis has been applied on condition that anyone who consults it is understood to recognise that its copyright rests with author and that no quotation from the thesis and no information derived from it may be published without prior written consent of the author. This thesis may be made available for consultation within the University Library and may be photocopied or lent to other libraries for the purposes of consultation.

For my parents

ACKNOWLEDGEMENTS

Several people and organizations have been instrumental in me producing this thesis. I could not have done this thesis without their influences and contributions.

First of all, I wish to acknowledge the Royal Thai Government and the Royal Thai Embassy in the U.S. and the U.K. for my financial support and taking care of me during my years of study abroad. I also wish to acknowledge the Department of Economics and International Development at University of Bath for giving me the opportunity to teach and learn economics.

I wish to acknowledge with deep gratitude my supervisors, Prof. John Hudson and Dr. Andrew Abbott for their immensely helpful suggestions, advice, comments and criticisms. More importantly, for their guidance to something that I could never imagined of.

I would like to thanks especially Khris Mattar, Mary Morgan and all Thai friends for their wonderful friendship, support, encouragement and patience throughout the duration of this thesis.

Finally, I have a personal acknowledgement to my mom. Throughout all of my endeavours, your endless love and never ending support have massively been inspirational to me.

ABSTRACT

This thesis documents the stylized facts of the business cycle in Thailand and analyzes the ability of real business cycle models to capture these facts. The models are solved by the method of finding a linear approximation to the first order condition proposed by King, Plosser and Rebelo (1988). By using the Baxter and King band pass filter to extract the cyclical component, we find that the volatility of investment and government spending are higher than that of aggregate output. The striking feature in developing countries, including Thailand, is that consumption is more volatile than output. These variables in general are pro-cyclical and highly persistent. Net exports are highly volatile and counter-cyclical. The business cycle features of developing countries tend to be more volatile than those of developed countries. The output fluctuations of the Asian countries are positively correlated.

A real business cycle model is constructed and it includes permanent, pure and realistic shocks to technology and government spending. The technology shock of Thai economy during 1993-2006 is significantly persistent. The government spending shock cannot generate the real business cycle properties. The multiple shocks and the shocks off steady state are introduced to alternatively study the effect of fiscal policy by replicating the 1997 Asian crisis. The government spending seems to have a limited applicability for this model. The model fails to explain a high volatility of consumption. The difference between theory and data is also present in the volatility and contemporaneous correlation with output of labour, wages and interest rate.

A one good two country international real business cycle model with complete market in line with Baxter and Crucini (1995) is built to explain the international facts of Thailand. The relationship between the Thai real aggregate fluctuations and those of the US from 1993-2006 is investigated. Technology spillovers significantly transfers from the US to Thailand, not another way around. The contemporaneous correlation of technology innovation of Thailand and the US is negative. The impulse response is done for permanent and realistic shocks of technology, government spending and taxation. The shocks off the steady state and the multiple shocks are also explored in the context of the open economy model. It is obvious from this analysis that large countries do not respond to small country shocks. Small countries, particular openness, are dominated by large

country shocks. The responses in Thailand are significant if the shocks are originated in the US. The model requires a high variance of technology innovation to explain the Thai facts. The shock in the US can explain the co-movement in Thailand better than the shock originates in Thailand itself. The model performs poorly to match the data in term of international co-movement and predicts that the cross correlation of consumption is higher than that of output.

TABLE OF CONTENTS

TABLE OF CONTENTS	1
LIST OF TABLES	6
LIST OF FIGURES	9

Chapter 1 : <i>Introduction</i>	16
--	-----------

Chapter 2 : *Literature Review*

2.1 Introduction	28
2.2 Business cycle facts	29
2.3 The neoclassical growth model	31
2.3.1 The basic neoclassical growth model	32
2.3.2 The optimal growth model	33
2.4 The real business cycle model (RBC Model)	35
2.4.1 The original model	35
2.4.2 A simple real business cycle model framework	37
2.4.3 The extending the basic model.....	39
2.4.3.1 Labour supply	39
2.4.3.2 Fiscal policy	41
2.4.3.3 Money	42
2.4.3.4 Multiple sector model	43
2.5 The international real business cycle model (IRBC Model)	44
The extension of the IRBC model	48
2.6 The business cycle models for a specific country	52
2.6.1 The RBC model for developed countries	52
2.6.2 The RBC model for developing countries	53
2.6.3 The IRBC model for developed countries	54
2.6.4 The IRBC model for developing countries	57
2.7 Conclusion	60

Chapter 3 : *Filter and Thai Facts*

3.1 Introduction	62
3.2 Filters	62
3.2.1 Time and frequency domains	63

3.2.2	The auto-covariance generating function and spectrum density	64
3.2.3	The first difference filter	66
3.2.4	The moving average filter	67
3.2.5	The Hodrick-Prescott filter	68
3.2.6	The Band-Pass filter	70
3.2.7	A gain in the filter	75
3.3	The business cycle regularities of Thai, Asian and developed countries ...	77
3.3.1	Decomposition of trend and business cycles of Thailand	78
3.3.2	Business cycle of Asian and developed countries	84
3.2.3	International business cycles	87
3.4	Conclusions	92

Chapter 4 : *Methodology*

4.1	Introduction	93
4.2	Overview of methodology	94
4.3	Dynamic programming	96
4.4	Computational method	98
4.4.1	General procedures for solving dynamic programming	99
4.4.2	Making the model stationary	104
4.4.3	Log-linearization of the model economy	106
4.5	Solving linear rational expectation models	110
4.5.1	The general representation	111
4.5.2	Blanchard and Khan (1980) method	112
4.5.3	King and Watson (1998) method	114
4.6	Solving for decision rules of real business cycle model	115
4.6.1	The method of undetermined coefficients	115
4.6.2	The method of finding a linear approximation to the first order conditions	118
4.7	Simulation and the shocks off the steady state	125
4.8	Conclusion	127

Chapter 5 : *A Closed Economy Model*

5.1	Introduction	128
5.2	Competitive equilibrium	129

5.2.1	Households	129
5.2.2	Firms	131
5.2.3	Government.....	132
5.2.4	Market clearing condition	133
5.2.5	Equilibrium	133
5.3	The social planner problem	134
5.4	Productivity disturbances	135
5.5	The steady state and the stationary economy	136
5.5.1	Necessary conditions	138
5.5.2	Steady state ratio	140
5.5.3	The local elasticities	140
5.5.4	Log-linearization around the steady state	142
5.6	Model calibration for Thai economy	143
5.6.1	Technology parameters	145
5.6.2	Preference parameters	146
5.6.3	The shock parameters	146
5.7	Model simulation for Thai economy	147
5.7.1	Capital stock shocks	148
5.7.2	Technology shocks	150
5.7.3	Government spending shocks	152
5.7.4	The multiple shocks	154
5.7.5	The shocks off the steady state	155
5.8	Model simulation with different parameter values	158
5.8.1	Pure temporary productivity shock	158
5.8.2	Permanent productivity shock.....	161
5.8.3	Pure temporary government spending shocks	163
5.8.4	High and low substitution for Thai economy	165
5.9	Model performance	167
5.9.1	Model performance with difference frequency	167
5.9.2	Model performance with difference parameters	179
5.10	Conclusion	183

Chapter 6 : *The open economy Model: Specification and calibration*

6.1	Introduction	186
6.2	Business cycles for OECD countries	187

6.3	International business cycles for the USA, Japan and Thailand	193
6.4	The basic IRBC model	199
6.4.1	Households	199
6.4.2	Technology	200
6.4.3	Government	201
6.4.4	Market structure	202
6.4.5	The forcing process specification	203
6.5	The equilibrium and solution algorithm	204
6.6	The model calibration	208
6.6.1	Preferences	209
6.6.2	Technology	209
6.6.3	Measuring productivity	212
6.6.4	Measuring government spending	216
6.7	Conclusion	217

Chapter 7 : *The open economy Model: Simulation and Performance*

7.1	Introduction	220
7.2	Model simulation	221
7.2.1	A permanent shock	222
7.2.1.1	Simulation US-Thailand: A shock in productivity	222
7.2.1.2	Simulation US-Thailand: A government spending shock	226
7.2.1.3	Simulation US-Thailand: A shock in taxes	229
7.2.2	A temporary shock	233
	Simulation US-Thailand: A shock in productivity	233
7.3	The shocks off the steady state in the open economy model	239
7.3.1	Simulation US-Thailand: The permanent shocks off the steady state	239
7.3.2	Simulation US-Thailand: The permanent shocks off the steady state and fiscal policies	242
7.3.3	Simulation US-Thailand: The temporary shocks off the steady state	245
7.3.4	Simulation US-Thailand: The temporary shocks off the steady state and fiscal policies	247
7.4	The multiple shocks in the open economy model.....	249
	Simulation US-Thailand: The multiple shocks in the open economy model	250

7.5	Model performance	254
7.5.1	The permanent technology shock; under scenario 1	255
7.5.2	The realistic technology shock without technology spillovers; under scenario 2.....	258
7.5.3	The realistic technology shock with altering variance of technology innovation; under scenario 3	259
7.5.4	The effect of the technology innovation correlation; under scenario 4	261
7.5.5	The effect of fiscal shocks; under scenario 4	262
7.5.6	The effect of the capital adjustment cost; under scenario 4	265
7.6	Conclusion	267
 Chapter 8 : <i>Filter Comparison</i>		
8.1	Introduction	270
8.2	The effects of alternative filters in Thai data	270
8.2.1	Inspecting the results for Thai GDP.....	272
8.2.2	Inspecting the results for Thai consumption.....	275
8.2.3	Inspecting the results for Thai investment.....	277
8.3	Moments analysis	280
8.3.1	Moments computed from The Thai data	280
8.3.2	Model moments.....	284
8.3.3	Moments comparison	286
8.4	Conclusion	288
 Chapter 9 : <i>Conclusion</i>		
 <i>Appendices</i>		
	Section A: Technical appendix	300
	Section B: Programming codes	314
	<i>Bibliography</i>.....	331

LIST OF TABLES

Table 1: The seven year duration of Thailand business cycles	18
Table 3.1: The stylized facts of Thai, Asian and developed countries	
Table 3.1 Panel A: Volatility.....	81
Table 3.1 Panel B: Volatility with respect to output	82
Table 3.1 Panel C: Co-movement with respect to output	83
Table 3.2: Cross-country co-movement with the same Thai variables	88
Table 3.3: Cross-country correlation of Thai, Asian and developed countries	
Table 3.3 Panel A: Cross-country correlation during 1965-2004	89
Table 3.3 Panel B: Cross-country correlation during 1965-1990	90
Table 3.3 Panel C: Cross-country correlation during 1991-2004	91
Table 5.1: The list of parameters	145
Table 5.2: Calibrated parameters for Thailand 1976-2005 quarterly	148
Table 5.3: Cyclical behaviour of the band pass filtered model economy from 1976-2005 annually	
Table 5.3 Panel A: Volatility and persistence	168
Table 5.3 Panel B: Co-movement	169
Table 5.4: Cyclical behaviour of the HP filtered model economy from 1976-2005 annually	
Table 5.4 Panel A: Volatility and persistence	169
Table 5.4 Panel B: Co-movement	170
Table 5.5: The band-pass filtered cyclical statistics for Thai key variables from 1976- 2005 annually	
Table 5.5 Panel A: Volatility and persistence	171
Table 5.5 Panel B: Co-movement	172
Table 5.6: The HP filtered cyclical statistics for Thai key variables from 1976-2005 annually	
Table 5.6 Panel A: Volatility and persistence	172
Table 5.6 Panel B: Co-movement	173
Table 5.7: Calibrated parameters for Thailand 1993q1-2006q3	174
Table 5.8: The band pass filtered cyclical statistics of the model economy from 1993q1- 2006q3	
Table 5.8 Panel A: Volatility and persistence	174

Table 5.8 Panel B: Co-movement	175
Table 5.9: The HP filtered cyclical statistics of the model economy from 1993q1- 2006q3	
Table 5.9 Panel A: Volatility and persistence	175
Table 5.9 Panel B: Co-movement	176
Table 5.10: The band-pass filtered cyclical statistics of the Thai key variables from 1993q1- 2006q3	
Table 5.10 Panel A: Volatility and persistence	176
Table 5.10 Panel B: Co-movement	177
Table 5.11: The HP filtered cyclical statistics of the Thai key variables from 1993q1- 2006q3	
Table 5.11 Panel A: Volatility and persistence	178
Table 5.11 Panel B: Co-movement	178
Table 5.12: The band-pass filtered cyclical statistics of the Thai key variables 1993q1- 2006q3	179
Table 5.13: The model with different values of international substitution	180
Table 5.14: The model with different values of inter-temporal substitution in fixed labour	182
Table 6.1: Business cycle properties of 7 OECD countries	
Table 6.1 A: Volatility	188
Table 6.1 B: Persistence	189
Table 6.1 C: Co-movement	190
Table 6.1 D: International co-movement	190
Table 6.2 The business cycle features of US, Japan and Thailand 1993q1-2006q4	
Table 6.2 A: Volatility	194
Table 6.2 B: Persistence	194
Table 6.2 C: Co-movement	195
Table 6.2 D: International co-movement	196
Table 6.3 Augmented Dickey-Fuller unit root test on total factor productivity of Thailand and the USA	215
Table 7.1: The model economy with permanent technology shock for Thailand and the USA under scenario 1	
Table 7.1 A: The volatility, persistence and co-movement	256
Table 7.1 B: The international co-movement	257

Table 7.2: The model economy with realistic shock without technology spillovers for Thailand and US under scenario 2	
Table 7.2 A: The volatility, persistence and co-movement.....	258
Table 7.2 B: The international co-movement	259
Table 7.3: The model economy with realistic technology shock with variance 0.5% and 1% for Thailand and the USA under scenario 3	
Table 7.3 A: The volatility, persistence and co-movement	260
Table 7.3 B: The international co-movement.....	261
Table 7.4: The model economy with varying correlation of technology innovation for Thailand and US	262
Table 7.5: The response of Thai's variables for a 1% increase in government spending	263
Table 7.6: The response of Thai's variables for a 1% cut in taxes	264
Table 7.7: The behaviour of Thai key variables with the alternate values of the marginal adjustment cost function for Thailand and US	265
Table 7.7 A: The volatility, persistence and co-movement	265
Table 7.7 B: The international co-movement	266
Table 8.1: The effect of filtering on moments from the Thai data; volatility	281
Table 8.2: The effect of filtering on moments from the Thai data; persistence	282
Table 8.3: The effect of filtering on moments from the Thai data; co-movement with respect to output	283
Table 8.4: The effect of filtering on moments from the model; volatility	284
Table 8.5: The effect of filtering on moments from the model; persistence	285
Table 8.6: The effect of filtering on moments from the model; co-movement with respect to output	285
Table 8.7: The deviation of the model's predictions from the fact; volatility	286
Table 8.8: The deviation of the model's predictions from the fact; Persistence	287
Table 8.9: The deviation of the model's predictions from the fact; Co-movement	287

LIST OF FIGURES

Figure1: The business cycle in Thailand 1965-2007	18
Figure 3.1: Transfer function of different filters	
Figure 3.1 Panel A: First difference filter	74
Figure 3.1 Panel B: Moving average filter at point 3	74
Figure 3.1 Panel C: HP filter lamda = 14400	75
Figure 3.1 Panel D: HP filter lamda = 1600	75
Figure 3.1 Panel E: HP filter lamda = 100	75
Figure 3.1 Panel F: Band Pass filter	75
Figure 3.1: A gain in different filters	
Figure 3.2 Panel A: First difference filter	76
Figure 3.2 Panel B: Moving average filter at point 3	76
Figure 3.2 Panel C: HP filter lamda = 14400	76
Figure 3.2 Panel D: HP filter lamda = 1600	76
Figure 3.2 Panel E: HP filter lamda = 100	76
Figure 3.2 Panel F: Band Pass filter	76
Figure 3.3 Percentage deviation from trend of Thai real variables	
Figure 3.3 Panel A: GDP	78
Figure 3.3 Panel B: Consumption	79
Figure 3.3 Panel C: Investment	79
Figure 3.3 Panel D: Government spending	79
Figure 3.3 Panel E: Net export	79
Figure 3.4 Log of Thai real variables and their growth component	
Figure 3.4 Panel A: GDP	80
Figure 3.4 Panel B: Consumption	80
Figure 3.4 Panel C: Investment	80
Figure 3.4 Panel D: Government spending	80
Figure 3.4 Panel E: Net export	80
Figure 4.1: The road map of methodology	95
Figure 4.2: The logic of the solution	126
Figure 5.1: Capital stock shocks	
Figure 5.1 Panel A: The response of capital, technology and labour	149
Figure 5.1 Panel B: The response of output, consumption and investment	149

Figure 5.1 Panel C: The response of real wage and rental price of capital	150
Figure 5.2: Technology shocks	
Figure 5.2 Panel A: The response of capital, total factor productivity and labour	151
Figure 5.2 Panel B: The response of output, consumption and investment	151
Figure 5.2 Panel C: The response of real wage and rental price of capital	152
Figure 5.3: Government spending shocks	
Figure 5.3 Panel A: The response of capital, total factor productivity and labour	153
Figure 5.3 Panel B: The response of output, consumption and investment	153
Figure 5.3 Panel C: The response of real wage and rental price of capital	154
Figure 5.4: The shocks off the steady state	
Figure 5.4 Panel A: A 1% negative technology shock followed in the following period by 1% positive government spending shock	156
Figure 5.4 Panel B: A 1% negative government spending shock followed in the following period by 1% positive technology shock	156
Figure 5.5: The shocks off the steady state (replicating the 1997 crisis)	
Figure 5.5 Panel A: A 5.8% negative technology shock followed in the following period by 300% positive government spending shock	156
Figure 5.5 Panel B: A 5.8% techonology shock following by a series of 100% positive government spending shock	156
Figure 5.5 Panel C: A 5.8% negative government spending shock followed in the following period by 30% positive capital shock	157
Figure 5.6: A pure temporary productivity shock	
Figure 5.6 Panel A: The response of capital, total factor productivity, output and consumption	159
Figure 5.6 Panel B: The response of investment, labour, real wage and rental price of capital	160
Figure 5.7: A permanent productivity shock	
Figure 5.7 Panel A: The response of capital, total factor productivity, output and consumption	161
Figure 5.7 Panel B: The response of investment, labour, real wage and rental price of capital	162
Figure 5.8: A pure temporary government spending shock	
Figure 5.8 Panel A: The response of capital, total factor productivity, output and consumption	164

Figure 5.8 Panel B: The response of investment, labour, real wage and rental price of capital	165
Figure 5.9: The low and high substitution economy	
Figure 5.9 Panel A: The low substitution economy	166
Figure 5.9 Panel B: The high substitution economy	166
Figure 6.1: The cyclical component of real output for the USA, Japan, Canada, Australia, UK, EU and Korea	192
Figure 6.2: The cyclical component of main macro variables for the USA, Japan and Thailand	197
Figure 6.3: The total factor productivity for US, Japan and Thailand 1993-2006	
Figure 6.3 A: US	214
Figure 6.3 B: Thailand	214
Figure 6.3 B: Japan	214
Figure 7.1: A 1 % increase in Thailand productivity	
Figure 7.1 A: The response of variables in Thailand	222
Figure 7.1 B: The response of variables in the USA	223
Figure 7.2: A 1 % increase in US productivity	
Figure 7.2 A: The response of variables in the USA	224
Figure 7.2 B: The response of variables in Thailand	225
Figure 7.3: A 1% increase in US government spending (closed economy)	226
Figure 7.4: A 1% increase in US government spending	
Figure 7.4 A: The response of variables in USA.....	227
Figure 7.4 B: The response of marginal product of capital for the USA and Thailand	228
Figure 7.4 C: The response of variables in Thailand.....	228
Figure 7.5: A 1% decline in US tax rate (closed economy)	230
Figure 7.6: A 1% permanent tax cut in Thailand	
Figure 7.6 A: The response of variables in Thailand	230
Figure 7.6 B: The response of variables in the USA.....	231
Figure 7.7: A 1% permanent tax cut in US	
Figure 7.7 A: The response of variables in the USA	232
Figure 7.7 B: The response of variables in Thailand	232
Figure 7.8: A temporary shock in productivity with technology spillovers in Thailand	
Figure 7.8 A: The response of variables in Thailand	234

Figure 7.8 B: The response of variables in the USA	235
Figure 7.9: A temporary shock in productivity with technology spillovers in the USA	
Figure 7.9 A: The response of variables in the USA	235
Figure 7.9 B: The response of variables in Thailand	236
Figure 7.10: A temporary shock in productivity with several values of technology spillovers in the USA	
Figure 7.10 A: A temporary shock in productivity in the USA with technology spillovers 0.20	237
Figure 7.10 B: A temporary shock in productivity in the USA with technology spillovers 0.15	237
Figure 7.10 C: A temporary shock in productivity in the USA with technology spillovers 0.10	237
Figure 7.10 D: A temporary shock in productivity in the USA with technology spillovers 0.05	238
Figure 7.10 E: A temporary shock in productivity in the USA without technology spillovers	238
Figure 7.11: A 1% increase in a permanent technology shock after another 1% increase in a permanent technology shock in a large open economy (the USA)	
Figure 7.11 A: Output	240
Figure 7.11 B: Consumption	240
Figure 7.11 C: Investment	240
Figure 7.12: A 1% increase in a permanent technology shock after another 1% increase in a permanent technology shock in a small open economy (Thailand)	
Figure 7.12 A: Output	241
Figure 7.12 B: Consumption	241
Figure 7.12 C: Investment	242
Figure 7.13: A 1% decrease in a permanent technology shock after another 22% increase in a permanent government spending shock in a large open economy (the USA)	243
Figure 7.14: A 1% decrease in a permanent technology shock after another 1.9% permanent cut in tax in a large open economy (the USA)	244

Figure 7.15: A 1% decrease in a permanent technology shock after another 2.3% permanent cut in tax in a small open economy (Thailand)	245
Figure 7.16: A 1% increase in temporary technology shock followed in the following period by 1% increase in temporary technology shock in a large open economy (the USA)	246
Figure 7.17: A 1% increase in temporary technology shock followed in the following period by 1% increase in temporary technology shock in a small open economy (Thailand)	247
Figure 7.18: A 1% decrease in temporary technology shock followed in the following period by 18% increase in permanent government spending shock in a large open economy (the USA)	248
Figure 7.19: A 1% decrease in temporary technology shock followed in the following period by 1.2% permanent cut in tax in a large open economy (the USA)	248
Figure 7.20: A 1% decrease in temporary technology shock followed in the following period by 1.6% permanent cut in tax in a small open economy (Thailand)	249
Figure 7.21: A 1% increase in permanent technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a large open economy (the USA)	250
Figure 7.22: A 1% decrease in permanent technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a large open economy (the USA)	251
Figure 7.23: A 1% decrease in permanent technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a small open economy (Thailand)	251
Figure 7.24: A 1% decrease in permanent technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a small open economy (Thailand)	252
Figure 7.25: A 1% increase in temporary technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a large open economy (the USA)	253
Figure 7.26: A 1% increase in temporary technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a small open economy	

(Thailand)	253
Figure 8.1 The effects of alternative filters on Thai GDP	
Figure 8.1 A: Top panel: Real GDP and its trend	
Bottom panel: The first difference filter	
and the Hodrick-Prescott filter	272
Figure 8.1 B: Top panel: The moving average and the Hodrick-Prescott filter	
Bottom panel: The band pass filter by Baxter-King	
and the Hodrick-Prescott filter	273
Figure 8.1 C: Top panel: The band pass filter by Christiano-Fitzgerald	
and the Hodrick-Prescott filter	
Bottom panel: The band pass filter by Baxter-King	
, the band pass filter by Christiano-Fitzgerald	
and the Hodrick-Prescott filter	274
Figure 8.2 The effects of alternative filters on Thai consumption	
Figure 8.2 A: Top panel: Real consumption and its trend	
Bottom panel: The first difference filter	
and the Hodrick-Prescott filter	275
Figure 8.2 B: Top panel: The moving average and the Hodrick-Prescott filter	
Bottom panel: The band pass filter by Baxter-King	
and the Hodrick-Prescott filter	276
Figure 8.2 C: Top panel: The band pass filter by Christiano-Fitzgerald	
and the Hodrick-Prescott filter	
Bottom panel: The band pass filter by Baxter-King	
, the band pass filter by Christiano-Fitzgerald	
and the Hodrick-Prescott filter	277
Figure 8.3 The effects of alternative filters on Thai investment	
Figure 8.3 A: Top panel: Real investment and its trend	
Bottom panel: The first difference filter	
and the Hodrick-Prescott filter	278
Figure 8.3 B: Top panel: The moving average and the Hodrick-Prescott filter	
Bottom panel: The band pass filter by Baxter-King	
and the Hodrick-Prescott filter	279
Figure 8.3 C: Top panel: The band pass filter by Christiano-Fitzgerald	
and the Hodrick-Prescott filter	

Bottom panel: The band pass filter by Baxter-King	
, the band pass filter by Christiano-Fitzgerald	
and the Hodrick-Prescott filter	279

Chapter 1

Introduction

In 1973, the real growth rate of Thailand was 10.24% per year and it declined to 4.47% the year after. The economy started to pick up again in 1975 at 4.97% growth. In the late 1970s, Thailand for the first time since 1965 had achieved a consistent growth for four years in a row which reached the highest peak in 1978 with 10.30% GDP growth. The Thai economy came into the period of relative economic decline with the average real growth rate of GDP 5.40% per year from 1979 to 1985. However, the Thai economy started to recovery after 1985 and from 1986 to 1995, the Thai economy grew rapidly on average at a rate of 9.48%. Growth peaked in 1988 with 13.29% of the growth. Under these circumstances Thailand was called one of the Asian tigers and with rising economic prosperity there was an increase in consumption and investment. The investment and output ratio started to increase in 1987. While the average investment-output ratio during the last four decades was 0.29, it rose to 0.37 during the economic boom. Thai people at that time consumed a greater variety of goods and services and a greater quantity than previous generations. The experience of Thailand during this period was quite remarkable. The booming stock market and the capital inflow seemed to ensure this optimistic perspective would continue. The Thai stock market hit an all time high in 1993 with 8,984 millions Baht in trading volumes and the market index at 1,682.85 points, compared to 893.42 points in 1992. The average of net foreign direct investment during the 1990s was, US\$ 2.74 billion dollars per annum, 5.68 times higher than that of the 1980s. Not only did output rise, but also employment rose too. During these remarkable years, the unemployment rate was low and from 1989 to 1996 averaged only 1.66% of the labour force, or about 537,000 workers. Each 1% point rise in the rate of unemployment could be approximately interpreted to 325,000 additional people out of work, during the past two decades. The growth pushed the unemployment rate down but at the same time it also resulted in inflation. The average inflation rate during 1989 to 1996 was 5.15% per year.

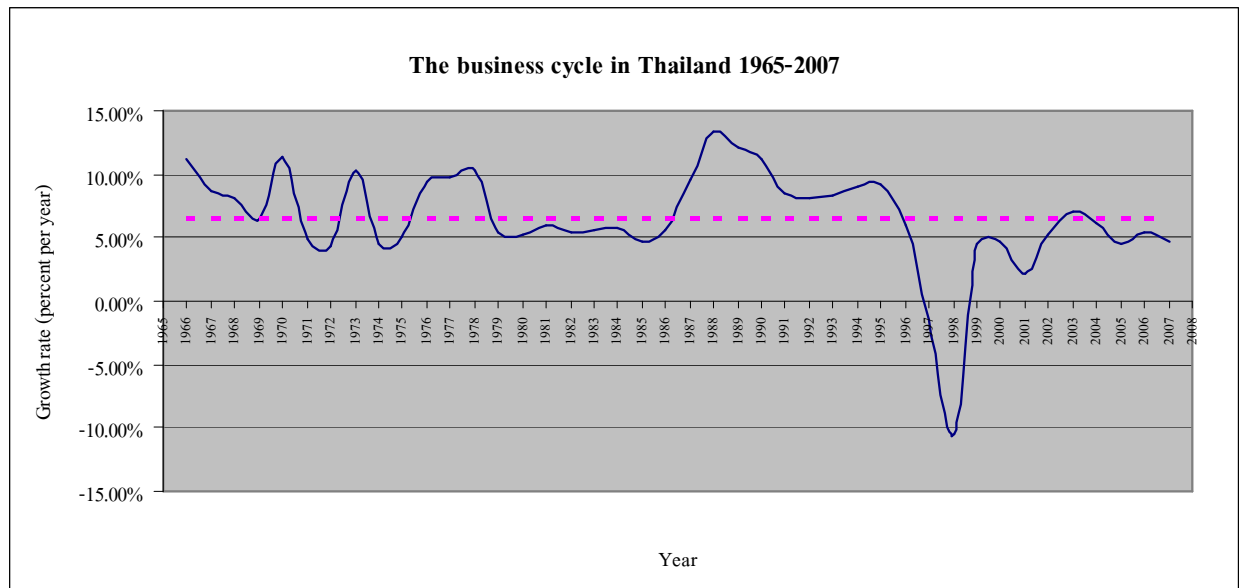
Then the economy began to decline again in 1996 and it became much worse in 1997. The most prolonged departure from zero growth occurred during the Asian crisis of

1997. The sustained period of economic growth ended abruptly on July 2, 1997 where the Thai Baht had to be devalued.¹ In the following days the stock market became to plummet and the foreign capital continued to flow out. The Thai stock market index dropped to 372.69 points in 1997. Investment growth fell by 20.54% in 1997 and hit the lowest point in Thailand's economic history at 44.32% in 1998. Consumption exhibited an identical pattern and fell by 11.51% in 1998. Unemployment was on the rise while output was falling. The real growth of Thailand reached the lowest point in Thai history in 1998 with -10.51%. The number of unemployed workers rose from 292,500 in 1997 to 1,137,900 in 1998. Many banks were forced to close. Millions of people remained out of work until 2002. The unemployment rate actually started to decline in 2003 with just 760,600 unemployed workers. The inflation rate on average was low at 1.03% per year from 1999 to 2002. In 1999 the rate of inflation reached its lowest level at 0.30%. Before 1997, Thailand experienced a trade deficit on average. The trade deficit in 1996 at 224.80 billions of US dollars became a surplus in 1997 and continued in surplus to 454 billions of US dollars in 1998. This was because the Baht had been devalued. Then the economy once more grew. In early 2000s, the average Thai growth was a relatively modest 5.02% per year.

The Thai economy oscillated between periods of high and low activity or the period of economic expansions and contractions from time to time but of varying frequency, intensity and duration. Figure 1 exhibits the basic features of the Thai business cycle. From 1965 to today, real GDP of Thailand has increased 13.97 times. Thai people are now producing about 14 times as many goods and services as Thai people did back in 1965.

¹ The crisis in 1997 came about by two important facts, one is the poor economic fundamentals in Thailand and another is the institutional problems in the financial sector. Thailand had fixed its currency to the US dollars since 1985. After the financial liberalisation in 1992 which allowed capital to freely flows in and out, Thailand experienced the massive foreign capital inflows, generating the rapid economic expansion and in turn a high price level. With the appreciation in US dollars during 1995-1996, the consequences were Thailand lost its export competitiveness. The external debts had increased substantially during this periods because financial institutions borrowed cheap dollars and converted to the Thai Baht. The excessive investments also placed in the unproductive projects. The Thai Baht was dramatically overvalued. In 1996, Thai exports had been dropped for the first time since 1988 because such a strong currency made their goods less competitive on world markets. See also Kaminsky and Reinhart (1998, 1996) Krugman (1997,1998), Cole and Kehoe (1996), Obstfeld (1995, 1996), Obstfeld and Rogoff (1995).

Figure 1 The business cycle in Thailand 1965-2007



From 1965 to 2007, real GDP increased by an average 6.56% per year. However, the annual growth rates have fluctuated widely from the average : Source World Bank World Development Indicators.

The growth has come in relatively small annual increments. On average, real GDP growth has grown by only 6.56% per year, indicated by the dotted line in figure 1. In just 40 years, the 6.56% annual growth led to the economy being 14 times its previous size. However, the growth path of the Thai economy is not as smooth. Even though the long term or average growth rate of the Thai economy is 6.56% per year, a year to year real GDP growth has shown a significant variation. Apparently, it failed to attain 6.56% growth every year. In some years the Thai economy was below the zero growth line, indicating that output decreased from one year to the next.

Nonetheless, figure 1 suggests the evidence that consistent growth, full employment and a stable price level are difficult to achieve. By comparison, the period 1973-2007, the alternating economic expansion and contraction was approximately 7 years. The real growth as in table 1 during 1966-1972 was virtually identical to that during 1973-1979 at approximately 7.80% per year.

Table 1 The seven year duration of Thailand business cycle

		1966- 1972	1973- 1979	1980- 1986	1987- 1993	1994- 2000	2001- 2007
Real growth	change	-6.84%	-4.86%	0.36%	-1.27%	-4.24%	2.60%
	mean	7.86%	7.79%	5.42%	10.15%	3.06%	5.05%

Change is the change over subperiod. It is defined by the difference between the beginning and the end of the real Growth. Mean is the mean annual percentage change.

However, the economy alternated between expansion and contraction 3 times over 1973-2007. After the seven year period of 1973-1979, the economy declined to grow at an average of 5.42% from 1980 to 1986, then grew very fast up to 10.15% on average from 1987-1993. During the economic crisis of 1997, the average growth rate hit the lowest level at 3.06%. The economy began to recover in 1999, but the rate of expansion was slow. The economic growth has continued through 2001-2007 at only 5.05% per year.

The fluctuation of output is one of the major problems in macroeconomics. One of the basic goals of macroeconomics is to understand and explain the business cycle, how stable the economy is and what the causes of the instability are. The business cycle can happen because of shocks to the economy such as with the government spending. An increase in government spending leads to an increase in output. Identically, the situation that causes the firms to produce more and consumers to spend more than usual will create the economic upswing. If the same conditions are in reverse, the economy could enter the contraction periods. Money supply and interest rates can also be considered as a source of the fluctuation. The interest rate is the key factor to specify how much people want to consume and firms want to invest. The political situation is also important to the fluctuation of output, particularly in the developing countries where the politics are really not stable. In addition, the aggregate fluctuations seem not to be completely independent of policy decisions. With more effective policies, the periods of economic expansions are likely to be more frequent and longer. In contrast, with poor policies, the recession could stay longer and be more severe. Therefore, when the shocks occur, the other economic variables fluctuate accordingly. The national variables such as consumption, investment, government spending, the level of taxes, national debts, budget deficits, trade, even the political situation and policy decisions are all suggested as responding to the business cycle.

The recession by the Keynesian explanation is the failure to clear the markets because the prices and wages are unable to adjust instantly. The main argument of Keynesian theory is that prices and wages are sticky. The deficiency of spending would tend to push the economy away from the market equilibrium. But the wages and prices are so sticky that they cannot adjust accordingly. As a result, the market is not always perfect. In the Keynesian point of view, the government spending and money supply shocks can generate the fluctuation in output when prices and wages are sticky. Keynes (1936)

suggested that after the great depression in the USA the government should intervene to limit the instability of the economy. When the economy slows down, the government should buy more goods and services, employ more workers, transfer more income to people and put more money in the economy. When the economy is growing too fast, the government should slow it down by cutting spending and raising taxes. Arguably too this is what we are seeing now in 2009 in response to the credit crisis.

Nonetheless, there is the real business cycle theory which believes that the economic fluctuations are caused by the real supply shocks, mainly productivity shocks. The real business cycle theory is developed under the new classical economic school of thoughts. The productivity shock is measured by the Solow residual or the change in output that cannot be explained by changes in labour and capital. In the real business cycle theory, prices adjust to clear the market instantly. Therefore, the market always continuously clears both in long run and short run. If firms are unable to sell their products at current prices, they could either reduce their rate of output or cut their price. Thus, the quantity of demand is increased. This price adjustment process is able to be implemented in the short run. Another crucial hypothesis confirming the markets always clear is the inter-temporal labour substitution. Indeed, agents would prefer working during the higher wage periods to during the lower wage periods. Similarly, they could take their leisure during the lower wage periods and shift their labours to the higher pay periods. Therefore, in this theory, the voluntary inter-temporal substitution between leisure and hour worked causes the variations in unemployment. The money market in the real business cycle theory can be ignored in the sense that the change in the money market will influence only nominal variables not real variables. The variation in the money stock will alter only price, it will have no effect on the real interest rate and in turn investment. The notion of rational expectations is also incorporated as an essential feature in the real business cycle theory. Households make their decisions rationally; thereby, the rational behaviour of agents also confirms the neutrality of the money. When output rises because of a productivity shock, the quantity of real money demand rises. The central bank may respond by raising the real money supply to accommodate the greater demand.² The shocks in money stock are not a driving force for generating

² The money balance in real business cycle model refers to the real money balance not nominal money balance. The real business cycle theory explains the economic fluctuations by real changes in the economy such as changes in technology and without any role for nominal variables such as the money supply.

the fluctuations in output. The money stock is endogenous in this theory. When the economy faces a positive technology shock, the real interest rate and real wages are both increased. It is a good time for people to supply their labour force and it is also a good time for firms to expand their businesses. The economic expansion is the result of the positive technology shock. In contrast, if the economy experiences a negative productivity shock, the economic responses are reversed. Therefore, in the business cycle theory point of view, the economic fluctuations mainly stem from the rate of real shocks or productivity shocks.

The Keynesian economists have claimed that there are many weak points in each main argument in the real business cycle theory such as the inter-temporal substitution of hours worked, the neutral of money and the voluntary nature of employment.³ They claim that the real business cycle theory is unable to capture the reality well. It is hard to believe that the unobserved technology shock is the core idea of the whole explanation. It is also difficult to be convinced that during recessions the high unemployment is largely voluntary. The labour supply is not very sensitive to the inter-temporal real wage. The period of high output and low unemployment are almost always related to the increase in money growth and inflation. Mankiw (2006), one of the current proponents of Keynesian economics, argued that the new classical theory was not ready for policy makers. He also claims that currently many economists coming from the new classical tradition are willing to concede to the idea of sticky prices and as long as this assumption can be blended in a suitably rigorous model with rational expectation and forward looking.⁴ Nonetheless, Keynesian economists are unable to end up convincingly with the conclusion and having a massively strong support that the real business cycle theory is so entirely wrong that the first year undergraduate economic students should not study it. Further, the new classical economists can exhibit that their theory is able to predict the

³ The Keynesian theory is also called the demand side theory where as the New classical theory is called the supply side theory. However, the debate and the comparison of the Keynesian and New classical economic school of thoughts to explain the aggregate fluctuation is far beyond the goal of this thesis. The famous advocates of the New classical economist are, for example, Friedman, Lucas, Sargent, Kydland, Prescott and etc. The notable proponents of the Keynesian are, such as, Samuelson, Modigliani, Tobin, Barro, Mankiw and etc.

⁴ In addition, while the New Classical theory suggests that the business cycle results from the intertemporal substitution, Campbell and Mankiw (1989) claim that the intertemporal substitution elasticity is close to zero. Plosser (1989) argue that the important drawback of the Keynesian theory in explaining business cycles is the absence of a consistent foundation based on microeconomic framework. The conflict between the Neoclassical and the Keynesian theories of business cycle can also be found in Mankiw (1985 and 1989).

real world quite remarkably well in several ways. Many renowned economists nowadays do not entirely take on one side but instead they study the combination of both theories. An increasing amount of research incorporates insights from both school of thought is called the new neoclassical synthesis, to advance our study of aggregate fluctuations.⁵

Even though there are many critics regarding the real business cycle theory, the theory introduces a solid framework to study the fluctuations of the economy. The real business cycle theory in which agents are rationally forward looking is another vital way to understand and explain the business cycle features. This thesis intends to present the theory, the real business cycle theory (RBC theory), regarding the causes and the nature of the aggregate fluctuation. Most of the real business cycle model primarily focuses on the business cycle frequencies of developed countries. However, the application of the real business cycle model for developing countries still remains largely to be explored. It is essential to explain the business cycle of the emerging countries as well. To be concrete and to see how well the theory can explain the business cycle properties, the focus for this study is on the Thai economy with the real business cycle theory.

This thesis has been written to achieve two main objectives. One is to study the characteristics of the business cycle of Thailand and another is to explain them by applying the real business cycle theory. The literature review is in the chapter 2. Chapter 2 starts with a review of the traditional measure of the business cycle by Burn and Mitchell (1946), followed by the basic business cycle facts for developed and developing economies. Since the business cycle model is further developed from the neoclassical growth model, the growth models are also discussed here. The original real business cycle model by Kydland and Prescott (1982) is revisited. This model serves as the foundation of many real business cycle models. The simple model and the extending models are introduced including to the international real business cycle model (IRBC model). This chapter ends up with the literature concerning the application of the RBC and IRBC model for specific countries.

⁵ As in Mankiw (2006), the works of the new neoclassical synthesis attempt to merge the strengths of the competing approaches such as Goodfriend and King (1997). They have been commonly involved in the monetary policy such as Clarida, Gala and Gertler (1999), McCallum and Nelson (1999).

Chapter 3 is important for both the RBC model and the stylized facts. It describes the concept of filters which are necessary to remove the trend and extract the proper cycles from a time series. It is important to filter a time series before evaluating the business cycle properties such as volatility, persistence, co-movement and even the international co-movement. The main focus is on the Hodrick-Prescott filter and Band Pass filter. Then the chapter presents the business cycle regularities of the Thai, Asian and developed countries. Since the real business cycle theory is laid on the foundation of the dynamic general equilibrium framework, the tools to solve the model are necessary to discuss. The dynamic programming approach is illustrated and described in chapter 4 because it is a tool to formulate and solve the general equilibrium framework. One major tool that this thesis uses is the method of solving linear rational expectation models. The method to make the model stationary and to log-linearise the model economy are also depicted.

The thesis then presents through chapters 5 to 7 the application of the real business cycle model and international real business cycle model in the case of the Thai economy. In the other words, the closed economy model is in chapter 5 and the open economy model is described in chapter 6 and chapter 7. In chapter 5, the competitive equilibrium and the social planning problems are investigated for the economy without any distortions. The productivity disturbance which is the heart of the real business cycle theory is described. The crucial procedure, the calibration, is then described. The technology, preferences and technology shock are calibrated for Thai economy. After calibrating the model economy for Thailand, the model is simulated using capital stock, government spending and technology shocks. In addition, this chapter describes the effect of the multiple shocks simultaneously and the response of the aggregate fluctuation multiple shocks one after another. Sensitivity analysis is also done for the closed economy model. The main concern of the macroeconomic theory is how well it can explain the real world. The closed economy model performance is evaluated by comparing the Thai business cycle properties and the model predictions.

Chapter 6 extends from the closed economy model to study the effect of the transmission of the shocks to variables between two countries. This chapter focuses on the model specification and calibration. It begins with the study of the international business cycles for OECD countries. Then the business cycles features of The USA, Japan and Thailand

are described because the first two countries are the main trading partners for Thailand. The one good two country IRBC model in line with Baxter and Crucini (1995) is introduced in this chapter. Then again the model is calibrated for the USA and the Thai economy. Each country has its own set of preferences, technology and productivities. Unlike the technology shock process in the closed economy framework, the technology shock includes technology spillover parameters between the two countries. This potentially the Thai shocks impact in the US economy and vice-versa. Moving on from the model specification and calibration, chapter 7 investigates the model simulation and performance. The model once again is simulated for both temporary and permanent productivity shocks. The fiscal policy shocks, government spending and tax shock, are also illustrated for the open economy model. The shocks off the steady state and the multiple shocks are also explored in the context of the open economy model. This chapter is completed by the sensitivity analysis and model performance.

Chapter 8 reviews the properties of different kind of filters. Thai data and the real business cycle model are used to study the effect of different filters on the business cycle properties. This chapter explores how the five different filters, the first difference filter, the equally weighted moving average, the Hodrick-Prescott filter, the band pass filter by Baxter and King and the band pass filter by Christiano and Fitzgerald could yield different results in terms of business cycle properties. The band pass filters retains the frequency between 6 and 32 quarters which is the frequency range for the purpose of business cycle analysis. The two band pass filters are similar in volatility, persistence and co-movement. The first difference and the moving average filters are not desirable filters. The Hodrick-Prescott filter is approximately a high pass filter, which removes only the slow moving component or the low frequency. For this reason we felt justified in using the standard band pass Baxter and King in our analysis.

Chapter 9 is the conclusion. Perhaps the main contribution lies in building a real business cycle a developing country, namely Thailand. The main findings of this thesis are that there are many striking business cycle fluctuations in the developing countries. In our small sample of countries, in particular Thailand, the volatility of investment and government spending are higher than that of aggregate output. Consumption is more volatile than output. These variables in general are pro-cyclical and are highly persistent. However, net exports are highly volatile and countercyclical. There are some crucial

similarities between the cyclical fluctuation in developing countries and those of the developed country. Co-movement and persistence for consumption, investment and government spending are quite similar. Nonetheless, the business cycle features of the developing countries fluctuate more than those of the developed countries. The business cycle regularities in the Asian countries exhibit a high degree of co-movement. We also find that the business cycle fluctuations of output among developing countries are positively correlated.

The closed economy model was built and simulated with three different kinds of driving parameters, capital, productivity and government spending. Other shocks cannot generate the real business cycle properties except for the productivity shock. The model is also simulated by multiple shocks and also by shocks off the steady state. The idea of the shocks off the steady state is introduced to simulate the impact of a shock after another shock has already occurred. This thesis to an extent replicates the 1997 economic crisis in Thailand and tries to minimize the cycle by using the government spending shock. Compared to the technology shock, the government spending shock has a minimal effect on the aggregation fluctuations. In this model economy, the volatility of output is higher than in the Thai economy, suggesting that the productivity shock can be one reason for the output fluctuations, but not the only reason. Similar to the Thai facts, investment in the model economy fluctuates much more than output. However, the model fails to explain the volatility of consumption relative to output for Thai economy. The model predicts that consumption fluctuates less than output does while the Thai facts show the opposite. The volatility and the contemporaneous correlation with real output of labour, wages and interest rate are substantially lower in the data than in the models.

For the open economy model, special attention is given to the implication of the calibrated parameters for Thailand and the USA by using the one good two country model with complete markets in line with Baxter and Crucini (1995). The calibrated parameters imply that the movements in productivity are highly persistent in both Thailand and the USA. There is evidence of transmission of shocks across countries. The technology spillovers significantly transfers from the USA to Thailand at 26.1%, but only 2.2% from Thailand to the USA. The contemporaneous correlation of technology innovation for Thailand and the USA is -0.186. The total factor productivity of each country shows evidence of a random walk. Therefore it is possible to assume and examine the effects on the Thai business cycle of a permanent shock. Three forcing

processes, technology, government spending and taxes, are examined. The technology shock in Thailand increases investment, consumption and output in Thailand. The investment and output in the USA declines on the impact because capital is shifted to Thailand. The increase in consumption is negligible in both countries because of the small country specific shock location. The responses in Thailand are significant if the shock originates in the USA. Consumption noticeably increases in both countries in this case. Government spending creates a pure wealth effects while tax shocks generates both wealth and substitution effect. The higher the persistence is, the higher the wealth effect is. The shocks off the steady state are also explored in the context of the open economy model. Comparing to the technology and tax shocks, the government spending shock has a small impact to the variables in the model economy. The USA does not respond to a shock generated in Thailand. Nonetheless, Thailand is dominated by a shock originating in the USA. The multiple shocks are once again the combination of several shocks.

Comparing the predictions of the open economy model with the data, the realistic technology shock with technology spillover with high variance (1%) captures the Thai stylized facts better than other specifications. The model requires a high variance of technology innovation (1%) to explain the Thai facts. Even though, the model cannot exactly match the data, the model can capture the business cycle pattern of volatility. It performs quite well in replicating the persistence. The shock in the USA can explain the co-movement in Thailand better than the shock originating in Thailand itself. However, the model fails to explain the counter-cyclical behavior of trade in Thailand and in the USA. In term of international co-movement, the model performs poorly in exactly matching the data. It can only match the sign of some variables. It fails to mimic the negative cross country correlation of consumption of the USA and Thailand. The shock in the USA can capture the negative sign of investment cross correlation, but not the shock originating in Thailand. The model predicts that the cross country correlation of consumption is higher than that of output. However, the facts show the opposite.

The economic implications of the work are also discussed. Thailand is susceptible to US shocks but not vice versa. The economic shocks in a large economy dominate the economic activities in a small economy. However, a large economy does not respond to the shocks in a small country. This suggests that in building a real business cycle model of a small open economy, in particular a developing country, are cognizance has to be

taken of shocks in other countries. But this is not so important in modeling the real business cycle for a large country. This chapter considers the policy options regarding the business cycle as well. Fiscal policy whilst stabilizing in the home country can add to initial productivity shocks impact on other countries. The negative technology shock in the home country causes the movement of capital to foreign country. Therefore, investment and output in foreign countries increase. The fiscal policy, for example an increase in government spending shock, induces people in home country to work more. The output in home country increases. The government spending shock in home country drives up the world interest rate, inducing people in foreign country to work harder. The output in foreign country also increases. The model also suggests that a shock impact can last a long time.

This thesis also describes the future research concerning the business cycle for Thailand. Finally, the technical appendix and the programming code for Thai real business cycle model are illustrated in the appendices.

Chapter 2

Literature Review

2.1 Introduction

One of the most difficult challenges in macroeconomic research is to understand the aggregate fluctuation of economic activities. Business cycle theory attempts to explain the fluctuation of macroeconomic aggregate around its trend. From the neoclassical framework and with dynamic general equilibrium, aggregate economic fluctuations are accounted for by productivity disturbance as a driving process. The “real” in real business cycle theory emphasises the role of real shocks, specifically technology shocks.

The pioneering work by Finn Kydland and Edward Prescott (1982) documents the breakthrough idea that real business cycle can be studied based on the neoclassical growth theory with a dynamic stochastic general equilibrium framework. Literature has focused on documenting the dynamic stochastic general equilibrium models in an attempt to explain economic variation. Much of the business cycle research has been conducted within a closed economy framework. However, the literature has recently built on the original work to construct an open economy model, namely international real business cycle (IRBC) model such as Backus, Kehoe and Kydland (1992, 1995), Blackburn and Ravn (1992), Baxter and Crucini (1995) and Correia, Neves and Rebelo (1995) that explain fluctuations of real macroeconomic aggregates through a cross country environment. In general, the models are evaluated by their performance to replicate the business cycle facts. Therefore, many literatures related to the real business cycle have only investigated business cycle features without imposing any theory.

The remainders of the chapter are as follows. Section 2.2 presents the literature review of business cycle facts for both developed and developing countries. The neoclassical growth model is discussed in section 2.3 because the real business cycle model is laid by the foundation of the growth theory. Section 2.4 provides the review of real business cycle model. Section 2.5 discusses the international real business cycle model. Since the business cycle model has also been applied to a particular country, section 2.6 considers the models for a specific country. Section 2.7 of this chapter concludes.

2.2 Business Cycle Facts

A macroeconomic time series consists of three main components: cycle, trend and seasonality. The main purpose of the business cycle analysis is to measure the fluctuation of the cyclical component of key macroeconomic variables. Following the traditional definition of the business cycle developed by Burn and Mitchell (1946), a business cycle is defined as the cyclical component within six and thirty two quarters. The cyclical component must be extracted from the macroeconomic time series before analyzing its cyclical properties. Hodrick and Prescott (1997) develop the well known HP filter to remove a time trend or a slow moving component. Recently, Baxter and King (1999) and Christiano and Fitzgerald (2003) introduce the band-pass filter to retain the cyclical component in a specific period¹. Having extracted the cyclical component, the business cycle facts are focus on three features, volatility, persistence and correlation.²

Understanding business cycle facts helps the development of the business cycle theory and helps formulate policy. Therefore, some researchers document only facts in detail before imposing theory to capture them later on³. This section highlights the business cycle facts of the key variables for developed and developing countries. Kydland and Prescott (1990) report the US business cycle fact during 1954-1989. Aggregate hours worked are strongly procyclical and its volatility is as much as that of real output. Approximately, two-thirds of the volatility in aggregate hours can be accounted for by the volatility of employment. The capital stock has no contemporaneous correlation with output. In fact the capital stock lags the output cycle by at least a year. Consumption and investment are strongly procyclical. Investment is approximately three times more volatile than output, while consumption volatility is less than output volatility. Government spending exhibits no pattern related to output. Imports and exports are procyclical. Imports show no phase shift with output but exports lags the cycle by from half year to a year. The US price level, either measured by the implicit GNP deflator or

¹ The filter issue is discussed in further details on chapter 3 “Filter and Thai Facts”.

² Volatility is measured by the standard deviation. Persistence is measured by the first-order autocorrelation coefficient. Correlation is measured by correlation coefficient.

³ Generally, most of the business cycle and international business cycle literatures document both facts and model performance.

by the consumer price index, is obviously countercyclical, while the monetary base or M1 generally procyclical during the observation period.⁴

Another important issue is the business cycle in developing countries. The literature mostly conduct for the comparison between the business cycle in developed and developing countries. Kim, Kose and Plummer (2003) examine the extent of similarities and differences of business cycle features for some of the Asian countries and compares with those of the G-7 countries during 1960-1996. They find that for the magnitude and persistence of output fluctuations, the Asian economies are approximately 35% more volatile than the G-7 countries for the entire sample period. Investment is the most volatile variable and consumption is the least volatile in the Asian and G-7 countries. Volatility properties of investment and government spending differ significantly across the sample of countries. In Asia, investment and government expenditure are more volatile than aggregate output in all of the Asian countries during the entire period. Specifically, investment is on average four times more volatile than output. Consumption is on average almost as volatile as output in the most cases. Both investment and consumption are on average contemporaneously positively correlated with output. For trade components in both the Asia and G-7 countries, both export and import are pro-cyclical, but net export is counter-cyclical. For the cross-country correlations of output fluctuations, the results suggest that business cycle fluctuations in the Asian economies exhibit high degree of co-movement. The cross-country correlations of government spending do not exhibit any particular pattern. The cross-country correlations of money stock (M2) vary considerably across country-pairs. Crucini (1997), Neumeyer and Perri (2005) and Calderon and Fuentes (2006) currently confirm the characteristics of the business cycle between developing and developed countries with those of Kim *et al.* (2003).⁵ Baxter and Kouparitsas (2005) recently use

⁴ The evidence of Prescott (1986), Cooley and Prescott (1995) and Backus and Kehoe (1992) are consistent with those of Kydland and Prescott (1990) for the US economy. Stock and Watson (1998) also investigate the US business cycle and show that the US trade balance is countercyclical as the same for many other developed countries. Real wages exhibit no sign of contemporaneous comovement with output. For other developed countries, Schlitzner (1995) shows that the economic fluctuations in Italy conform to the general features of the business cycle in developed countries.

⁵ Neumeyer and Perri (2005) focus on the fluctuation in Argentina and consider the role of the interest rate to the economic variation. Calderon and Fuentes (2006) investigate the business cycle of 7 Latin-American and 7 Asian countries, comparing to 14 developed economies. Akin and Kose (2008) examines the hanging nature of growth spillovers between developed countries and developing countries and find that the impact of the developed economic activity on the developing countries has declined during 1986-2005.

the method of Learner to search for the robust variables in explaining the co-movement. They find that bilateral trade is robust to explain the co-movement. Canova and Gianni (2003) examine sources of cyclical movements in output in G-7 countries and conclude that demand shocks are the dominant source of output variation. Rand and Trap (2002) also investigate the business cycle in developing countries. They find that output is more volatile in developing countries than industrialized countries. Consumption and investment are strongly procyclical. They suggest that supply shocks are often the main source of short-run output fluctuations in developing countries.

2.3 The Neoclassical Growth Model

The neoclassical growth model was originally developed by Solow (1956) and Swan (1964) and it has become the most important foundation of dynamic economic analysis, including a framework of the real business cycle model. The neoclassical growth model proposed by Solow and Swan concerns the properties of the production function, which assumes constant return to scale and diminishing returns to each input, and of a law of motion of capital stock over time.⁶ The production function consists of the evolution of the three inputs, capital, labour and labour augmenting technology. They assume that a constant fraction of output is invested in a new capital every period. Therefore, it refers to the neoclassical model of capital accumulation with exogenous saving rates. Regardless of where the starting point of capital is, the model implies that the economy converges to a steady state, a situation in which the various quantities grow at a constant rate as the rate of technology progresses which is determined outside the model.

Nevertheless, there is no economic decision being made by the work of Solow and Swan. Alternatively Cass (1965) and Koopmans (1965) introduce preferences over consumption for a representative agent and the agents optimally make their decisions.⁷ In this way, the neoclassical growth model provides for an endogenous determination of the saving rates. Specially, Cass and Koopman consider a simple economic environment which is populated by identical agents who have the same preferences and their decisions are

⁶ Solow (1994) considers the neoclassical growth model as the second wave of interest in growth theory in the past 50 years. The first was the work of Harrod and Domar. The third wave began as a reaction of omissions and deficiencies in the neoclassical model.

⁷ The representative agents' decision is a main component in the Ramsey growth model as proposed by Ramsey (1928).

coordinated by a benevolent social planner.⁸ In general, the work of Cass and Koopmans completes the basic neoclassical growth model and is therefore considered as the benchmark model to understand growth and transitional dynamics. However, even though the Cass and Koopmans version of the growth model displays the endogeneity of saving rates, the model in the steady state still requires exogenous technological progress⁹. Recent research extends the basic neoclassical growth model to construct the more complicated models for example the real business cycle model, which is the main focus of this thesis. In this section, the basic neoclassical growth model of capital accumulation is discussed before extending it to allow for real business cycle analysis in the next section.

2.3.1 The basic neoclassical growth model

This section investigates the structure of the basic neoclassical model of capital accumulation in discrete time. The model starts from the neoclassical aggregate production function:

$$Y_t = F(K_t, N_t X_t) \quad (2.1)$$

where Y_t is output, K_t is capital, N_t is labour input and X_t is labour augmenting technology. The production function exhibits constant return to scale and this property is also known as homogeneity of degree one in capital and labour input. It assumes that holding constant levels of N_t and X_t , each additional input of K_t delivers additional positive but diminishing returns to output. The same argument is applied for N_t .¹⁰ The production function also satisfies the Inada condition in which the marginal product of capital approaches infinity as capital goes to 0 and approaches 0 as capital goes to infinity. The same property is assumed for labour.¹¹ For the case of the famous Cobb-Douglas production function, it can be written in the functional form of $Y_t = K_t^{1-\alpha} (N_t X_t)^\alpha$ where α determines labour share.

⁸ The equilibrium of Cass and Koopmans model can be supported by decentralized framework. In the other words, the decentralized allocations are Pareto optimal.

⁹ Bernake and Gurkaynak (2001) argue that the long run growth is significantly correlated with saving rate. It is not easy to explain this correlation with the exogenous growth model.

¹⁰ $F_K(.) > 0$, $F_{KK}(. < 0$ and $F_N(.) > 0$, $F_{NN}(. < 0$

¹¹ $\lim_{K \rightarrow 0} F_K(.) = \lim_{N \rightarrow 0} F_N(.) = \infty$ and $\lim_{K \rightarrow \infty} F_K(.) = \lim_{N \rightarrow \infty} F_N(.) = 0$, following Inada (1963).

The next important building block of the model is the law of motion of capital. The physical capital stock evolves according to the first difference equation in K_t :

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (2.2)$$

where δ is the depreciation rate of capital stock and I_t is investment. The economy as a whole can only consume and invest when it has produced. Therefore, this results in the additional equation for resource constraints:

$$Y_t = C_t + I_t \quad (2.3)$$

The model of Solow and Swan assumes that representative agents save or invest a fixed fraction σ of income, $I_t = \sigma Y_t$. Growth in labour input γ_N , and labour augmenting technology, γ_X , is assumed to be constant and exogenous. Mathematically, $N_{t+1} = N_t \gamma_N$ and $X_{t+1} = X_t \gamma_X$ where γ_Z is the growth rate of Z and $\gamma_Z - 1 = (Z_t - Z_{t-1})/Z_{t-1}$. In the steady state, capital, output and consumption grow at rate $\gamma_N \gamma_X$ or $\gamma_N \gamma_X = \gamma_Y = \gamma_K = \gamma_C$.

2.3.2 The optimal growth model

This section extends the Solow and Swan model described in 2.3.1 by including individual decisions at the microeconomic level. This model, first developed by Ramsey (1928) and refined later by Cass (1965) and Koopmans (1965, 1967), continues to assume that the growth rate of labour input and labour augmenting technology are exogenous. Nonetheless, the saving rate is endogenous because the solution of capital stock results from optimizing representative agents' preferences over the consumption path with respect to the economy's resource constraints.¹²

The representative household has the following utility over consumption path:

¹² Recently the study of the growth also considers the human capital. Barro (1991) provides the facts that the growth rate of real per capita GDP is positively related to initial human capital and negatively related to the initial level of real per capita GDP. Barro, Mankiw and Sala-I-Martin (1995) study the capital mobility in neoclassical growth model. Cooley and Ohanian (1997) use the neoclassical growth model to study the policies designed by Keynes for the post war period of the UK. They conclude that the high tax rates levied on capital income causes, in the early post war period, Britain's poor macroeconomic performance.

$$u(C_t) = \begin{cases} \log(C_t) & \text{for } \sigma = 1 \\ \frac{C_t^{1-\sigma} - 1}{1-\sigma} & \text{for } \sigma \neq 1 \end{cases} \quad (2.4)$$

Most of the models use time separable utility functions, providing a constant relative risk aversion (CRRA). σ is called a coefficient of relative risk aversion and measures the degree of risk aversion of the agents. It is important to note that $1/\sigma$ is the elasticity of intertemporal substitution. The utility has positive but diminishing returns from an additional unit of consumption and it also satisfies the Inada condition. In the context of the Cass and Koopmans neoclassical growth model, the representative household wants to maximize its lifetime utility subject to its budget constraint.¹³

$$\begin{aligned} \text{Max } E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \\ \text{Subject to } K_{t+1} = (1-\delta)K_t + F(K_t, N_t X_t) - C_t \end{aligned} \quad (2.5)$$

where β is the time discount factor and $\beta \in (0,1)$. The high value of β implies that the representative households are impatient and less willing to push their consumption to the next period. However, the neoclassical growth models are not appropriate to study the macroeconomic issues such as real business cycles because the growth model assumes that labour input is fixed and the model is deterministic. The real business cycle model incorporates neoclassical factors in determining the economic fluctuation. Basically, the real business cycle model remedies these flaws by allowing a variation in labour input in the representative agent's preferences and adding stochastic technology shocks to the model.¹⁴ The next section explores the real business cycle model and focuses on the original works by Kydland and Prescott (1982) and the basic real business cycle model.

¹³ The method to do optimization and to solve the model is described in chapter 4 in more detail.

¹⁴ Since Brock and Mirman (1972, 1973) and Danthine and Donaldson (1981) considered the neoclassical growth model with the inclusion of the random aggregate shocks to productivity, a substantial body of business cycle literatures has been followed these ideas to make the growth model stochastic.

2.4 The Real Business Cycle Model

After the study to determine the characteristics of the equilibrium paths by Lucas and Prescott (1971), Finn Kydland and Edward Prescott (1982) built on the equilibrium growth model and that prior research the “Time to Build and Aggregate Fluctuations”.¹⁵ The work of Kydland and Prescott was the pioneering research in the business cycle field. They created the concept of how the business cycle can be studied by using dynamic general equilibrium models. These models allow agents in the competitive market to form rational expectations about the future. One principle drawback of the literature using static computational general equilibrium is that it rules out capital accumulation. The dynamic model, technically, overcomes this particular weakness. Consequently, the paper has changed the direction of research in this field. Since Kydland and Prescott (1982) studied the aggregate fluctuation within the closed economy framework using the technology shock as a driving process, their work was referred to as the “real business cycle” model. Many research programs that have built up on the foundation laid by this paper have grown spectacularly for the last three decades. A number of papers have tested the ability of the models to match the fluctuation.

2.4.1 The original models

The initial set of dynamic general equilibrium models of the 1980s focused on productivity shocks as a source of economic fluctuations. Kydland and Prescott (1982) and Long and Plosser (1983) described how a well-functioning market economy would respond to productivity shocks, concluding that it displays important characteristics of business cycles in the real world. Kydland and Prescott studied a variant of the one sector neoclassical growth model, augmented to include variable labour supply, invest according to a so call “time to build” investment process and various other elements aimed at producing a modern business cycle model. Their model was driven by an aggregate productivity shock. Long and Plosser constructed a multi-sector neoclassical growth model, in which changes in sectoral productivity responses in other sectors and ultimately at the aggregate level in the economy. These papers stimulated much research into the influence of productivity on macroeconomics activity including work on its measurement, its origin and its consequences.

¹⁵ Blanchard and Fischer (1989) claim that the technology shock is uncertainty. The presence of uncertainty effects the behaviour of agents. The shocks themselves causes agents to constantly revise their optimal plans.

Kydland and Prescott assume that multiple periods are required to build new productive capital. There are $s_{j,t}$ projects which need j periods to completion. Therefore, $s_{j,t+1} = s_{j+1,t}$ where $J = 1, \dots, J-1$. The capital accumulation for this economy is governed by:

$$k_{t+1} = (1 - \delta)k_t + s_{1,t} \quad (2.6)$$

The fraction of the resources allocated to the investment project in the j th period from the last is denoted by ϕ_j . Total non-inventory investment in the current period t is $\sum_{j=1}^J \phi_j s_{j,t}$.

Another capital goods is the inventory stock of capital, u_t which inherited from the previous period. Total investment i_t can therefore be written as:

$$i_t = \sum_{j=1}^J \phi_j s_{j,t} + u_{t+1} - u_t \quad (2.7)$$

Since output can be either consumed or invested for this economy, the resource constraint of the model can be expressed as follow:

$$c_t + i_t \leq y_t \quad (2.8)$$

where $y_t = f(a_t, k_t, n_t, u_t)$. a_t is a shock to productivity and n_t is the labour input, and $f(\cdot)$ is a constant-returns-to-scale production function. The production function takes the form $f(a_t, k_t, n_t, u_t) = an^\theta [(1 - \sigma)k^{-\nu} + \sigma u^{-\nu}]^{\frac{-(1-\theta)}{\nu}}$ where $\theta, \sigma \in (0, 1)$ and $\nu \in (0, \infty)$. This production function results in a share, θ , for labour input in the steady state. The elasticity of substitution between capital and inventory is $1/(1 + \nu)$. The preference takes the non-time separable form, $u(c_t, \alpha(L)l_t) = [c_t^{1/3} (\alpha(L)l_t)^{2/3}]^\gamma / \gamma$ because it admits greater intertemporal substitution of leisure. l_t is leisure and L is the lag operator, and $\alpha(L) = \sum_{i=0}^{\infty} \alpha_i L^i$. This preference is the constant relative risk aversion with the degree of risk aversion being $1 - \gamma$.

It is important to point out the central implication of the original model. Kydland and Prescott argued that private agents are rational and thus expect the central bank and government to re-optimize its policy in the future. Therefore, the policies in the model are ineffective, meaning that any anticipated monetary policy will not effect output and it is optimal not to have the government spending in the model. Kydland and Prescott evaluate how well their model does at replicating certain U.S. post war facts about business cycle. They begin by choosing most of the parameters of their model to match macro observations and steady state facts, a method of parameter selection which is usually called “calibration”. Kydland and Prescott (1982) argue that their model is consistent with high variability in investment and the low variability in consumption and their high correlation with output.

2.4.2 A simple real business cycle framework

The real business cycle model uses the a neoclassical growth model of the form developed by Solow (1956), Cass (1965) and Koopmans (1965), but it views cycles arising through the reactions of optimizing agents to real disturbance for example the random shocks in technology. The core element of the business cycle and growth theory is the aggregate production function, relating output to labour and capital input. The real business cycle theory makes exogenous stochastic fluctuation in productivity the predominant cause of variation in economic activity. A standard real business cycle model is an extended version of the growth model cycle by adding two important features, variation in labour input and random shocks to productivity. With the assumption on technology, endowment and preferences, the model is constructed to capture the main features of growth and business cycle. This section discusses the structure of the basic real business cycle model and later on this chapter investigates the extensions that have been made on the simple model.

Labour and capital input are employed in the production function, generating the output of the economy. The production function is described by the neoclassical production function with an improvement in factor productivity.¹⁶

$$Y_t = A_t F(K_t, N_t X_t) \quad (2.9)$$

¹⁶ The assumptions of the production function and also preferences of this model ensure that steady state features are feasible. See King *et al.*(1988) for detail.

where A_t represents a random shock in productivity, N_t is defined as number of per capital hours work and X_t is the deterministic part of production. X_t is assume to grow at a constant rate γ_X and $\gamma_X > 1$. For the Cobb-Douglas production function, it can be written as $Y_t = A_t K_t^{1-\alpha} (N_t X_t)^\alpha$. The law of motion of capital stock evolves according to:

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (2.10)$$

The shock process in this economy is assumed to follow an AR(1) process:

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t \quad (2.11)$$

where ρ measures the persistence of the shock and ε_t represents the innovation of technology. The innovation is iid with means 0 and variance σ_A^2 or $\varepsilon_t \sim N(0, \sigma_A^2)$. To replicate the aggregate fluctuation, the model in general requires a high variance of technology innovation.¹⁷ The economy is populated by a large number of infinitely lived representative agents who have the expected utility function of the form:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \quad (2.12)$$

where L_t is leisure. The time endowment is 1, $1 = N_t + L_t$. The individuals can share their time between work, N_t , and leisure, L_t . To ensure that the preference is consistent with the steady state path, the additional assumption must be imposed to the preference. King *et al.* (1988) shows that the period utility function must take the form:

$$u(C, L) = \frac{\{Cv(L)\}^{1-\sigma} - 1}{1-\sigma} \quad (2.13)$$

¹⁷ Burnside, Eichenbuam and Rebelo (1993) estimate that the variance of innovations to technology is roughly 50 percent less than what implied by standard real business cycle models using the structural model of labor hoarding. Burnside and Eichenbuam (1996) analyze the role of the variable capital utilization rates in propagating shocks over business cycles. After including the variable capital utilization, the variance of technology shocks can be reduced significantly in explain the business cycles.

This class of utility implies that income and substitution effects of wage changes on labour supply exactly offset each other. Keeping the economy closed and ignoring the public sector, the output of the economy is the sum of consumption and investment:

$$Y_t = C_t + I_t \quad (2.14)$$

The findings from the standard model of Cooley and Prescott (1995) are that the output in the model fluctuates less than in the US data. Investment in the model economy fluctuates much more than it does in the US data. In the model, consumption fluctuates much less than does output. Consumption, investment and hours in the model are all highly procyclical as they are in the US economy. The salient property of the model is that all variables in the model exhibit a high correlation with output. Nonetheless, the shortcomings of the standard model are focused in its labour market implications. The volatility of aggregate hours in the model is less than the volatility of output, while in the data they display the similar magnitude. The variation in aggregate hours in the data can mostly be explained by variation in employment as opposed to the variation in hours per worker. In the next section, the extensions to the basic model are reviewed.

2.4.3 Extending the basic model

There are several areas in which extensive contributions have been made. This section focuses on four main strands of the research, specifically the behaviour of the labour input, the inclusion of fiscal policies; the inclusion of monetary policy; and the extension to a multisector model. The last essential extension is the application of the model in a specific country.

2.4.3.1 Labour supply

The basic real business cycle model, workers are paid their marginal product of labour as wage and this wage clears the labour market. Therefore, at the market clearing wage, everyone who wants to work can do. The predicted outcome of the simple model is the intensive margin or number of per capita hours worked. The basic model cannot take into account the situation in which households who do not find it preferable to work may not supply their labour. RBC researchers have explored the way to remedy this failure. An employment lotteries developed by Rogerson (1988) and apply to business cycle

model by Hansen (1985), called the indivisible labour, are introduced to the basic model. The main idea of indivisible labour is that changes in labour input come from movements into and out of employment. The production side of the economy is the same as the basic model, but the separable preference is used and given by:

$$u(c_t) - v(N_t) \quad (2.15)$$

In this case, households can either work full time (have a job), $N_t = 1$ or not work at all (have no job), $N_t = 0$. Two important assumptions for this model are the social planner provides full insurance against being unemployed and all people get selected to work with the same probability. Therefore, let p is the fraction of the population that the planner picks to work, p also represent the probability that a particular agent gets selected to work.

The expected utility of the representative household is:¹⁸

$$p_t[u(c_t) - v(1)] + (1 - p_t)[u(c_t) - v(0)] \quad (2.16)$$

In equilibrium of this model, representative households are assigned to employ or to unemploy each period randomly. p_t is the choice variable of the planner each period.

The fluctuation in the aggregate hours in the data mostly comes from the variation in the employment, whereas in the simple model those variation stems from the hours per workers. The indivisible labour version of the business cycle model can correct this problem. Hansen and Wright (1992) document that the indivisible labour model raises the output fluctuation and the ratio of the standard deviation of total hours to that of output.¹⁹ Christiano, Eichenbaum and Vigfuss (2003) recently report that the technology

¹⁸ $E[u(c_t) - v(N_t)] = p_t[u(c_t) - v(N_t = 1)] + (1 - p_t)[u(c_t) - v(N_t = 0)]$
 $= u(c_t) - p_t[v(1) - v(0)] - v(0)$

It is possible to ignore $v(1)$ and $v(0)$ because they are just a constant number. Ignoring constants added to the utility function does not change the necessary condition. Therefore, we can write

$E[u(c_t) - v(N_t)] = u(c_t) - \theta p_t$ where $\theta = v(1) - v(0) > 0$. It is possible to generalize the preference to be a multiplicative case as well.

¹⁹ Also see Hansen and Wright (1992) for further details.

shock drives up the per capita hour worked. They claim that it is a specification error to model the low frequency component of per capita hour worked.

2.4.3.2 Fiscal policy

There are a large number of extensions of real business cycle models by including fiscal policy to the standard model. These literatures introduce the additional shock, a fiscal shock, to study the dynamic effect of the shock on the economic fluctuation. Christiano and Eichenbuam (1992) introduce the fiscal shock to the basic model. They assume that the government spending shock follow an AR(1) process. Government spending is financed by lump-sum or non-distortionary taxes levied on household. The government's budget is balanced in each period. The resource constraint of the economy is:

$$Y_t = C_t + I_t + G_t \quad (2.17)$$

where G_t represents government spending at period t . Government spending does not generate any utility for the household and it cannot be used in production. An increase in government spending results in a negative wealth effect on representative agents and induces them to supply more labour. Their model solves the empirical problem of low correlation between total employment and productivity observed in actual US data.

The fiscal shocks such as distortionary taxes and government spending are also introduced to the model by Baxter and King (1993), Braun (1994) and McGrattan (1994). The general idea is that the total consumption of goods cannot exceed the disposable income:

$$C_t + I_t \leq (1 - \tau_t)Y_t + T_t \quad (2.18)$$

where τ_t is the tax rate on output at period t and T_t represent the transfer payment at date t . It is possible to extend the model to consider different tax rates on capital and labour income. The government uses tax revenue to fund government purchases and lump-sum transfers:

$$\tau_t Y_t = G_t + T_t \quad (2.19)$$

Baxter and King (1993) investigate the difference between the effect of temporary and permanent fiscal policies. They show that the impact of multiplier for output depends on the duration of the fiscal policy shock. McGrattan (1994) uses different tax rates on capital and labour income. By allowing the fiscal shock with the indivisible labour model, the representative agents adjust their decision according to changes in tax rates. These changes cause the household to make intertemporal substitution and result in the fluctuation in consumption, hours worked, investment and income. McGrattan (1994) also evaluate the Kydland-Presscott model extension by Hansen and by Braun. They improve the model performance which assumes the fiscal shock. McGrattan, Rogerson and Wright (1997) examine the model with U.S. economy with including the explicit household production and fiscal shocks. They find that the model with household production generates different prediction for the tax changes.

2.4.3.3 Money

Another major extension of the model of fluctuation is to include money in the model. The literature has seen the development of models with technology not being the only driving force of the economy. Cooley and Hansen (1989, 1995, 1997, 1998) developed the real business cycle model with a cash in advance constraint. In this case, money can be required to purchase consumption goods. They introduce the cash in advance motive for holding money into the simple indivisible labour model.

In the basic version of their model, representative agents' consumption decisions must satisfy the cash in advance constraint:

$$P_t c_t \leq m_{t-1} + gM_{t-1} \quad (2.20)$$

The nominal value of consumption purchase, $P_t c_t$, must not exceed money holdings of a particular household carried over from last period, m_{t-1} , plus the lump sum transfer of seigniorage revenue, gM_{t-1} . The money supply, M_t , evolves according to:

$$M_t = (1 + g)M_{t-1} \quad (2.21)$$

The money supply is assumed to grow at a constant rate, g , over time. The household budget constraint is:

$$c_t + i_t + \frac{m_t}{P_t} = W_t N_t + R_t k_t + \frac{m_{t-1}}{P_t} + \frac{gM_{t-1}}{P_t} \quad (2.22)$$

where P_t denotes the aggregate price level. The model solution requires that all variables fluctuate around a constant mean. Because the positive money growth results in inflation, it is necessary to make a change of variable in this model. This motivates introducing the following change of variables in the steady state, $\hat{m}_t = m_t/M_t$ and $\hat{P}_t = P_t/M_t$. The money supply in this model is an exogenous state variable. Christiano and Eichenbaum (1995) also examine the role of money in general equilibrium and find that a positive money supply shock generates a large drop in the interest rate.

2.4.3.4 Multiple sector models

The extension of the real business cycle model by including multiple sectors and sector specific shocks was introduced by Long and Plosser (1983). Long-Plosser considered the idea that consumers generally desire a variety of goods and so will spread increments to wealth across many different commodities at many different dates could leading to business cycle phenomena. They focused on particular aspects of the business cycles, persistence in the sense that business cycle involves strong positive serial correlation of aggregates and co-movement in the sense that, over the course of business cycles, expansions and contractions, many different sectors rise or fall together. They concluded that real shocks and real mechanisms may be important for the business cycle. Technical progress is possible in obtaining solutions to the model with alternative specifications of preferences and technology. Recently, Kouparitsas (2002) shows that the high degree of business cycle co-movement across U.S. regions over the last 40 years reflects the fact that regions are influenced by common sources of shock, not any significant spillover of shocks across regions. Talmain and Abadir (2002) investigate firm heterogeneity into the standard monopolistically competitive RBC model. In this economy, the final good

are composed of the intermediate good. They found that the aggregate output has different properties from log-linear time series models such as auto-regressive model.

However, the majority of the literature follows the aggregated model in line with Kydland and Prescott but a significant minority of the literature uses the model proposed by Long and Plosser.

2.5 The International Real Business Cycle Model

Afterward researchers have been tempted to construct the open economy model to account for business cycle aspects. The original work of the international real business cycle theory is explored by Backus, Kehoe and Kydland in 1992. Backus and Kehoe (1992) obtained the facts of the OECD countries for within and cross country correlations. Backus, Kehoe and Kydland (1992), and Kydland (1992) attempted to answer how a two-country real business cycle model is able to capture the features of the domestic and international business cycles. The open-economy model allows us to consider the correlations across countries. This model is the extension of Kydland and Prescott (1982). The role of financial markets in allocating risk has been introduced. The model has the assumption of a single good with complete markets for a state contingent claim. Unlike the Kydland and Prescott (1982), this model specifies the technology shock process for the two countries as a bivariate auto-regression and the model allows agents to participate in international capital markets:

$$A_{t+1} = \rho A_t + \varepsilon_{A,t+1} \quad (2.23)$$

where $A_t \equiv (A_t^H, A_t^F)$ and A_t^H, A_t^F denotes the technology in the home and foreign country respectively. ρ is a matrix of coefficients, and $\varepsilon_{A,t} = (\varepsilon_{A,t}^H, \varepsilon_{A,t}^F)$ and $\varepsilon_{A,t}^H, \varepsilon_{A,t}^F$ are the innovation of technology in the home and foreign country respectively. The innovation $\varepsilon_{A,t}$ allows contemporaneous correlation between the home and foreign innovations. The model allows innovation in the shocks to be correlated across countries and also allows the diffusion of technological shocks between countries, as technological change is transmitted across borders.

The studies of closed economies suggest that a neoclassical growth model with an aggregate technology shock can account for the fluctuation in consumption and investment relative to output and the correlation of these fluctuations with output. Consumption is smoother in the open economy theoretical environment than it is in the data. In contrast, investment is more volatile in the model. The contemporaneous cross-correlation between investment and output is substantially smaller in the model than in the facts. The closed-economy model is superior for each of those properties. The two country model mimics poorly in explaining the international correlations. The model fails to explain the correlations of consumption and output across countries. In theoretical model, there is a higher cross country correlation for consumption than there is for output. The actual data documents that the opposite is true. They call this discrepancy as “the consumption/output anomaly”.

Backus, Kehoe and Kydland (1994, 1995) modify the model so that the two countries produce different, imperfectly substitutable goods. They changed the technology equation so that the quantity of total output in each country measured in units of local good can be used by domestic or foreign consumers. The elasticity of substitution between foreign and domestic goods is also incorporated into this model. Backus, Kehoe and Kydland (1994) have constructed two-good dynamic models of international trade which they use to explore the relationship between the term of trade and the balance of trade.

The structure of the two good-two country model of Backus, Kehoe and Kydland basically consists of two sets of preferences one for country 1 and another for country 2. The representative agents in each country i maximize their expected utility as below:

$$U_i = E_0 \sum_{t=0}^{\infty} \beta^t u(C_{i,t}, 1 - N_{i,t}) \quad (2.24)$$

where $C_{i,t}$ and $N_{i,t}$ are consumption and employment in country i . The period utility function is written as $U(C, 1 - N) = [C^\theta (1 - N)^{1-\theta}]^{1-\sigma} / (1 - \sigma)$. Technology in the model must be modified. Each country now specializes in the production of a single good, labelled a for country 1 and b for country 2. Each good is produced using capital, K ,

and labour N , with production function as the same form, $F(K, N) = K^{1-\alpha} N^\alpha$. Therefore, the resource constraints can be expressed as:

$$a_{1,t} + a_{2,t} = Y_{1,t} = A_{1,t} F(K_{1,t}, N_{1,t}) \quad (2.25)$$

$$b_{1,t} + b_{2,t} = Y_{2,t} = A_{2,t} F(K_{2,t}, N_{2,t}) \quad (2.26)$$

where $Y_{i,t}$ represents total output in country i , measured in units of the local good, and $a_{i,t}$ and $b_{i,t}$ denotes uses of the two goods in country i . The capital accumulation equation is simplified by setting the time to build parameter equal to 1. Therefore, the law of motion of capital is written according to:

$$K_{i,t+1} = (1 - \delta)K_{i,t} + I_{i,t} \quad (2.27)$$

Consumption, investment and government spending in each country are composites of the foreign and domestic goods:

$$C_{1,t} + I_{1,t} + G_{1,t} = \Omega(a_{1,t}, b_{1,t}) \quad (2.28)$$

$$C_{2,t} + I_{2,t} + G_{2,t} = \Omega(b_{2,t}, a_{2,t}) \quad (2.29)$$

where $\Omega(a, b) = [\omega a^{1-\gamma} + b^{1-\gamma}]^{\frac{1}{1-\gamma}}$ and Ω is called Armington aggregator.²⁰ The parameter ω and γ are both positive, and the elasticity of substitution between foreign and domestic goods is $\sigma = 1/\gamma$. The terms of trade are defined by:

$$P_t = \frac{q_{2,t}}{q_{1,t}} \quad (2.30)$$

²⁰ The Armington aggregator is the constant elasticity of substitution aggregator, and thus allow the flexibility over the choice of both share parameters, ω , and the elasticity of substitution, $1/1-\gamma$.

where $q_{1,t}$ and $q_{2,t}$ are the prices of the domestic and foreign goods, respectively. In equilibrium, this relative price can be calculated from the marginal rate of substitution in the Armington aggregator evaluated at equilibrium quantities:

$$P_t = \frac{[\partial\Omega(a_{1,t}, b_{1,t})/\partial b_{1,t}]}{[\partial\Omega(a_{1,t}, b_{1,t})/\partial a_{1,t}]} = \omega^{-1} \left[\frac{a_{1,t}}{b_{1,t}} \right]^{\frac{1}{\sigma}} \quad (2.31)$$

The trade balance of country 1, expressed in units of the domestic good, is:

$$TB = a_{2,t} - P_t b_{1,t} \quad (2.32)$$

The ratio of net exports to domestic output, $Y_{1,t}$ is:

$$NX_{1,t} = \frac{a_{2,t} - P_t b_{1,t}}{Y_{1,t}} \quad (2.33)$$

They documented the two features of international data, the countercyclical movements in the net exports and the tendency for the trade balance to be negatively correlated with current and future movement in the term of trade, but positively correlated with past movement. They also documented that the lagged cross-correlation function between terms of trade and net exports performs a “J-curve” relationship that is similar to the data.²¹ Backus, Kehoe and Kydland (1995) also extend their paper in 1992 by eliminating the inventory accumulation and leisure durability. The model replicated either the consumption correlation or the output correlation in the data, but not the two together. The discrepancy between theory and data is the relative size of the consumption and output correlation, rather than either one separately. Once again, they refer to this discrepancy as “the consumption/output anomaly”. For the two goods-two country model, the volatility of the term of trade substantially lower than that of data. They refer to this problem as the “price anomaly”. They concluded that the international business cycle focuses on these two striking differences between data and theory.

²¹ Kose and Yi (2001) examine whether the stronger international trade linkages of the pairs of countries tend to have more highly correlated business cycles by using standard IRBC models along the line of Backus et al.(1994) where trade is induced by an Armington aggregator. They find that the model cannot explain results that are consistent with the facts.

Extension of the international real business cycle models

As in Backus, Kehoe and Kydland (1995), there are generally five ways in literatures to help account for these two anomalies. Firstly, the non-traded goods are introduced into the model as the work of Stockman and Tesar (1995). The result of adding non-traded goods is that traded goods consumption, rather than total consumption, is more highly correlated across countries in the model than in the data. They solved one problem at the cost of another. The anomaly is pushed onto the traded component of consumption.

Secondly, the impulse generating fluctuations can influence the international business cycle. Costello and Praschnik(1992) introduced oil price shocks. In this work, the terms of trade for manufactured goods remain less variable in the model than in the data and the cross-country correlation of manufactured goods consumption is much higher than in the data. The consumption/output anomaly therefore remains unsolved. Nevertheless, the variability of the term of trade in oil-importing countries increases and the correlation of consumption across countries decreases. Backus and Crucini (2000) extend the two-good, two-country, stochastic growth model of Backus et al. (1994) to incorporate a third country that sells oil. They control the sources of shocks driving the terms of trade. As a result, the co-movement and volatility of the terms of trade, output, and the trade balance become less puzzling.

Thirdly, the restriction on the asset market is to introduce to theory. The complete markets model has been heavily criticized for its counterfactual forecasting of consumption and output correlation across countries. With incomplete markets, the model has the potential to account for the consumption/output discrepancy. Apart from complete markets, Baxter and Crucini (1993, 1995) also introduced incomplete market, which agents trade a single risk free bond. The complete market assumes that there is frictionless international trade in output, so that there is unified world resource constraint for the single produced good:

$$\pi(Y_{1,t} - C_{1,t} - I_{1,t} - G_{1,t}) + (1 - \pi)(Y_{2,t} - C_{2,t} - I_{2,t} - G_{2,t}) \geq 0 \quad (2.34)$$

where π represent fraction of population the country 1 and 2. For incomplete markets, the financial trade is restricted to a non-contingent real bond. They consider one period

risk free discount bonds. The flow budget constraints for the market structure in which the financial trade is limited to the real bond are given by:

$$P_{i,t}^B B_{i,t+1} + C_{i,t} + I_{i,t} = Y_{i,t} + B_{i,t} \quad (2.35)$$

where $B_{i,t}$ denotes the per capita quantity of these discount bonds purchased by country i which mature in period $t+1$. Let r_t denote the world rate of return on these risk free securities, and therefore, $P_{i,t}^B = (1-r_t)^{-1}$ and it is the price of a discount bond purchased in period t . The world marketing clearing condition is given by:

$$\pi B_{1,t} + (1-\pi) B_{2,t} = 0 \quad (2.36)$$

Importantly, they found out that in order to distinguish the complete from incomplete market, the productivity in each country must follow the random walk without spillovers but with correlated innovations. Moreover, Baxter and Crucini (1993) originally proposed an adjustment cost function to mitigate the response of investment to the location-specific shocks. The capital accumulation is given by:

$$K_{i,t+1} = (1-\delta)K_{i,t} + \phi(I_{i,t}/K_{i,t})K_t \quad (2.37)$$

where $\phi(I_{i,t}/K_{i,t})$ is the adjustment cost function in country i . The function ϕ controls the number of units of output that must be foregone to increase the capital stock in a particular location by one unit. Since there is a single good in the two countries, capital owners have a strong incentive to locate new investment to where productivity shocks are persistent. The model would perform a very high volatility of investment without some friction in the capital accumulation. With random walk shocks, the incomplete market can solve the consumption/output anomaly. However, the correlations of consumption, investment, and employment are negative. Arvanitis and Mikkola (1996) confirm that the incomplete market models can explain the facts better the complete market models as suggested by Baxter and Crucini (1993, 1995). Kose and Yi (2006) use a three country model with transportation costs and simulate the effects of increased goods market integration under complete markets and international financial autarky.

They find that under both asset market structures the model can create stronger correlations for pairs of countries that trade more, but the increased correlation falls far short of the facts.

Fourthly, the money is added to the theoretical economy to help account for the term of trade anomaly. Grilli and Roubini (1992, 1996) and Schlagenhauf and Wrase (1995) applied the liquidity model to the open economy. In these works, the asset and good markets are separated for one period, and shocks to the stock of money have a one-period effect on interest rate, currency prices, and relative prices. Consequently, without imposing different kind of market structure, the model generates the high volatility of term of trade. Nevertheless, the model fails to mimic one of the important features of data, which is the persistence of relative price movement.

The last extension of the theory is to introduce imperfect competition. The idea is that imperfectly competitive firms sell their output in markets that are internationally segmented, the price discrimination might lead to greater changes in relative prices than we see with perfect competition. Giovannini (1988) shows that this change can lead to persistent movements in relative prices across countries. However, it is necessary for the model to be improved for an aggregation.

Recently, a large and growing number of papers in the international real business cycle involve adding more complexity in the benchmark models to fix consumption/output and term of trade anomalies. Nevertheless, there is one important additional puzzle as in Baxter (1995), “international co-movement” which refers to the cross country correlations of factor inputs. The models tend to have a negative correlations while, in the data, investment and employment are positively correlated across countries. Boileau (1996) reproduces the observed empirical evidence in a two-country real business cycle model with endogenous growth. The model includes a non-market sector and international externalities in production. It generates realistic cross-country correlations for output, consumption, and productivity with standard parameter values. Boileau (1999) analyzes whether trade in capital goods can explain the underpredicted volatility of the term of trade. Models with trade in capital goods generate a volatility between 0.55 and 0.98 for net exports and between 1.23 and 3.24 for the terms of trade.

Roche (1996) develops the model of the world economy with introduction of government sector. The countries produce a non-specialized traded good and agents derive utility from the consumption of private and public goods and services. With this modification, the model has a potential explanation for the low cross-country correlations in private consumption and output.

Pakko (1997, 2002) works on the baseline model economy used by Backus *et al.* (1994,1995) along with imposing the investment adjustment cost for capital accumulation. Pakko (1997) shows that including asset market restrictions has little effect on the model's prediction. Pakko (2002) shows how the introduction of investment adjustment costs affects cross country investment correlations and the dynamics of the term of trade and net exports. The elasticity of substitution between foreign and domestic goods is the crucial parameter for generating strong implications of the model. With low substitution elasticity, the model with investment adjustment cost displays the positive cross-country correlations of investment and employment, as well as countercyclical net exports and a negative correlation between net exports and term of trade, retaining the J-curve pattern. Pakko (1997) claim that in a particular condition such as the aversion to compositional consumption risk be sufficiently stronger than the aggregate consumption risk the complete market can be associated with cross country consumption correlations that are lower than cross country output correlations.

The standard preference commonly used in RBC and IRBC literature is time separable preferences with a fixed discount factor. Kim and Kose (2003) examines the dynamic implications of different preference formulations in the IRBC model under incomplete market. When the models are solved using the normal linear approximation approach, Kim and Kose (2003) assert that it is impossible to generate stationary state variables and a well-defined wealth distribution in an open economy under incomplete market with time separable preferences and a fixed discount factor. Kim and Kose compare the impulse responses and moments of the time separable preferences with fixed discount factor and time non-separable preferences with an endogenous discount factor. However, they concluded that the results of the two models are quite identical under traditional parameter values.

Generally, in the models, cross-country correlations are much higher for consumption than for output, while in the data the opposite is true; and cross country correlations of employment and investment are negative, while in the data they are positive. Kehoe and Perri (2000) attempt to remedy the anomalies by introducing the open business cycle model with endogenous incomplete market. This paper incorporates friction to the model that helps to resolve these flaws. The friction is that international loans are imperfectly enforceable; any country can default on its debts and has a penalty by suffering the consequences for future borrowing. This friction captures the difficulties of enforcing contracts between sovereign nations that involve large transfers of resources which are backed only by promises to repay later. The paper focuses on the difficulties of enforcing contracts between agents in different sovereign nations when sovereign governments can abrogate contracts. Kehoe and Perri concluded that including these enforcement difficulties in business cycle models resolve the anomalies by reducing the gap between the cross-country correlation of consumption and that of output and making employment and investment co-move positively in the two countries. Nonetheless, in this model, the correlation between net exports and output is positive instead of negative.

2.6 The Business Cycle Model for a specific country

The business cycle model can also be applied to specific countries. The real business cycle theory is design for the developed countries, specifically the US, and the model performs well to capture the facts of many developed countries. However, studies in real business cycle, to date, have primarily focused on business cycle characteristics of major developed countries and a limited number of developing countries. It is essential to evaluate the model for emerging economies both real business cycle and international real business cycle. This section discusses the model application for a particular country by starting with the real business cycle and extending to international real business cycle.

2.6.1 The real business cycle model for developed countries

Section 2.4 has discussed the benchmark model by Kydland and Prescott (1982) and also the basic model. The extensions of the basic model have also been explored on section 2.4, which are the labour supply, fiscal policies, monetary policies and a multisector model. Most of these models are designed for explaining the US economic fluctuation.

Recently, the literatures that are worth to mention in this section are King and Rebelo (1994) discuss the structure of the real business cycle model with the extensions of the model. They conclude that a large technology shock is required to generate a realistic variation. A small technology shock can produce a realistic fluctuation, if the model is modified for varying capital utilization. Ravn (1997) examines the role of permanent and transitory shocks of the real business cycle model for the UK business cycle. The model can account for many features of the UK data and the temporary shocks are pertinent to explain the business cycles. Rebelo (2005) reviews the contribution of the business cycle model and the open research area of business cycle. The paper investigates the challenges to the RBC model such as the behaviour of asset pricing. The common utility function used in the RBC model displays counterfactual implication of asset pricing. The role of alternative shocks, such as oil and fiscal shocks, are discussed. Both oil and fiscal shocks generate significantly low business cycle fluctuations.

2.6.2 The real business cycle model for developing countries

Although business cycle models have been successfully applied to developed economies, their abilities to mimic the observed data of emerging countries remain largely unexplored. The contributions for the real business cycle model have been made for developing countries are also in the main areas of the behaviour of the labour input, the inclusion of fiscal policies and monetary policies.

Ellery, Gomes and Sachisda (2002) documents the empirical real business cycle in postwar Brazil using two dynamic general equilibrium models, one a standard real business cycle model and another the indivisible labour model following Hansen (1985).²² They use two filters to extract the cyclical component, one a widely used filter Hodrick-Prescott and another a band-pass filter. They compare the actual facts with the predictions from the theory. Both models fail to explain the high volatility of consumption, hours and productivity when compared with the volatility of the GNP. The models also fail to replicate the low correlation between productivity and GNP.

²² The indivisible labour has a special feature where all variations in labour input reflect adjustment along the extensive margin. The model restricts the consumption possibilities set so that individuals can either work full time or not at all. Consequently, fluctuations in the total hours are due to employment rather than hours per worker.

Bergoeing and Soto (2005) test the real business cycle model for the Chilean economy. They analyze the importance of technology, fiscal, and monetary shocks as the sources of aggregate fluctuation in Chile. Typically, they construct the economy characterized by a government that engages in fiscal and monetary policy, a large number of identical firms, and a large number of identical consumers. In their general model, money is held because it is required to buy consumption goods. They introduce the cash-in-advance motive for holding money into the basic indivisible labour real business cycle model following Cooley and Hansen (1995). Money is created by government according to an exogenous law of motion. Furthermore, they modify the cash-in-advance model as mentioned by imposing the constraint that the nominal wage rate for period t be agreed one period in advance. The main finding is that business cycle models are able to replicate much of observed fluctuations of both the real and monetary sides of the economy. An economy with government expenditures and labour indivisibility are able to account for the short-run fluctuations in Chile. Monetary shocks and nominal contracts generate excessive volatility or fail to explain the data in labour market behavior. The model requires adding some features on the optimising behavior of agents to mimic closer the fluctuation in consumption.

2.6.3 The international real business cycle model for developed countries

The growing number of papers involves modifying the transmission channels and introducing more possible sources of aggregate fluctuation to the international real business cycle model in order to explain the business cycle in specific countries. The introduction of the GHH utility function has also been considered to solve the problem of low volatility of consumption and the procyclical of the trade balance in the model. Additionally, the economic methodology such as a dynamic factor model is employed to identify different sources of cyclical fluctuations and relationships among different variables.

Ortega and Estudios and Espana (1998) studies the relationship between the Spanish real aggregate fluctuations of the trade weighted average of the other four EU countries. They follow a standard two good-two country model based on Backus *et al.* (1994). However, they introduce possible sources of aggregate fluctuations are both government spending shocks and technology shocks. They modify certain key parameters and derive four different model specifications, autarky, autarky with common shock, trade only, and

full interdependence. Autarky refers to no trade nor spillovers in the shock processes and uncorrelated shocks. The temporarily correlated technology shocks are added to the previous model to be autarky with common shocks. For trade only model, there are no common shocks nor spillovers effects but trade in final goods and services is allowed. The common and country-specific shocks transmitted through trade and spillovers between across technology disturbances. They find that the existence of trade in consumption goods by itself does not enough to replicate the data joint fluctuation. However, the existence of common technology shocks is essential to mimic the observed data, output cross correlation is significant due to common shocks.

Schlitz (1995) analyse statistically the post-war stylized facts in Italy. The Hodrick-Prescott filter is employed to de-trend the economics time series of Italy. The paper shows that the business cycle in Italy conforms qualitatively to the general character of that in developed countries. Povoledo (2004) applies several business cycle model to the Italian economy to see whether they can match the fluctuations observed data. First, a standard real business cycle is employed and calibrated for Italy to evaluate its performance to match the Italian business cycle. Povoledo asserts that the ability of the model to capture the real-world statistics is always limited since all standard deviations are drawn from the standard deviation of single shock. Therefore, second, Italy is modified as a small open economy and introduces more than one shock, technology shocks and the world rate of return on bonds. There is a single asset which is an internationally traded bond. The rate of return on bond is exogenous. With respect to investment, the open economy model is satisfied. The volatility of investments is higher in the open economy. For the wages, the open economy performs similarly to the closed one. In general, the introduction of more than one shock can enhance the ability of the model. However, there are some flaws. First, the model fails replicate the very large volatility of hours observed in the data. Second, as the volatility of consumption is reduced, it is unavoidable for an increase volatility of hours. Finally, the standard deviation of wages is too low to capture the Italian data. Povelodo introduces both firing cost and hiring costs in the model to see if the previous model can match the fluctuation of hours worked and employment of the Italian data. This model increases the volatility of hours. However, she said that to closer match with data will require a competitive labour market.

Grohe (1998) compare international real business cycle models with two different classes of market transmission channel of the US business cycle to the Canadian economy. First, Grohe represents a single international traded good and perfect competition through financial market. Second, to analyze whether business cycle are transmitted through variations in export demand, the previous model is extended to include two goods, one produced exclusively in Canada and one supplied elasticity to Canada on the world market. For transmission through financial market only, the predicted output and employment responses were too small compared to the data for the small-large country pair Canada and the US. When allowing for transmission through variations in export demand, the impulse responses for output and hours were closer to the estimated ones, but the predicted output and employment increases still fell short of the observed response. Additionally, the predicted appreciation of the terms of trade was much stronger than the one observed and the predicted export response was wrong even in direction.

Researchers have also been considered another class of utility function to remedy the problem of the volatility of consumption that was much lower than that of the data, and the balance of trade that was procyclical instead of countercyclical. This utility is called GHH preference as it is proposed by Greenwood, Hercowitz and Huffman (1988):

$$u(C_t, 1 - N_t) = [C_t - \psi X_t N_t^\nu]^{1-\sigma} \quad (2.38)$$

where $\nu > 0$ and $\psi > 0$. The GHH preference implies that the elasticity of substitution associated with leisure is zero. With the condition of the disutility of work in the market has to increase with the level of technical progress, this utility can be consistent with steady-state growth. Correia, Neves and Rebelo (1995) use the GHH utility function to explain the business cycle in Portugal. Gail (1998) also uses the GHH utility for the German economy.

Correia, Neves and Rebelo (1995) study the small open economy, Portugal, and discuss its main features of business cycle. They construct one good-two country model related to Mendoza (1991), Backus *et al.* (1992) and Baxter and Crucini (1993). However, they consider two different momentary utility functions. One is the most common preference

specification as they refer to it as “standard preferences” and it is described by King, Plosser and Rebelo (1988). The equation (2.38) represents the GHH utility function used in Correia *et al* (1995). Their model with the momentary GHH utility is consistent with the relative variability and co-movement found in the data. In addition, like many business cycle literatures, Correia *et al.*(1995) claim that shocks to productivity are still unavoidable to create a recognizable business cycle.

Gail (1998) studies the business cycle in Germany in line with Baxter and Crucini (1995). GHH-preferences with taste shocks resulting from government consumption is used to represent the utilities of agents for two countries. Government consumption represents the shock through the possibility to be a substitute for private consumption. The cyclical component of the German data is isolated by Hodrick and Prescott filter and the model is calibrated to German data. Gail find that, for the shock process of Germany and the US, investment and hours worked are positively correlated internationally and the consumption correlation is well below unity and the output correlation is positive. However, the consumption/output anomaly cannot be solved.

Kose, Otrak and Prasad (2008) use the dynamic factor model to analyze the evolution of the degree of global cyclical interdependence over the period of 1960-2005. The data is categorized into three groups, industrial countries, emerging countries and developing economies. They concluded that there is the evidence of business cycle fluctuations among the group of industrial economies and among the group of emerging market economies. However, there is evidence of business cycle convergence within each of these two groups of countries but divergence between them.

2.6.4 The international real business cycle model for developing countries

Although most of the international real business cycle focuses on the business cycle in developed countries, the trend of doing research for developing countries is similar. The transmission channels are modified. The possible sources of aggregate fluctuation are added, particularly the world interest rate and the terms of trade, to explain the business cycle in specific countries. This is because the developing countries are in general small open economies and vulnerable to the external factors. The GHH preferences are also used to represent the utilities of agents. To analyze the open economy model for developing countries, researchers also constructed the world region model economy.

The region represents the group of developed and developing countries. Recently, a modern dynamic stochastic general equilibrium called the global economy model (GEM) developed by IMF is also used for the developing economy.

Ahn and Kim (2003) construct the dynamic general equilibrium for a small open economy and calibrate the model to capture the model features to those observed in the Korean data. There are three goods consumed in this small open economy; exportable goods, non-traded goods and importable goods. However, two goods, exportable and non-traded goods are produced. GHH-preferences are used in this paper. Agents supply labour input to perfectly competitive firms, which operate in the exportable and non-traded sectors. The capital accumulation process is dynamic in each sector and the production in the exportable in the exportable sector also uses imported intermediate goods. All labour and leisure decisions are endogenously determined in the model. The model formulates world financial markets where domestic households can buy and sell risk-free international bonds. Domestic shocks are government expenditures and changing in productivities in exportable and non-traded sectors. There are two external shocks, terms of trade and world real interest rate shocks. In relative volatility, the model produces more volatile investment and less volatile consumption than output, which replicate correctly the Korean data statistically observed. The export series from the model is not as volatile as the data, partly because GHH preference where the amount of production and export are directly determined by the amount of labour input. The model also predicts an excessively high correlation between consumption and output, and between export and output. They said that this phenomena due to the GHH preferences. The model with GHH preference generates a negative correlation between output and net exports. However, it is surprising that the Korean data shows a positive correlation between these two variables. For impulse response analysis, the aggregate variables respond in a similar manner to the productivity shocks in the exportable and non-traded sectors, the price variables respond in an absolutely opposite direction.

Neumeyer and Perri (2005) tests the role of interest rates to the business cycles in emerging economies by modifying a standard one good small open economy where the only asset traded is a non-contingent real bond. Neumeyer and Perri compare business cycles in a set of small open emerging economies, Argentina, Brazil, Mexico, Korea and Philippines, and those of a small open developed countries, Australia, Canada,

Netherlands, New Zealand and Sweden. They document that many features of business cycles are similar in the two sets of economies, but that there are some strikingly differences. Even though the real interest rate is countercyclical for the two groups of countries, it leads the business cycle in emerging countries but lag in developed countries. Also emerging countries performs relatively high output volatility to developed countries. They modify a standard real business cycle in two ways. First, firms have to pay for part of the factors of production before production takes place, creating a need for working capital. Second, they consider preferences which generate labour supply that is independent of consumption. To explore the role of interest rate as in driving business cycles, they calibrate their model to Argentina's economy for the period 1983-2001. They assume that a large mass of international investors is willing to lend to the emerging economies. Loans to the domestic economy are risky assets because they assume that there can be default on payments to foreigners. These assumptions create the sources of volatility in interest rate and they built the model to capture this particular feature. Neumeyer and Perri summarize that real interest rates are induced by fundamental shocks and amplify the effect on business cycles, contributing to the high volatility. They find that eliminating default risk in emerging economies can reduce about 27% of their output volatility.

Cicco, Pancrazi and Uribe (2006) investigate the performance of the model for Argentina. They show that the model fails to explain the number of important aspects of emerging business cycles, including volatilities of output, consumption, investment and trade balance.

However, it is feasible, and desirable, to construct particular versions of these models for application to particular countries. Canova (1993) has constructed a three-region world which he has parameterised to correspond to the USA, Germany, and Japan. He focuses on the understanding of mechanisms by which various shocks are transmitted across countries. Kouparitsas (2001) constructs a two region world, in which one region, the North, imports raw materials and export manufactured goods. The second region, the South, exports raw materials and imports manufactured goods. The North is parameterised to resemble major industrial countries, while the South is parameterised to correspond to non-oil, commodity-exporting, developing countries. Kouparitsas examines the extent to which fluctuations in the Northern economy cause fluctuations the

terms of trade and induce business cycles in the South economy. He concluded that the model contains a strong mechanism for the transmission of business cycles from one region to another. The model suggests that fluctuations in the Northern aggregate output account for about 70 percent of the variation in Southern consumption.

Elekdag and Tchakarov (2006) employ the Global Economy Model (GEM) developed by IMF to study the role of interest rates in business cycle fluctuation in Thailand. The GEM is a modern dynamic stochastic general equilibrium (DSGE) model. The standard version of GEM is augmented by including balance sheet related credit market frictions by incorporating a financial accelerator mechanism. The paper shows that the Thai monetary authorities may best respond to the foreign interest rate shocks. They suggest that the fully flexible exchange rate regime stabilizes real variable best. The depreciation of the exchange rate creates expenditure switching effects that dominate the balance sheet effects. They also recommend that a flexible exchange rate regime is better to manage the challenges of an external environment characterized by rising global interest rate.

Even though a number of papers have analyzed the business cycle properties of developed countries using closed economy model and open economy models, not many papers have analyzed the business cycle in developing countries using dynamic stochastic general equilibrium and in Thailand particularly. The main purpose of this thesis is to test how well the business cycle models capture the facts of Thai economy.

2.7 Conclusion

This chapter presents the development of the business cycle model. The real business cycle model is laid by the foundation of the growth model. To construct the business cycle model, two important things include to the growth model which are the variable of labour input and the driving process. The famous model by Kydland and Prescott (1982) and the basic real business cycle framework are discussed in this chapter. In the basic model, all variables exhibit a strong correlation with output. In general consumption volatility less fluctuate than does output, while investment fluctuate much higher than does output. The drawback of the standard model is the labour implication. This chapter also reviews the extension of the basic model to cope with that flaw by using the

indivisible labour framework. Other driving variables, such as monetary and fiscal policy shocks, rather than technology are discussed in this chapter as well. The models with other shocks allow us to analyse the effect of monetary and fiscal policy on the economic fluctuation.

To completely analyse the business cycle model, the international real business cycle model or open economy model are presented in this chapter. The benchmark international real business cycle model by Backus, Kehoe and Kydland (1992) basically extends the closed economy model by Kydland and Prescott (1982). Backus *et al.* (1992) introduce one good-two country model with complete market. The shortcoming of this model is that consumptions are strongly correlated across countries than does output. They call this flaw, the consumption/output anomaly. Therefore, the models of Backus *et al.* (1994, 1995), two goods-two country version, are constructed to deal with the problem. The model mimics either consumption correlation or output correlation in the data, but not the two together. The problem of the consumption/output anomaly still exists. For the two goods-two countries model, term of trade fluctuates substantially less than that of data. They refer to price anomaly. A number of papers in the international real business cycle involve adding more complexity in the benchmark models to fix consumption/output and term of trade anomalies.

Chapter 3

Filter and Thai facts

3.1 Introduction

Generally, the macroeconomics time series are composed of a cyclical and a growth component.¹ The time series usually display a cycle around its growth trend. The cyclical part of the series is often referred to as business cycle component. To measure the main features of business cycle, the series must be de-trended and this usually achieved using a filter. The process of de-trended is what we usually call filter. To achieve the goal of this chapter, the Thai economics time series are filtered to eliminate of the trends and identify the properties of Thai business cycle. The filters are used both to real Thai data and to the theoretical economy later in chapter 5 and chapter 6 because the performance of the model is measured by the comparison of the model with data moments. This chapter describes the filter issues, both in time and frequency domains, and the characteristics of the Thai business cycle. More importantly, it reports the properties of business cycles between Thai, Asian countries and the major developed countries.

3.2 Filters

The non-stationary time series cannot yield their second moments. Therefore, the time series must be changed to a stationary series before measuring the features of the business cycle.² Since many macro-economics time series are non-stationary, it is necessary to get rid of the non-stationary movement. Basically, removing the non-stationary time series is achieved by removing the trends. A trend is the long-term movement that represents the growth or decline of a variable over time. The objective of the real business cycle model is to explain the business cycle frequency, not the long-run growth or low frequency data. Therefore, the macro-economics time series have to be

¹ There is in fact also a seasonal component, but as the data are seasonally adjusted, this component has already been removed by those preparing the data series.

² As in Hamilton (1994), the process for y_t is stationary if its probability distribution does not change over time, that is the joint distribution does not depend on t . Typically, a stationary series has a characteristic of constant mean and variance over time. Nelson and Plosser (1982) provide the evidence supporting the existence of an autoregressive unit root in a variety of the U.S. macroeconomic time series.

eliminated a very slow moving component. There are broadly two ways to extract the trends from the time series. One approach is what so called filters, which are employed to the time series to remove trends. Hodrick and Prescott (1980) establish the method, which is commonly known as HP filter, to separate the low frequency movements. The use of HP filter in the empirical data has grown dramatically in recent years. Baxter and King (1999) propose the band-pass filter based on the definition of business cycle by Burns and Mitchell (1946).³ Also, one traditional approach to separate trends is to apply the first difference to time series. Another way of separating the trends is curve fitting, like splines. The idea is to fit some functions, which have a capability to mimic the trends, to the series. When the appropriate function is fitted to the series, it may be use to forecast the trends. In fact, it is quite difficult and complicated to derive the function to replicate the trends. Therefore, this alternative is not widely used by economists. The economists rather separate growth and cyclical component of an observable time series by using a filter.

Filters are used to extract the trends from the time series in this chapter. For measuring and comparing business cycle from the actual and model economy, the real Thai and model data are filtered by taking logarithms and de-trended with the filters. Since the filter heavily relies on the concept of time series analysis, before describing a property of a filter, it is important to discuss some useful concepts in time series analysis.

3.2.1 Time and frequency domain

Macroeconomics time series is broadly viewed in two ways, a time and a frequency domain. Generally, the time domain approach views a time series as discrete while the frequency domain admits to the time series having a frequency.⁴ A series x_t , which can be characterized as either an auto-regressive or moving average process, is described as a

sequences of innovations $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ as $x_t = \mu + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$. The time domain properties of

x_t are described by $x_t = \mu + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$ in term of expectations, variances and auto-

³ Burns and Mitchell estimate the business cycle by excluding all data before the 6th quarter and after the 32th quarter. So the length of business cycle is no more than 26 quarter in length.

⁴ The boarder definition can be found in Hayashi (2002).

covariance. A time series can be also analyzed in the frequency domain in which x_t can be regarded as consisting of a weighted sum of periodic components shown below:

$$x_t = \mu + \int_0^\pi \alpha(\omega) \cos(\omega t) d\omega + \int_0^\pi \delta(\omega) \sin(\omega t) d\omega \quad (3.1)$$

Therefore, any time series may be represented in the frequency domain, as sum of an infinite number of sines and cosines whereas in the time domain a time series may be regarded as a process with an infinite number of auto-covariances.⁵ Specifically, the time domain approach concentrates on the auto-covariance generating function whereas the frequency domain focuses on the spectral density.⁶ These two methods are somehow related to each other. Any features of the data that can be explained by one way can be similarly well explained by the other. However, the time domain representation might be simpler for some features, while for some features the frequency domain description may be easier.

3.2.2 The auto-covariance generating function and spectrum density

In time series, value of x_t in one period typically is correlated with its own value in the next period. The correlation with its own lagged values is called the autocorrelation or serial correlation. Therefore, the first order autocorrelation is the correlation between x_t and x_{t-1} .⁷ Similar argument with auto-covariance, the j^{th} auto-covariance of a series x_t is the covariance between x_t and its j^{th} lag, x_{t-j} . If $x_t = \mu + \varepsilon_t$ and ε_t is iid with $\varepsilon_t \sim (0, \sigma^2)$, then the j^{th} auto-covariance of a series x_t can be written as:

$$r_j = \text{cov}(x_t, x_{t-j}) = E(x_t - \mu)(x_{t-j} - \mu) \quad \text{for } \forall t \quad (3.2)$$

x_t is covariance stationary since its mean, $E(x_t) = \mu$, and its auto-covariance, $r_j = E(\varepsilon_t \varepsilon_{t-j})$ is independent to time.

⁵ See also Chartfield (2004) and Hamilton (1994).

⁶ The spectrum representation is shown on the Appendices, Section A.

⁷ The covariance and correlation definition can also be found in Gallant (1997).

For a covariance stationary process, it can be shown that r_j and r_{-j} would exhibit the same value.⁸

The entire set of auto-covariance of a covariance stationary process x_t can be described by the auto-covariance generating function $g_x(z)$. If the series $\{\gamma_j\}_{j=-\infty}^{\infty}$ has an absolute sum auto-covariance or $\sum_{j=0}^{\infty} |\gamma_j| < \infty$, then one way of representing it is through a scalar value:

$$g_x(z) = \sum_{j=-\infty}^{\infty} \gamma_j z^j = \sum_{j=1}^{\infty} \gamma_j (z^j + z^{-j}) \quad (3.3)$$

where z is a complex scalar.⁹ If the function $g_x(z)$ is transformed by dividing it by 2π and setting z in polar coordinate form as $z = e^{-i\omega} = \cos(\omega) - i \sin(\omega)$, we will have a spectral density.¹⁰ The spectral density function or power spectrum of x_t can be given by:

$$\begin{aligned} S_x(\omega) &= \frac{1}{2\pi} g_x(e^{-i\omega}) \\ &= \frac{1}{2\pi} g_x[\cos(\omega) - i \sin(\omega)] \\ &= \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j e^{-i\omega j} \quad \text{where } -\pi < \omega \leq \pi \end{aligned} \quad (3.4)$$

Since the data must be filtered or treated in a particular way before analyzing it, it is possible to describe the effects of this process on the auto-covariances. Due to the fact that the population spectrum and the sequence of auto-covariances contain the same

⁸ $r_j = E(x_{t+j} - \mu)(x_{t+j-j} - \mu) = E(x_{t+j} - \mu)(x_t - \mu) = E(x_t - \mu)(x_{t+j} - \mu) = r_{-j}$

⁹ $z = a + bi$ and the modulus of a complex number, $|z| = \sqrt{z\bar{z}} = \sqrt{a^2 + b^2} = 1$, where $\bar{z} = a - bi$, a is a real part and b is an imaginary part of z .

¹⁰ The details in polar coordinate can be found in Rade and Westergren (2004).

information, we can analyze the effects of the filter on spectrum density as well.¹¹ Let y_t be a filtered time series which is generated by applying the filter, $h(L)$, to a time series $x_t = \mu + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$.

It follows that:

$$y_t = h(L)x_t \quad \text{where} \quad h(L) = \sum_{j=-\infty}^{\infty} h_j L^j \quad (3.5)$$

Therefore:

$$\begin{aligned} y_t &= h(L)[\mu + \psi(L)\varepsilon_t] \\ &= h(L)\mu + h(L)\psi(L)\varepsilon_t \\ &= \mu^* + \psi^*(L)\varepsilon_t \end{aligned} \quad (3.6)$$

Since $g_y(z) = \sigma^2 \psi^*(z) \psi^*(z^{-1})$ and $g_x(z) = \sigma^2 \psi(z) \psi(z^{-1})$, $g_y(z) = h(z)h(z^{-1})g_x(z)$, where $h(z)h(z^{-1})$ is transfer function of the filter $h(L)$. Because the auto-covariance generating function can be easily altered to spectrum density, the transfer function is applied to the spectrum density of the original time series to generate a new spectrum density of a new filtered series in frequency domain. Consequently, to filter the model economy, we apply the transfer function of each filter to the spectrum density of a data generated by the model. Afterward, the population moments of the model economy can be derived from a new spectrum density for a filtered series. Next we turn our attention to different kinds of filter.

3.2.3 The first difference filter

Economists usually apply the logarithms to a time series to eliminate a linear trend. Nonetheless, a problem for this approach is that a stochastic trend component or unit root cannot be removed. Economists solve that problem by simply introducing the first difference approach. The difference in the value of x between t and $t-1$ is formally called the first difference. It is convenient to use the lag operator, where $L^j x_t = x_{t-j}$, to show the filtered time series, y_t :

¹¹ See the Appendices, Section A.3 for details.

$$y_t = (1 - L)x_t \quad (3.7)$$

It can be easily shown that the transfer function of the first difference filter is $(1 - z)(1 - z^{-1})$. Figure 3.1 A depicts the first difference transfer function. Many economic time series displays growth that is approximately exponential. One way to eliminate the growth component is to take logarithms difference. Therefore, economic time series are simply filtered by the first difference approach. In this case, y_t is assumed to be a cyclical movement of a time series. Although the first difference can remove a stochastic trend, there are several problems involved to this filter. One major problem is that there is a phase shift in the filtered series because this filter is not symmetric. Another problem is that this filter cannot separate the cyclical variation within a particular band frequency.

3.2.4 The moving average filter

Economists use a simple smoothing technique called the moving average to help estimate the underlying trends. The moving average is essentially a linear transformation to the original time series. To eliminate trends, we apply a moving average to a time series x_t , obtaining a time series y_t :

$$y_t = \sum_{j=-J}^J b_j x_{t-j} \quad (3.8)$$

The moving average, in fact, takes a weighted average of successive value of an interesting series. The weighted average is represented by $\sum_{j=-J}^J b_j$ and it is again convenient to use the lag operator to express the weighted average. Let:

$$b(L) = b_{-J}L^{-J} + b_{-J+1}L^{-J+1} + \dots + b_{J-1}L^{J-1} + b_J L^J = \sum_{j=-J}^J b_j L^j \quad (3.9)$$

We can express the new cyclical time series as $b(L)x_t = \sum_{j=-J}^J b_j x_{t-j}$. $b(L)x_t$ is a linear combination of present and past values of x .¹² This moving average serves to isolate trends if it is symmetric, $b_{-j} = b_j$ for $j=1,2,\dots,J$. The symmetric moving average is sometimes called two sided moving average.¹³ Since this filter is symmetric, there is no phase shift in the filtered time series. To keep it simple, this study applied a 3 point moving average in which $J=1$. For a 3 point moving average, its transfer function can be represented by $\frac{1}{9}[1+2\cos(\omega)]^2$ and this transfer function is illustrated in figure 3.2 B.

3.2.5 The HP filter

Hodrick and Prescott (1980) develop the HP filter to estimate a slow moving component of a time series. The data used in that research is seasonally adjusted; therefore, an observable time series, y_t , is composed of two main parts, cyclical component, y_t^c , and growth component, y_t^g :

$$y_t = y_t^c + y_t^g \quad (3.10)$$

y_t is a stationary observable time series and it may be interpreted as noise from the non-stationary time trend, y_t^g . y_t^c and y_t^g are unobservable time series. To decompose an observable time series into a trend and a stationary component, the filter allocates some weight to a linear time trend. Specifically, the filter chooses the optimal growth component by solving the minimization problem below:

¹² $b(L)$ can also be represented as the sum of a geometric series.

That is $b(L) = 1/(1-\lambda L) = 1 + \lambda L + \lambda^2 L^2 + \dots$

Therefore, $b(L)x_t = 1/(1-\lambda L)x_t = x_t + \lambda x_{t-1} + \lambda^2 x_{t-2} + \dots = \sum_{s=0}^{\infty} \lambda^s x_{t-s}$

Because the new sequence is created with past values, the operator $(1-\lambda L)^{-1}$ with $|\lambda| < 1$ is typically called the backward expansion.

¹³ Another way to express a moving average is as follow $y_t = 1/2J+1 \sum_{j=-J}^J x_{t-j}$ where y_t is the estimated growth component of the original series, x_t .

$$\text{Min}_{\{y_t^g\}_{t=1}^T} \left\{ \sum_{t=1}^T (y_t - y_t^g)^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2 \right\} \quad (3.11)$$

The measure of the smoothness of $\{y_t^g\}$ path is the sum of the squares of its second difference, representing a linear time trend. The trends have to be a smooth function while the trend components have to mimic the real data. The parameter, λ , plays an essential role in the cost and benefit of choosing the trend.¹⁴ The larger the smoothing parameter, the more and more weight is allocated to a linear time trend and the smoother is the solution series. In the extreme case, if $\lambda \rightarrow \infty$, it is optimal to assume that $(y_{t+1}^g - y_t^g) = (y_t^g - y_{t-1}^g)$ which arbitrarily near some constant number (β). Therefore the growth component acts as a linear time trend or $y_t^g = y_0^g + \beta t$. In contrast, if $\lambda \rightarrow 0$, $y_t = y_t^g$. In this unusual case, the trend component becomes the original series or the cyclical component does not exist.¹⁵

By taking the derivative of the minimization problem with respect to $\{y_t^g\}_{t=1}^T$ and rearrange them, the first order necessary conditions can be written in the matrix form as:

$$y_{T \times 1}^c = \lambda P_{T \times T} y_{T \times 1}^g \quad (3.12)$$

where P is the co-efficient matrix with $T \times T$ dimension¹⁶. By the construction of matrix P , the summation of each column becomes zero, and consequently, the summation of the estimated cyclical component equals to zero as well ($\sum_{t=1}^T y_t^c = 0$). We

can solve for the trend component of an observable time series as below:

$$\begin{aligned} \lambda P_{T \times T} y_{T \times 1}^g &= y_{T \times 1} - y_{T \times 1}^g \\ y_{T \times 1}^g &= (\lambda P_{T \times T} + I_{T \times T})^{-1} y_{T \times 1} \end{aligned} \quad (3.13)$$

¹⁴ Hodrick and Prescott show that under some conditions the best choice of λ is driven by the relative variances of y_t^c and the second difference of y_t^g .

¹⁵ The treatment in this issue can also be found in Enders (2004).

¹⁶ The details of the first order condition matrices are illustrated in the Appendices, Section A.1.

The value of λ involves some way to the goal of filtering out the business cycle fluctuations. The frequency of the data dictates the value of λ ; for quarter data, $\lambda = 1600$ is widely used which remove the frequencies of roughly 32 quarters or longer. The value of λ are 100 and 14400 for annual and monthly data respectively. The HP filter can remove unit root or stochastic growth components from the data. The filtered time series does not exhibit a phase shift since this filter is symmetric. Prescott (1986) also claims that the HP filter has almost the same characteristics as a high pass filter where the high frequency can pass through. The HP filter eliminates the periodicities lower than 32 quarters while retaining those higher than 32 quarters.

King and Rebelo (1993) have shown that for the HP filter the cyclical component, y_t^c , of a time series y_t is $y_t^c = \left[\frac{\lambda(1-L)^2(1-L^{-1})}{1 + \lambda(1-L)^2(1-L^{-1})^2} \right] y_t$ and the transfer function of this filter in the Fourier transform is $\hat{C}(\omega) = \frac{4\lambda[1 - \cos(\omega)]^2}{1 + 4\lambda[1 - \cos(\omega)]^2}$. Therefore, given the fact that $\cos(0) = 1$ and $\cos(\pi) = -1$, the HP filter places zero weight on the zero frequency or $\hat{C}(0) = 0$, while places near unit weight on high frequency or $\hat{C}(\pi) = \frac{16\lambda}{1 + 16\lambda}$. Graphs of a transfer function of the HP filter in different value of smoothing parameter, λ , are shown in figure 3.1 C, D and E.

3.2.6 The Band pass filter

Baxter and King (1999) introduce the band pass filter to separate a specific frequency from a time series. The band pass filter is fundamentally constructed on the concept of the moving average. This section heavily borrows from Baxter and King (1999).

This alternative filter has a trend-reducing properties and no phase shift in the filtered output time series. The general idea of the band pass filter is to apply the moving average to extract a periodic component of a time series within a particular frequency band. The band pass filter removes a slow moving and a very high-frequency component of a time series while retaining the frequency component in the between. The intermediate frequency is referred to as business cycle. Based on the study of Burns and Mitchell (1946), the business cycle has a periodicity no less than 6 quarters and no more

than 32 quarters. The band pass filter is constructed to pass through the cyclical component between six and thirty two quarters. This filter is built by the combination of low and high pass filters. A low pass filter removes a high frequency but passes through a low frequency components. On the other hand, a high pass filter eliminates a low frequency but retains only a high frequency component.

The frequency domain or spectral analysis provides a rigorous foundation to the band pass filtering. In spectral analysis, the time series are viewed as a weighted sum of periodic functions of cosine and sine of a given frequency. Granger and Hatanaka (1964) suggest that spectral analysis essentially decomposes a stationary series into a number of uncorrelated components, each associated with different frequency. To understand the effect of the band pass filter to a time series, we begin with the covariance-stationary spectral representation of a time series.

$$x_t = \mu + \int_0^\pi \alpha(\omega) \cos(\omega t) d\omega + \int_0^\pi \delta(\omega) \sin(\omega t) d\omega = \mu + \int_{-\pi}^\pi \xi(\omega) d\omega \quad (3.14)$$

For zero mean at frequency ω , we can write $x_t = \int_{-\pi}^\pi \xi(\omega) d\omega$ $\xi(\omega)$ are mutually orthogonal, so $E[\xi(\omega_i)\xi(\omega_j)'] = 0$, where $\omega_i \neq \omega_j$. It can be shown that variance of x_t is $\gamma_0 = \text{var}(x_t) = \int_{-\pi}^\pi S_y(\omega) d\omega$. $S_y(\omega)$ is the variance of the ω -frequency component of x_t and is formally called the spectral density function of x_t . Given the fact that $\text{var}(x_t)$ corresponds to the area under the spectrum and it is equivalent to the integral of the square of $\xi(\omega)$ which is uncorrelated at different frequency, it implies that the variance of a time series can be decomposed to the contribution of the different frequency. Furthermore, we know that the high frequencies are associated with short-term variation in x_t because of $\text{period} = \frac{2\pi}{\omega}$.

A series y_t formed by filtering the original series x_t can be expressed as:

$$y_t = \int_{-\pi}^\pi \alpha(\omega) \xi(\omega) d\omega. \quad (3.15)$$

$\alpha(\omega)$ is the frequency domain of representation of a filter. The filter in frequency domain is formally known as the frequency response function. Therefore, $\alpha(\omega)$ is the frequency responds function of one specific filter and it indicates the extent to which y_t response to x_t at frequency ω . The frequency response function $\alpha(\omega)$ of a linear system is a complex function and may be written in the Fourier form $\alpha(\omega) = \sum_{j=-\infty}^{\infty} b_j e^{i\omega j} = G(\omega) e^{i\phi(\omega)}$ where $G(\omega)$ and $\phi(\omega)$ are gain and phase respectively¹⁷.

To have trend elimination properties, the band pass filter has the sum of the filter weight is zero or $\sum_{j=-\infty}^{\infty} b_j = 0$ which exist as long as $\alpha(0) = 0$. Therefore, with $\alpha(0) = 0$ and the frequency domain implication of symmetry $\alpha(\omega) = \alpha(-\omega)$, the band pass filter can be applied to a non-stationary economics time series.

By using the concept of the Fourier transformation, the filter and its frequency response function are a Fourier transform pair:

$$b(L) = \sum_{j=-\infty}^{\infty} b_j L^j \quad \dots \text{filter} \quad (3.16)$$

$$\alpha(\omega) = \sum_{j=-\infty}^{\infty} b_j e^{i\omega j} \quad \dots \text{frequency response function} \quad (3.17)$$

The weight b_j can be obtained from the frequency response function by the Fourier integral $b_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} \alpha(\omega) e^{i\omega j} d\omega$.

Low pass filter is a trend estimator. It completely eliminates all the high frequency variation and passes through only low frequency movement of time series x_t . The low

¹⁷ The function indicating the amplitude of the wave is called the gain of the system and the function called the phase shift displays the movement of the wave. Let $\overline{\alpha(\omega)}$ be a complex conjugate of $\alpha(\omega)$. The gain $G(\omega)$, which is equivalent to $|\alpha(\omega)|$, may be calculated via $G^2(\omega) = |\alpha(\omega)|^2 = \alpha(\omega) \overline{\alpha(\omega)}$.

pass filter passes component of the data with periodicity more than or equal to 32 quarters. In this specific case, the frequency response function $\alpha(\omega)$ performs such that:

$$\alpha(\omega) = \begin{cases} 1 & \text{if } \omega \in [-\underline{\omega}, \underline{\omega}] \\ 0 & \text{otherwise} \end{cases} \quad (3.18)$$

It follows that:

$$\begin{aligned} b_0 &= \frac{1}{2\pi} \int_{-\underline{\omega}}^{\underline{\omega}} d\omega = \frac{\underline{\omega}}{\pi} \\ b_j &= \frac{1}{2\pi} \int_{-\underline{\omega}}^{\underline{\omega}} \alpha(\omega) e^{i\omega j} d\omega = \frac{1}{2\pi} \frac{1}{ij} [e^{i\underline{\omega}j} - e^{-i\underline{\omega}j}] \\ &= \frac{1}{\pi j} \sin(\underline{\omega}j) \quad \text{for } j = 1, 2, \dots \end{aligned} \quad (3.19)$$

For the finite order filter, $b(L) = \sum_{j=-J}^J b_j L^j$, the frequency response function is

$$\alpha_K(\omega) = \sum_{j=-J}^J b_j e^{i\omega j} . \quad \text{The idea for finite filter is to minimize the loss function, } \delta(\omega), \text{ by}$$

choosing the approximating weights.¹⁸ The loss function represents the difference between infinite and finite frequency response function, $\delta(\omega) = \alpha(\omega) - \alpha_K(\omega)$. The optimal approximating filter for J lag length is built by truncating the infinite filter weight at lag J . Baxter and King suggest that, for quarterly macroeconomic data, the truncating point J should equal 12 in order to retain the major business cycle movements which are not lower than 6 and higher than 32 quarters. For annual data, J should equal to 3 and the filter retains cycle in between 2 and 8 years.

High pass filter is a trend eliminator. It completely removes low frequency component of a time series. The high-pass filter passes through components of the data with periodicity less than or equal to 32 quarters. Since a time series can be viewed as the mixture of low and high frequency components, the high pass filter typically can be constructed from low pass filter. Generally speaking, the high pass filter equal to one

¹⁸ Baxter and King (1999) have shown that the approximation is to select the weights b_j to minimize

$$L = \frac{1}{2\pi} \int_{-\pi}^{\pi} |\delta(\omega)|^2 d\omega .$$

minus the finite low pass filter. Mathematically speaking, the high pass filter weight equal to $1 - b_0$ where $j = 0$ and equal to $-b_j$ where $j \in I$.

The band pass filter is constructed from two low pass filters with two different cutoff frequencies, $\underline{\omega}$ with frequency response function $\underline{\alpha}(\omega)$ and $\bar{\omega}$ with frequency response function $\bar{\alpha}(\omega)$. The frequency response functions of the band pass filter are built on the frequencies which lie in a particular frequency band, $\underline{\omega} \leq |\omega| \leq \bar{\omega}$ and zero elsewhere. To have a desired frequency response equal to 1 within $\underline{\omega} \leq |\omega| \leq \bar{\omega}$ and zero elsewhere, the frequency response function of band pass filter must be $\underline{\alpha}(\omega) - \bar{\alpha}(\omega)$. The filter weight of a band pass filter is also derived from the filter weights of the two low pass filters. If \underline{b}_j and \bar{b}_j are the filter weights of the low pass filter with cutoff frequency $\underline{\omega}$ and $\bar{\omega}$ respectively, the filter weight of a band pass filter can be expressed as $\underline{b}_j - \bar{b}_j$. Corresponding to Bruns and Mitchell (1964), figure 3.1 F depicts the transfer function of the band pass filter with two cutoff frequencies, $\underline{\omega} = 2\pi/32$ and $\bar{\omega} = 2\pi/6$. This band pass filter admits frequency components between 6 and 32 quarters. The band pass filter has several desirable properties for studying business cycle. It can eliminate trends, produce no phase shift and pass through a frequency in a particular band.

Figure 3.1 Transfer function of different filters

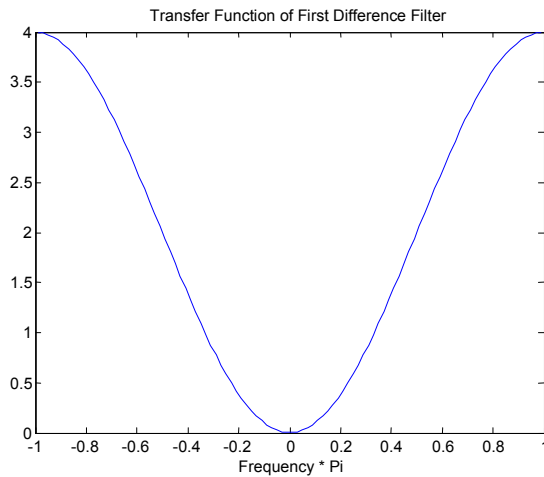


Figure 3.1 A

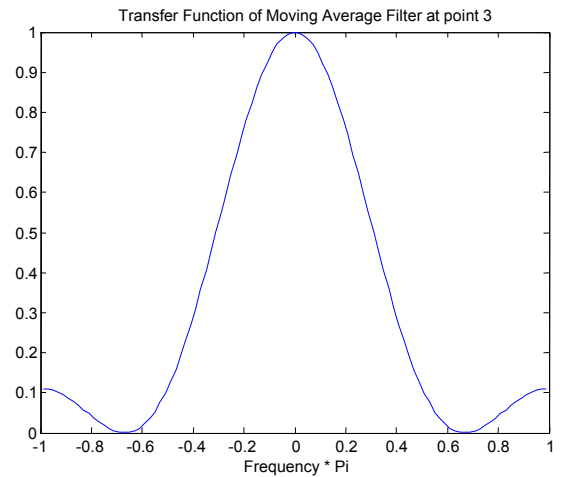


Figure 3.1 B

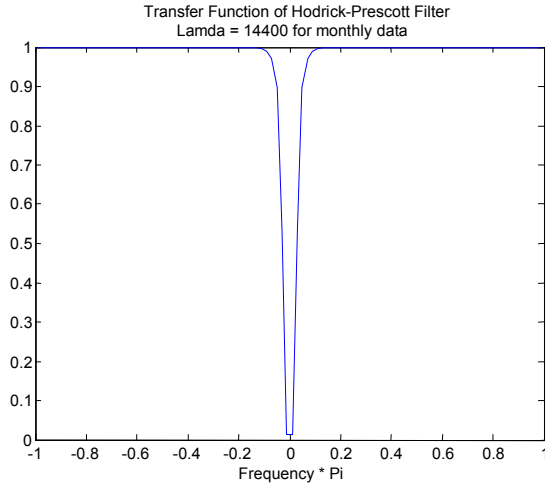


Figure 3.1 C

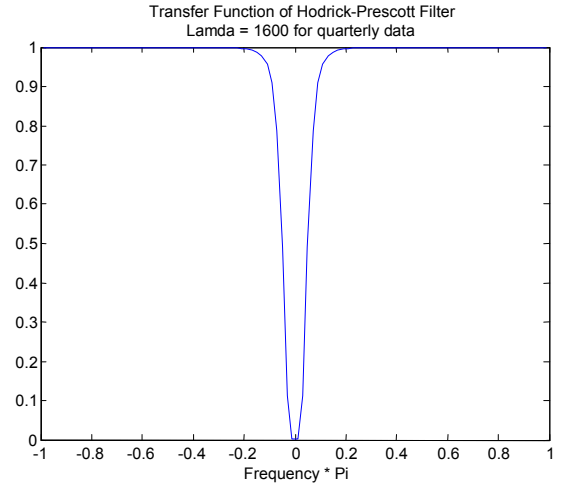


Figure 3.1 D

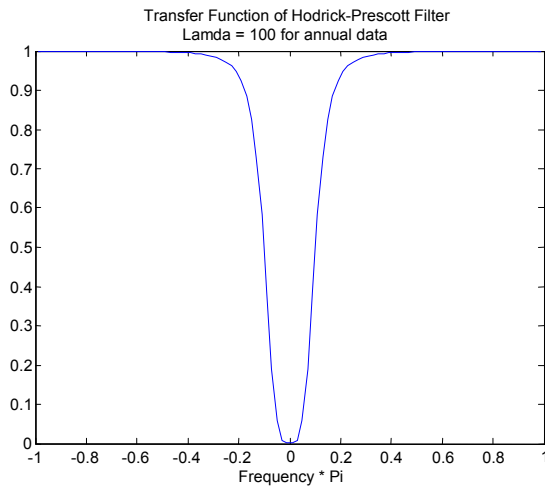


Figure 3.1 E

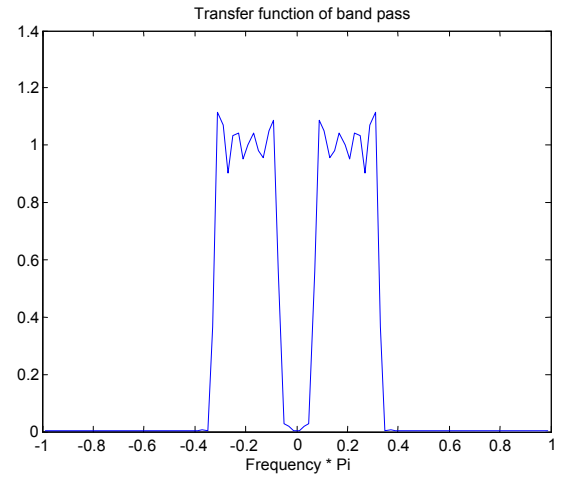


Figure 3.1 F

3.2.7 A gain in the filters

The property of the ideal filter follows Burn and Mitchell's business cycle definition in which the business cycles are the component with more than 6 quarters and less than 32 quarters. Therefore, the transfer function of the ideal filter has two cutoff frequencies, $2\pi/32$ and $2\pi/6$. The frequency response function of ideal filter is one in the period between 6 and 32 quarters and zero elsewhere. Figure 3.2 depicts the frequency response function of ideal filter which is shown by the dash lines. If the transfer function of a filter mimics the frequency response function of the ideal filter, that filter has high gain. Good filter minimizes the deviation of its transfer function and that of ideal filter. The gains of various filters are illustrated in Figure 3.2.

Figure 3.2 Transfer functions of different filters

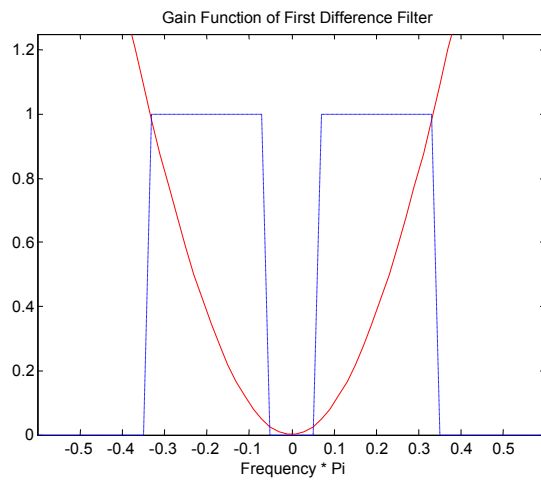


Figure 3.2 A

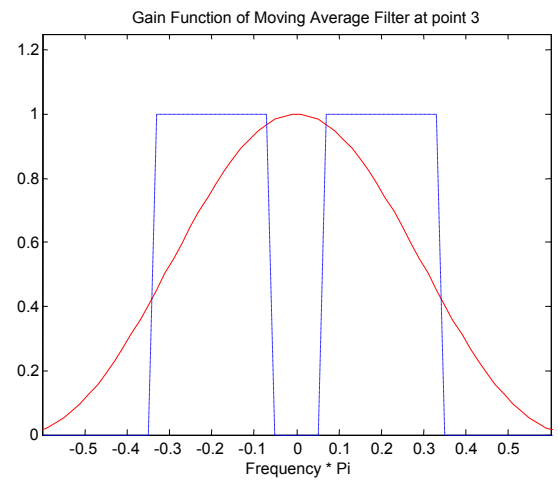


Figure 3.2 B

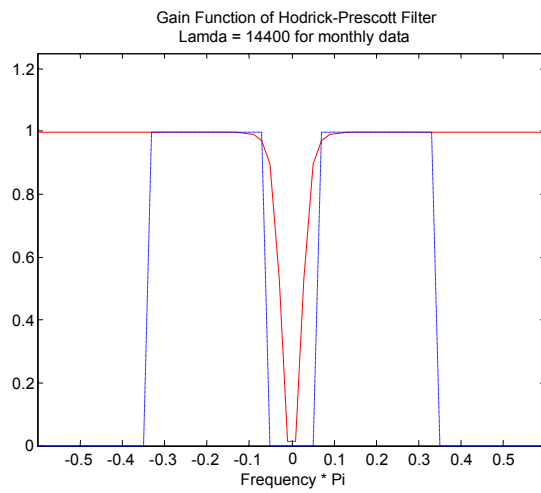


Figure 3.2 C

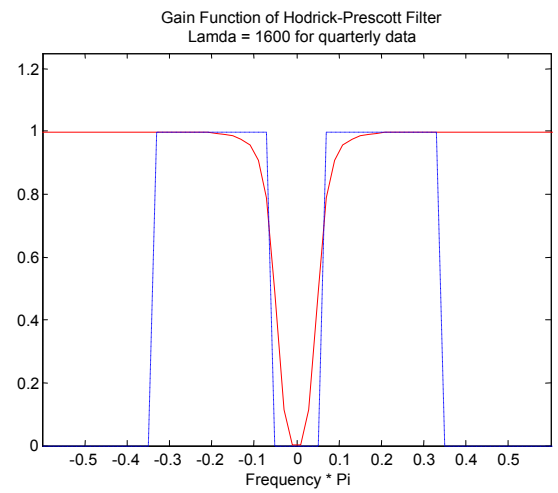


Figure 3.2 D

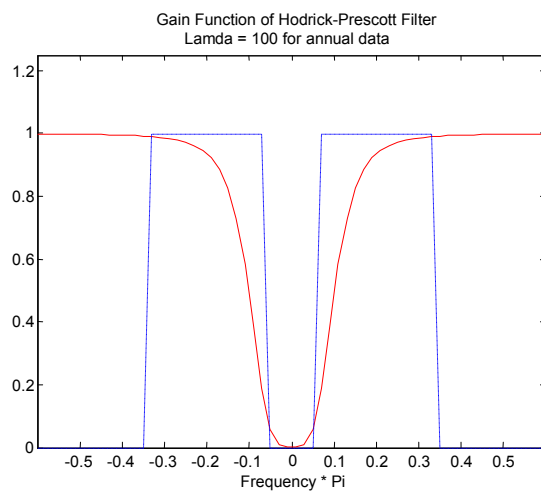


Figure 3.2 E

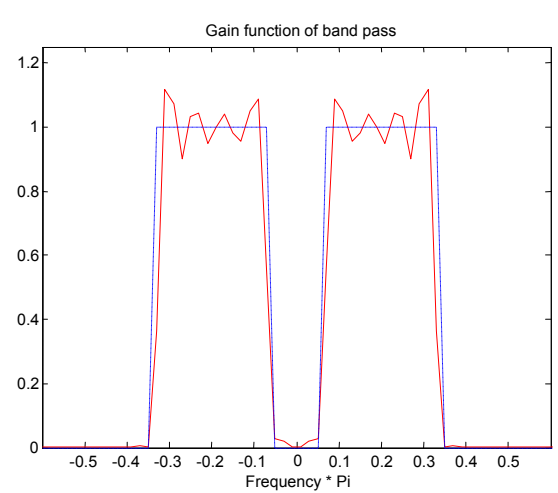


Figure 3.2 F

Obviously, the first difference and moving average has a low gain. These two filters are unable to capture the ideal filter. Among three value of smoothing parameters, the HP filter with a smoothing parameter 1600 behaves approximately as the high pass filter with cutoff period 6 quarters. However, the HP filter cannot describe the low pass ideal filter. The band pass filter has high potential to explain the ideal filter and, in turn, it has high gain.

3.3 The business cycles regularities of the Thai, Asian and developed countries

This chapter has documented how to measure business cycles. One of the main objectives of this chapter is to study the cyclical component of the filtered time series. Next the key business cycle regularities of Thailand, some of the Asian countries and developed countries will be investigated. This study also uses a regular practice in the business cycle literature by decomposing a time series into a trend and cyclical movement. Since it is quite difficult to have the quarterly data for a long period for many emerging economies, the annual data are used and they are obtained from the World Development Indicator published by the World Bank. To be consistent, all data are obtained only from one source, the World Bank. The range of data starts from 1965 and ends at 2004. The countries in this section can be broadly divided into two groups, Asian countries and developed countries. Asian countries consist of Thailand, Korea, Malaysia, Philippines, Indonesia and Singapore. Developed countries are USA, EU and Japan. The EU in this chapter means the 14 European Monetary Union.¹⁹ For Singapore, the World Bank data base does not provide the export and import variables. The trade variables of Singapore are obtained from the IMF-IFS. There are also few exceptions for investment variables for Indonesia and the EU. The range of investment variables for Indonesia starts from 1979 and for the EU from 1971. All data are real and seasonally adjusted. Base years vary for different countries.

The empirical data are divided into two time periods, 1965-1990 and 1991-2004. This two time periods are in particular interest to examine the main features of business cycles because in 1965-1990, there existed the global economic expansion in the 1960s, the first oil crisis in 1973, the second oil shocks in 1980 and the debt crisis in the early 1980s.

¹⁹ European Monetary Union aggregate include Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain.

During 1991-2004, it was the period of war such as Gulf war in the early 1990s and Iraq war in the early 2000s. The Asian financial crisis also originated from Thailand in July 1997. In 1997, Thai currency had been devalued more than 20% against the US dollar. To analyze the cyclical properties, the HP and Band pass filter are used for the data. For annual frequency, the HP smoothing parameter is 100. The band pass filter has a cut off periodicity at 2 and 8 quarter with truncated point at 3.

3.3.1 Decomposition of trend and business cycle of Thailand

Figure 3.3, the corresponding percentage deviations from trend of each variable for Thailand, have been demonstrated by three different kinds of filters, the first difference, the HP and the band pass filter.²⁰

Figure 3.3 Percentage deviation from trend of Thai real variables

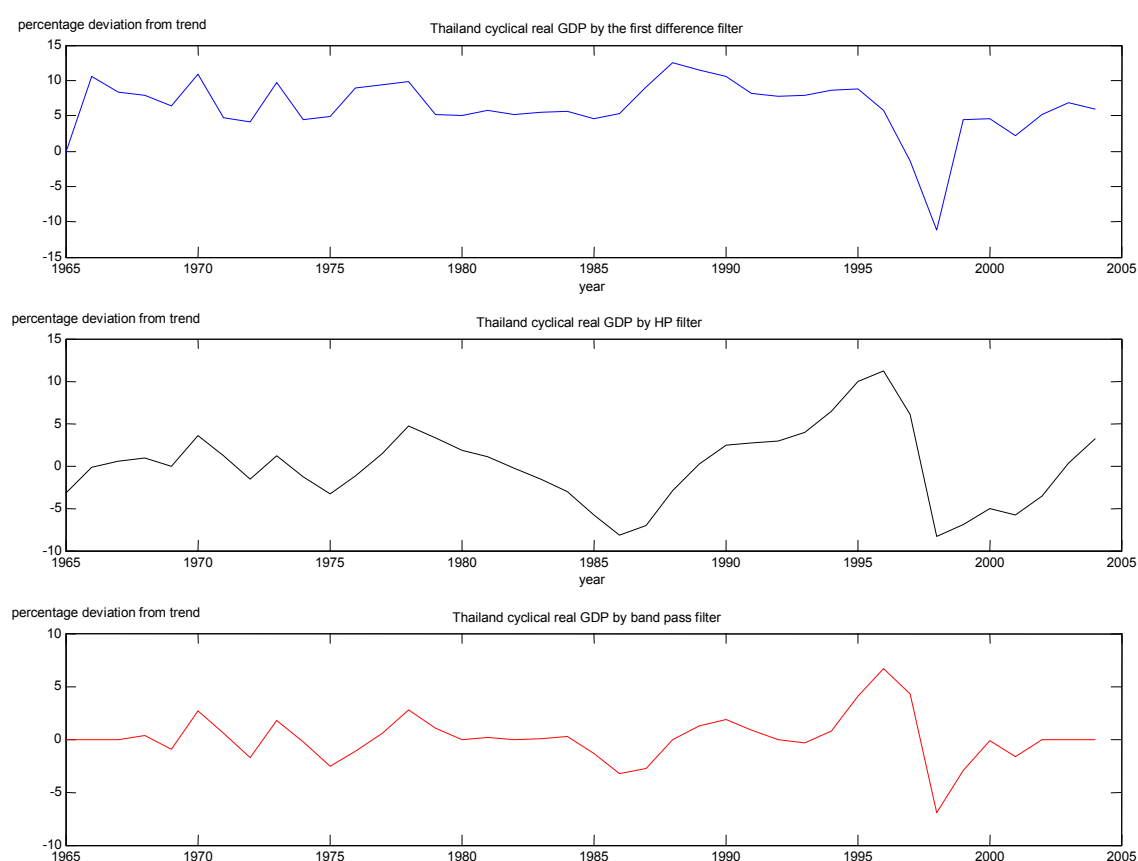


Figure 3.3 A Real GDP cyclical by the first different, HP and band pass filter

²⁰ The inclusion of the post 1997 data does not influence the estimates for the pre 1997 data for the first difference filter and the HP filter. Nonetheless, for the band pass filter, the pre 1997 data would change if the data stopped at the sample period at 1997. The band pass filter requires the truncation period from both ends of the data.

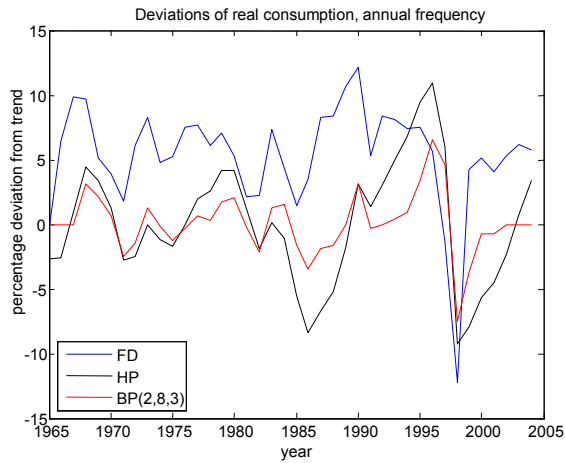


Figure 3.3 B Consumption

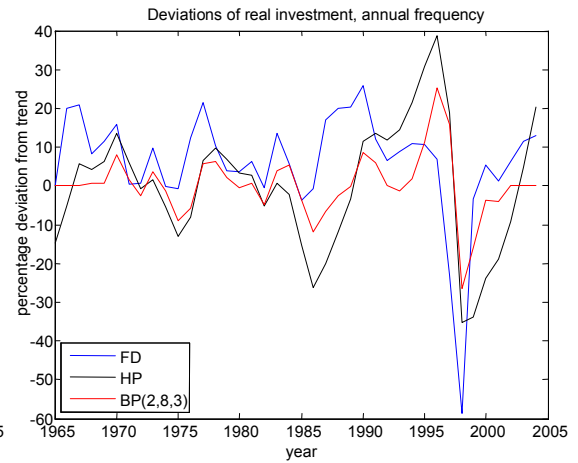


Figure 3.3 C Investment

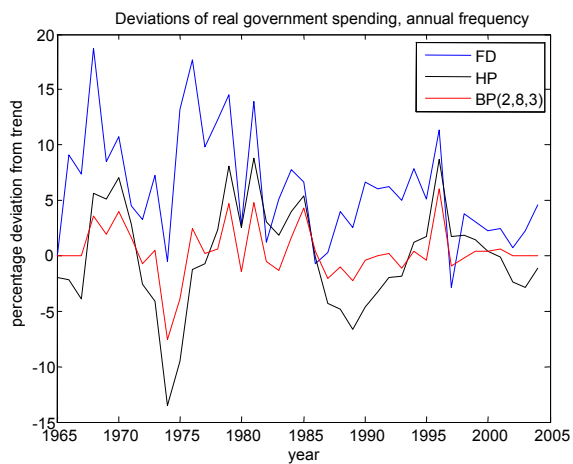


Figure 3.3 D Government spending

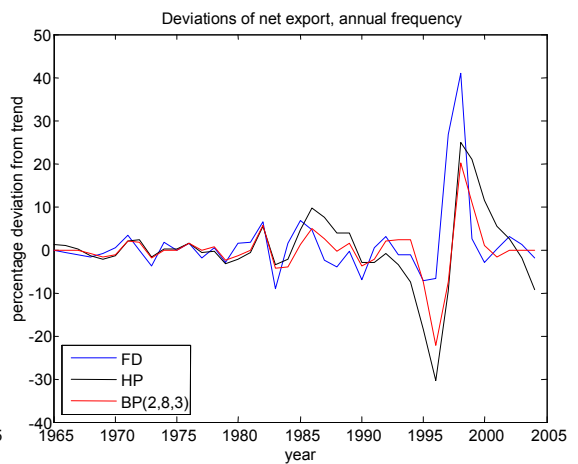


Figure 3.3 E Net export

Figure 3.4 plots the natural logarithm and trend of real major macroeconomics variables for Thailand from 1965 to 2004, annually. Since all variables except net export are in logarithms, the change in the trend component represents the growth rate. Because the growth component represents the slow moving component and the HP filter is the trend estimator, this section follow a common practice by using the HP filter to decompose the trend of a time series.

Figure 3.4 Log of Thai real variables and their growth component

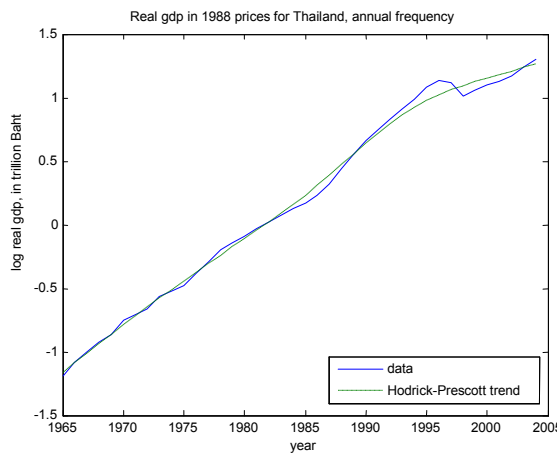


Figure 3.4 A GDP

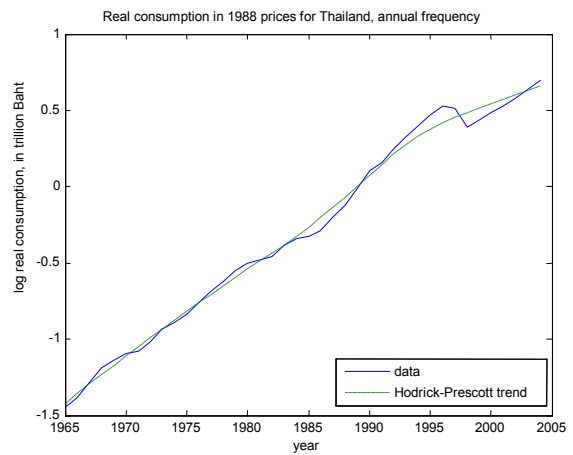


Figure 3.4 B Consumption

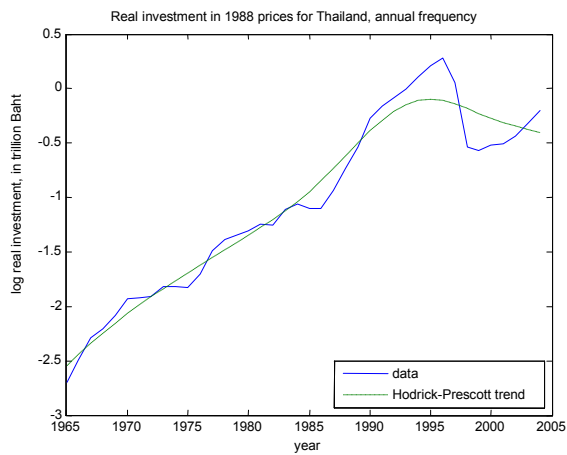


Figure 3.4 C Investment

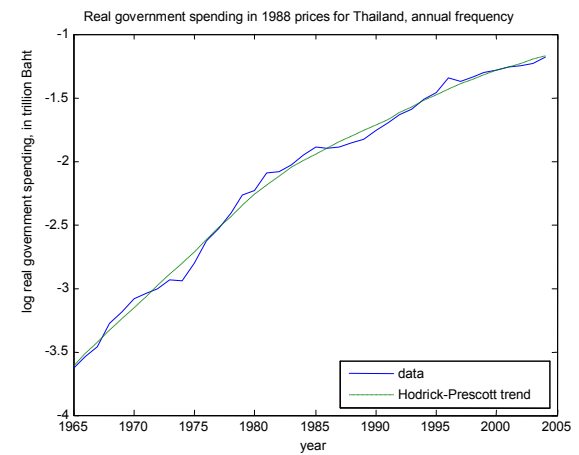


Figure 3.4 D Government spending

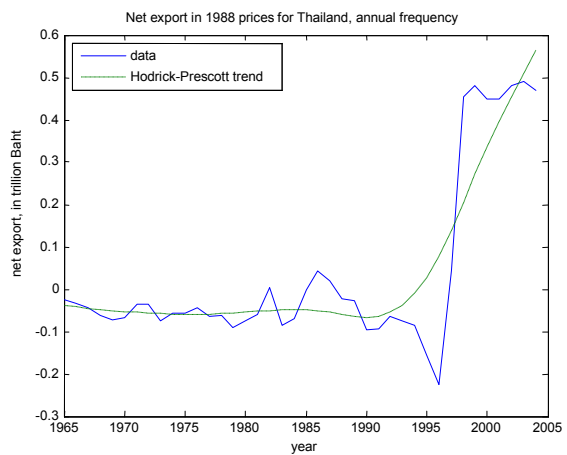


Figure 3.4 E Net Export

As depicted in figure 3.4, the Thai economy grows consistently during the entire observation period. Before 1997, the output increases sharply. However, during the financial crisis in July 1997, the output decreases dramatically below its trend. One can

obviously see from figure 3.4 A and B that consumption and output have a high correlation. Figure 3.4 C shows that investment has a significantly high deviation around its trend. Investment clearly fluctuates above its trend over 1990-1996 and suddenly drops in 1997. Similar to investment, Thailand net export has a negative trend over the long period 1965-1996 and increases significantly in 1997.

Table 3.1 The stylized facts of Thai, Asian and developed countries

Table 3.1 A; Volatility

	Tha	Kor	Mal	Phi	Ind	Sin	USA	EU	Jap
1965-2004									
Output	2.23	2.09	2.17	2.00	2.67	1.94	1.33	0.78	1.24
Consumption	2.38	2.69	3.37	0.84	3.19	2.10	1.04	0.64	1.00
Investment	8.25	6.85	9.39	8.11	7.90	5.09	3.86	1.93	2.84
Government spending	2.43	1.24	3.39	2.84	3.35	3.78	0.99	0.39	1.09
Export	4.30	5.65	3.69	6.26	6.54	6.09	3.14	1.97	3.58
Import	8.57	6.61	7.15	6.08	9.45	5.66	4.11	2.39	4.48
1965-1990									
Output	1.42	1.74	1.82	2.18	0.90	1.88	1.52	0.77	1.35
Consumption	1.56	1.73	3.09	0.94	1.33	1.57	1.17	0.58	1.18
Investment	4.67	6.82	7.13	8.72	2.98	5.19	4.38	1.55	3.04
Government spending	2.70	1.25	3.38	2.92	2.32	3.48	1.08	0.33	1.24
Export	4.34	6.21	3.40	6.01	4.30	6.52	3.42	1.95	3.84
Import	8.45	5.59	6.79	6.20	3.90	5.78	4.71	2.18	4.35
1991-2004									
Output	3.25	2.58	2.74	1.08	3.55	1.85	0.60	0.47	0.77
Consumption	3.36	3.92	3.60	0.43	2.43	2.67	0.51	0.47	0.43
Investment	12.38	6.43	12.42	4.98	10.34	4.50	1.74	1.46	1.81
Government spending	1.64	0.98	3.40	1.95	3.89	3.39	0.52	0.17	0.76
Export	3.85	3.06	3.84	6.29	7.75	3.68	2.10	1.49	2.79
Import	8.56	7.94	7.06	5.06	10.96	4.55	2.26	1.72	3.44

Notes: Volatility is measured by the percentage of standard deviation of the series. Tha is Thailand, Kor is Korea, Mal is Malaysia, Phi is Philippines, Ind is Indonesia, Sin is Singapore and Jap is Japan. All statistics are based on the band pass filter.

Table 3.1 A reports the amplitude of fluctuations measured by the percentage of standard deviation of the cyclical component of each series. The volatility and the correlation of each variable with respect to its own output are demonstrated in Table 3.1 B and C respectively. Before turning our intention to the business cycle features of the Asian and developed countries, this section first consider the properties of the Thai business cycle. Volatility of all Thai variables increases from the first to second period except for

government spending and export. The correlation with output for all Thai variables increases from the first to the second period for all variables, except for net export.

The interesting characteristics of the Thai business cycle are the high volatility of each series. The volatility of output in Thailand is much higher than the volatility of output in most of the developed countries for all sample periods. The volatility of output increases from the first period, 1.42% to 3.25 % in the second period, 1991-2004.

Table 3.1 B; Volatility with respect to output

	Tha	Kor	Mal	Phi	Ind	Sin	USA	EU	Jap
1965-2004									
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	1.07	1.29	1.55	0.42	1.19	1.08	0.78	0.82	0.81
Investment	3.70	3.28	4.33	4.06	2.96	2.62	2.90	2.47	2.29
Government spending	1.09	0.59	1.56	1.42	1.25	1.95	0.74	0.50	0.88
Export	1.93	2.70	1.70	3.13	2.45	3.14	2.36	2.53	2.89
Import	3.84	3.16	3.29	3.04	3.54	2.92	3.09	3.06	3.61
1965-1990									
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	1.10	0.99	1.70	1.48	1.48	0.84	0.77	0.75	0.87
Investment	3.29	3.92	3.92	3.31	3.31	2.76	2.88	2.01	2.25
Government spending	1.90	0.72	1.86	2.58	2.58	1.85	0.71	0.43	0.92
Export	3.06	3.57	1.87	4.78	4.78	3.47	2.25	2.53	2.84
Import	5.95	3.21	3.73	4.33	4.33	3.07	3.10	2.83	3.22
1991-2004									
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	1.03	1.52	1.31	0.40	0.68	1.44	0.85	1.00	0.56
Investment	3.81	2.49	4.53	4.61	2.91	2.43	2.90	3.11	2.35
Government spending	0.50	0.38	1.24	1.81	1.10	1.83	0.87	0.36	0.99
Export	1.18	1.19	1.40	5.82	2.18	1.99	3.50	3.17	3.62
Import	2.63	3.08	2.58	4.69	3.09	2.46	3.77	3.66	4.47

Notes: Volatility with respect to output is measured by the percentage of standard deviation of the series over the percentage of standard deviation of output. The abbreviations of the countries are the same as Table 3.1 A. All statistics are based on the band pass filter.

Unlike the volatility of consumption in developed countries, the volatility of consumption in Thailand is higher than that of output. One reason for this is that the consumption series for Thailand includes only household consumption whereas for the developed countries it is total private consumption. The variance of consumption with respect to output is bigger than one for all periods. The permanent income theory can account for a high variance with respect to output. As it is in most countries, household

consumption in Thailand is strongly pro-cyclical. The correlation of consumption with output increases from the first period at 0.55 to the second period, reaching 0.99, as shown in table 3.1 C.

Investment in Thailand is a high volatile series. With respect to output, investment volatilities are 3.70, 3.29 and 3.81 for the whole, the first and the second period respectively. Investment in Thailand is about three to four times more volatile than output in each period as in table 3.1 B. The investment cyclical component is also strongly pro-cyclical. This evidence has shown the similarity to the industrial countries. In Thailand, investment performs higher correlation to output than consumption does during 1965-1990 but they are identical at 0.99 during the second period 1991-2004.

Table 3.1 C; Co-movement with respect to output

	Tha	Kor	Mal	Phi	Ind	Sin	USA	EU	Jap
1965-2004									
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.85	0.87	0.80	0.81	0.55	0.80	0.88	0.80	0.81
Investment	0.95	0.84	0.82	0.85	0.86	0.65	0.96	0.89	0.96
Government spending	0.27	-0.03	0.30	0.65	0.72	-0.07	-0.07	-0.09	0.05
Net export	-0.84	-0.70	-0.73	-0.23	-0.56	-0.35	-0.52	-0.43	-0.35
1965-1990									
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.55	0.74	0.73	0.85	-0.45	0.85	0.88	0.68	0.83
Investment	0.88	0.81	0.69	0.85	-0.53	0.80	0.97	0.87	0.97
Government spending	0.32	-0.23	0.17	0.62	-0.54	-0.23	-0.07	-0.12	0.01
Net export	-0.44	-0.32	-0.62	-0.36	0.81	-0.19	-0.54	-0.30	-0.43
1991-2004									
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.99	0.99	0.94	0.43	0.93	0.82	0.86	0.92	0.67
Investment	0.99	0.99	0.94	0.90	0.95	0.55	0.90	0.90	0.93
Government spending	0.47	0.29	0.49	0.57	0.96	0.20	0.17	0.01	0.22
Net export	-0.95	-0.96	-0.88	0.23	-0.87	-0.64	-0.71	-0.32	-0.26

Notes: Co-movement with respect to output is measured by the correlation coefficient of the series of output and another series. The abbreviations of the countries are the same as Table 3.1 A. All statistics are based on the band pass filter.

The volatility of Thailand's government expenditure declines from the first to the second period, reaching 1.64%. For the entire period, the volatility of Thailand government spending is 2.43%. Figure 3.4 D shows that the government spending is largely uncorrelated with output. The table 3.1 C shows the similar results. The correlation

between government spending and output is quite weak for all periods. However, the government spending is slightly higher pro-cyclical from the first period at 0.32 to the second period at 0.47.

Consider the trade variables of Thailand, export and import in Thailand have a quite stable fluctuation with volatility of 4% and 8% respectively for all periods. Like many countries, Thailand trade variable displays a negative correlation with output. Net export has increased more than two times larger in negative correlation with output in the second period, reaching -0.95.

3.3.2 Business cycles of Asian countries and developed countries

One important feature of the Asian business cycle is a high volatility for the main macroeconomics variables. Mendoza (1995) reports output of developing countries is about two to four times more volatile than that of developed countries. Table 3.1 documents that the Asian business cycles are approximately 40% more volatile than that of the developed countries. Kim, Kose and Plummer (2003) suggest the higher volatility for the Asian countries could be due to such political instability, a weaker financial sector and institutional problem. Crucini (1997) and Kose (2002) document that the trade variables of the Asian countries are influenced by external factors such as world price, interest rate fluctuation and the demand of import and export from developed countries. Therefore, all these factors cause a higher degree of business cycle volatility in emerging economies. The volatility of output increases in the second period for all Asian economies except for Singapore and the Philippines. During 1965-1990, Philippines is the most volatile economy with an output volatility of 2.18% and Singapore is the second most volatile economy. Indonesia is the least volatile economy with output volatility less than 1%. During 1991-2004, Indonesia becomes the most volatile economy with output volatility of 3.55%. Philippines is the least volatile economy with output volatility slightly above 1%. Several factors can account for the highly volatile Asian economies in the second period. However, one major explanation is the Asian financial crisis in 1997.

Consumption volatility in the Asian economies has an important characteristic. Consumption volatility is on average for all Asian countries just slightly above the output volatility in most cases. Consumption is pro-cyclical for all Asian countries except for

Indonesia in the first period. For all Asian countries except for Philippines and Singapore, the correlation coefficient of consumption with output, range from 0.93-0.99, increases significantly in the second period. The higher co-movement in the second period could be explained by the rapid expansion. It was a good time to consume and invest before the crisis.

Volatility property of investment is high in all Asian countries, in particular during 1990-2004. On average for all Asian countries and for each observation period, investment volatility is about two to four times more volatile than output. Investment volatility is significantly different across the sample of countries. The volatility of investment in Malaysia is 7.31% while it is 2.98 % in Indonesia in the first period. Investment volatility in Thailand, Malaysia and Indonesia increases significantly while for Singapore and Korea it slightly decreases in the second period. Singapore and Korea might not be affected severely from the 1997 crisis. Investment volatility in the Philippines interestingly decreases substantially from 8.72% in the first period to 4.98% in the second period. The co-movement of investment with output is uniformly pro-cyclical except for Indonesia in the first period. If investment is pro-cyclical, then an increase in aggregate output coincides with an increase in investment. On average for Asian countries, investment is pro-cyclical for all cases. Particularly in the second period, the correlation coefficient increases strikingly, and is above 0.9 for all countries except for Singapore. It might be the case that the Asian countries had a tremendous growth rate before the crisis 1997. The high growth rate can account for the high investment during the second period.

In all Asian countries except Indonesia, the volatility of government spending decreases in the second period. The less volatile in government spending suggests a degree of stabilization in economic policy. Even though there was the global expansion, the oil price shocks and the debt crisis during 1965-1990, the output volatility of all Asian economies, interestingly except for Philippines, is not as high as the output volatility during 1991-2005. This finding has an essential implication that the Asian economic crisis has a very strong impact to business cycle of Asian countries. The variation in the world economy during the 1965-1990 period does not induce a highly volatile Asian economic environment. Talvi and Vegh (2000) report that government spending is pro-cyclical in many developing countries. Table 3.2 shows that the co-movement of

government spending with output is significantly different across the sample countries as the correlation coefficient ranges from -0.03 to 0.72 for the whole period and -0.23 to 0.62 in the first period. For the whole period, government spending is negatively correlated to the output level of Korea and Singapore but positive in all other cases. Government spending, in the first period, is countercyclical in Korea, Singapore and Indonesia where as all countries uniformly perform a positive correlation in the second period. The explanation for an increase in government spending could be, with an economic expansion, the government increases its expenditures on infrastructure project such as physical and human capital. Additionally, as growth increased or during a good time, government became easier to spend on industrial and other well-being projects.

For the trade variables, export and import are more volatile in the Asian countries than in developed countries for all cases. The volatility of import increase in the second period for all countries, and it increases substantially in Indonesia, reaching 10.96%. Export volatility also increases in the second period except for Korea and interestingly for Thailand. A lower volatility of trade variables in the first period for Asian economy can be partly described by risk sharing diversification of expanding their export and import markets and becoming less vulnerable to external shocks. The financial crisis in second period can mainly account for a high volatility of these trade variables in the Asian economies. The correlation properties of trade components is depicted in Table 3.1 C shows that net exports are uniformly counter cyclical in both Asian and developed countries except for Indonesia in the first period and Philippines in the second period.

For developed countries, on average, the volatility of output, consumption, investment, government spending, export and import less volatile than that of Asian countries. Moreover, the volatility decreases over time from the first to the second period. Stock and Watson (2002) confirm these facts for the United States and the G-7 countries. They reported that from 1960-1983, the standard deviation of annual growth rates in real GDP in the United States was 2.7%. From 1984 to 2001, the corresponding standard deviation was 1.6%. Stock and Watson (2005) also document that the volatility of economic activity in most G7 economies has moderated over the past forty years. It is very interesting to note that during the second period when many variables in Asian countries exhibit an increase in volatility, all variables in developed countries perform a decrease in volatility. Consumption and investment are positively correlated to output as

demonstrated in table 3.1 C. However, the investment series is more strongly pro-cyclical than the consumption series for all cases. This finding is consistent with Backus, Kehoe and Kydland (1995) that investment volatility is about two to three times more volatile than output for developed countries. This finding suggests the similar argument that the investment volatility of the USA, EU and Japan is roughly two to three times more volatile than output. Government spending volatility decreases in the second period. In the first period, government spending is counter-cyclical for USA and EU. However, all developed countries are pro-cyclical in the second period. Net export in developed countries is contemporaneously counter cyclical in all cases. It is strongly counter cyclical in the second period, except for Japan.

3.3.3 International business cycle

This section reports the properties of business cycle across the Asian countries by examining the cross-country correlations of the main macroeconomics series. Table 3.2 presents the contemporaneous cross-country correlation with the same Thai variables and Table 3.3 shows the contemporaneous cross-country co-movement in the Asian and developed countries for three periods. Table 3.2 illustrates the fact that Thai business cycles have a high degree of correlation with Asian countries and the degree of co-movement increases in the second period. This study suggests that Thai business cycle has a low degree of co-movement to developed countries. Nonetheless, among the developed countries, Thai business cycles are more correlated to Japan. Interestingly, in the second period, business cycle in Thailand exhibits a negative correlation with the USA and the EU, but not with Japan. The crisis in the second period might suggest that negative correlation.

The finding on Table 3.3 suggests that for the Asian countries output fluctuations displays a high degree of co-movement. The cross country correlations of output are positive for all Asian countries. The output fluctuations for all Asian countries are positively correlated with the Asian business cycle and they increase in the second period for all cases. Similar to developed countries, cross country correlation are positive for all cases for all developed countries except for Japan and the USA in the second period. Developed countries output fluctuations exhibit a high degree of correlation. The output fluctuations for all developed countries are positively correlated with the developed

business cycle. The output fluctuation of the Asian and developed economies displays the positive co-movement for the entire and the first period

Table 3.2 Cross-country co-movement with the same Thai variables

	Tha	Kor	Mal	Phi	Ind	Sin	USA	EU	Jap
1965-2004									
Output	1.00	0.74	0.72	0.42	0.76	0.51	0.04	0.19	0.42
Consumption	1.00	0.66	0.67	0.32	0.18	0.59	0.04	0.04	0.37
Investment	1.00	0.92	0.84	0.39	0.74	0.45	-0.21	-0.04	0.33
Government spending	1.00	0.06	-0.25	-0.39	0.29	0.15	0.02	-0.02	0.29
Net export	1.00	0.90	0.77	-0.05	0.29	0.45	-0.29	-0.20	0.52
1965-1990									
Output	1.00	0.49	0.51	0.41	0.35	0.49	0.25	0.52	0.50
Consumption	1.00	-0.03	0.41	0.58	-0.26	0.43	0.32	0.33	0.39
Investment	1.00	0.80	0.75	0.61	-0.06	0.76	0.03	0.31	-0.15
Government spending	1.00	-0.04	-0.38	-0.52	0.35	0.04	0.08	-0.11	0.27
Net export	1.00	0.44	0.34	0.04	-0.46	-0.24	-0.03	0.05	0.29
1991-2004									
Output	1.00	0.93	0.87	0.69	0.89	0.63	-0.45	-0.27	0.52
Consumption	1.00	0.94	0.92	-0.03	0.82	0.75	-0.51	-0.30	0.75
Investment	1.00	0.96	0.90	0.52	0.81	0.48	-0.53	-0.43	0.41
Government spending	1.00	0.61	0.03	0.11	0.25	0.35	-0.23	0.19	0.44
Net export	1.00	0.96	0.82	-0.12	0.61	0.65	-0.49	-0.38	0.70

Notes: The cross-country co-movement with the same Thai variables is measured by the correlation coefficient of each of Thai variables and each variable of another country. The abbreviations of the countries are the same as Table 3.1 A. All statistics are based on the band pass filter.

However, in the second period, the cross country correlation of the USA and the EU are negatively correlated with Thailand, Korea, Malaysia, Philippines and Indonesia. The Asian economic crisis can account for the negative co-movement between the Asian and developed countries in the second period.

The cross country consumption and government expenditure correlations of the Asian countries do not show any specific pattern. The consumption fluctuations for all developed countries are positively correlated with the developed business cycle, except for Japan in the second period. The cross country correlations of investment are positive for all cases for all Asian countries except for Thailand and Indonesia in the first period. The investment in the USA and the EU are positively correlated for all cases, except for the first period.

Table 3.3 Cross-country correlation during 1965-2004 of Thai, Asian and developed countries

Table 3.3 A; Cross-country correlation during 1965-2004

1965-2004		Tha	Kor	Mal	Phi	Ind	Sin	USA	EU
Output	Korea	0.74							
	Malaysia	0.72	0.54						
	Philippines	0.42	0.28	0.46					
	Indonesia	0.76	0.57	0.74	0.25				
	Singapore	0.51	0.32	0.76	0.53	0.43			
	USA	0.04	0.15	0.20	-0.08	0.00	0.23		
	EU	0.19	0.24	0.26	0.14	0.05	0.24	0.44	
	Japan	0.42	0.33	0.31	0.22	0.32	0.33	0.32	0.63
Consumption	Korea	0.66							
	Malaysia	0.67	0.53						
	Philippines	0.32	-0.04	0.27					
	Indonesia	0.18	0.20	0.18	-0.01				
	Singapore	0.59	0.51	0.59	0.27	-0.09			
	USA	0.04	-0.01	-0.14	-0.23	-0.46	0.04		
	EU	0.04	0.15	0.04	0.17	-0.48	0.11	0.31	
	Japan	0.37	0.32	0.24	0.23	-0.26	0.18	0.40	0.49
Investment	Korea	0.92							
	Malaysia	0.84	0.76						
	Philippines	0.39	0.36	0.46					
	Indonesia	0.74	0.68	0.79	0.39				
	Singapore	0.45	0.36	0.73	0.51	0.45			
	USA	-0.21	-0.13	-0.16	-0.29	-0.39	-0.06		
	EU	-0.04	0.15	-0.07	0.15	-0.14	-0.10	0.14	
	Japan	0.33	0.39	0.33	0.18	0.45	0.13	-0.03	0.61
Government spending	Korea	0.06							
	Malaysia	-0.25	0.25						
	Philippines	-0.39	0.11	0.28					
	Indonesia	0.29	0.12	0.30	0.04				
	Singapore	0.15	-0.09	-0.08	-0.17	0.10			
	USA	0.02	0.15	-0.27	-0.08	-0.12	-0.16		
	EU	-0.02	0.18	0.16	0.00	0.24	-0.07	0.14	
	Japan	0.29	-0.01	0.11	0.07	0.39	0.07	-0.24	0.12
Net export	Korea	0.90							
	Malaysia	0.77	0.80						
	Philippines	-0.05	-0.13	-0.10					
	Indonesia	0.29	0.43	0.56	0.24				
	Singapore	0.45	0.54	0.57	0.02	0.40			
	USA	-0.29	-0.29	-0.43	-0.41	-0.43	-0.42		
	EU	-0.20	-0.15	-0.46	-0.08	-0.46	-0.27	0.05	
	Japan	0.52	0.46	0.19	0.29	0.03	0.14	-0.17	0.19

Notes: The cross-country correlation 1965-2004 is measured by the correlation coefficient of the series of two countries. Tha is Thailand, Kor is Korea, Mal is Malaysia, Phi is Philippines, Ind is Indonesia, Sin is Singapore and Jap is Japan. All statistics are based on the band pass filter.

Table 3.3 B; Cross-country correlation during 1965-1990

1965-1990		Tha	Kor	Mal	Phi	Ind	Sin	USA	EU
Output	Korea	0.49							
	Malaysia	0.51	0.11						
	Philippines	0.41	0.14	0.43					
	Indonesia	0.35	0.01	0.49	0.07				
	Singapore	0.49	0.05	0.77	0.48	0.46			
	USA	0.25	0.33	0.38	-0.2	0.43	0.18		
	EU	0.52	0.34	0.53	0.11	0.41	0.4	0.55	
	Japan	0.5	0.24	0.22	0.08	0.19	0.36	0.43	0.69
Consumption	Korea	-0.03							
	Malaysia	0.41	0.03						
	Philippines	0.58	-0.08	0.31					
	Indonesia	-0.26	-0.15	-0.1	-0.06				
	Singapore	0.43	0.07	0.56	0.55	-0.28			
	USA	0.32	0.2	-0.06	-0.3	-0.42	-0.09		
	EU	0.33	0.32	0.04	0.28	-0.66	0.33	0.42	
	Japan	0.39	0.36	0.13	0.17	-0.46	0.23	0.55	0.64
Investment	Korea	0.8							
	Malaysia	0.75	0.26						
	Philippines	0.61	0.46	0.47					
	Indonesia	-0.06	-0.3	0.15	0.61				
	Singapore	0.76	0.36	0.92	0.56	0.02			
	USA	0.03	0.21	-0.05	-0.71	-0.88	-0.11		
	EU	0.31	0.3	0.04	0.22	0.29	-0.21	-0.02	
	Japan	-0.15	-0.45	0.00	-0.19	0.35	-0.29	-0.11	0.62
Government spending	Korea	-0.04							
	Malaysia	-0.38	0.32						
	Philippines	-0.52	0.23	0.43					
	Indonesia	0.35	-0.1	0.13	0.03				
	Singapore	0.04	-0.24	-0.06	-0.38	0.12			
	USA	0.08	0.09	-0.42	-0.08	-0.23	-0.09		
	EU	-0.11	0.19	0.09	0.41	0.27	-0.17	0.04	
	Japan	0.27	0.05	0.02	-0.03	0.47	-0.02	-0.28	0.13
Net export	Korea	0.44							
	Malaysia	0.34	0.47						
	Philippines	0.04	0.31	0.12					
	Indonesia	-0.46	-0.21	0.13	0.26				
	Singapore	-0.24	-0.02	0.44	0.51	0.50			
	USA	-0.03	-0.11	-0.43	-0.44	-0.25	-0.70		
	EU	0.05	-0.17	-0.38	0.17	-0.5	0.05	-0.26	
	Japan	0.29	0.36	-0.17	0.38	-0.32	0.02	-0.02	0.49

Notes: The cross-country correlation 1965-1990 is measured by the correlation coefficient of the series of two countries. Tha is Thailand, Kor is Korea, Mal is Malaysia, Phi is Philippines, Ind is Indonesia, Sin is Singapore and Jap is Japan. All statistics are based on the band pass filter.

Table 3.3 C; Cross-country correlation during 1991-2004

1991-2004		Tha	Kor	Mal	Phi	Ind	Sin	USA	EU
Output	Korea	0.93							
	Malaysia	0.87	0.94						
	Philippines	0.69	0.79	0.93					
	Indonesia	0.89	0.87	0.92	0.84				
	Singapore	0.63	0.75	0.85	0.76	0.6			
	USA	-0.45	-0.25	-0.15	-0.07	-0.5	0.31		
	EU	-0.27	-0.01	-0.09	-0.03	-0.31	0.15	0.44	
	Japan	0.52	0.62	0.78	0.91	0.75	0.55	-0.17	0.03
Consumption	Korea	0.94							
	Malaysia	0.92	0.92						
	Philippines	-0.03	-0.12	0.08					
	Indonesia	0.82	0.8	0.8	0.41				
	Singapore	0.75	0.82	0.81	-0.26	0.44			
	USA	-0.51	-0.33	-0.43	-0.56	-0.7	0.08		
	EU	-0.3	-0.06	-0.17	-0.65	-0.46	0.17	0.75	
	Japan	0.75	0.59	0.68	0.39	0.80	0.38	-0.80	-0.59
Investment	Korea	0.96							
	Malaysia	0.9	0.93						
	Philippines	0.52	0.64	0.75					
	Indonesia	0.81	0.77	0.93	0.67				
	Singapore	0.48	0.53	0.67	0.78	0.74			
	USA	-0.53	-0.32	-0.36	0.15	-0.53	0.06		
	EU	-0.43	-0.22	-0.32	-0.04	-0.53	-0.1	0.83	
	Japan	0.41	0.49	0.57	0.63	0.61	0.91	0.14	0.16
Government spending	Korea	0.61							
	Malaysia	0.03	0.23						
	Philippines	0.11	-0.31	0.14					
	Indonesia	0.25	0.56	0.68	0.17				
	Singapore	0.35	0.08	-0.22	0.34	0.11			
	USA	-0.23	0.02	0.15	-0.14	0.00	-0.59		
	EU	0.19	0.09	0.43	0.09	0.53	0.29	0.09	
	Japan	0.44	-0.17	0.35	0.63	0.26	0.43	-0.24	0.32
Net export	Korea	0.96							
	Malaysia	0.82	0.82						
	Philippines	-0.12	-0.23	-0.17					
	Indonesia	0.61	0.64	0.83	0.29				
	Singapore	0.65	0.76	0.73	-0.39	0.64			
	USA	-0.49	-0.41	-0.58	-0.62	-0.86	-0.42		
	EU	-0.38	-0.3	-0.66	-0.28	-0.7	-0.42	0.76	
	Japan	0.7	0.64	0.32	0.18	0.34	0.36	-0.44	-0.11

Notes: The cross-country correlation 1991-2004 is measured by the correlation coefficient of the series of two countries. Tha is Thailand, Kor is Korea, Mal is Malaysia, Phi is Philippines, Ind is Indonesia, Sin is Singapore and Jap is Japan. All statistics are based on the band pass filter.

3.4 Conclusions

The chapter investigates the business cycle properties within and cross country of Thai economy. It documents the cyclical features of the Asian countries, including Thailand, and compared them with those of the developed countries, the USA, EU and Japan. Because the main concern in this chapter is to document the empirical information, the empirical methodology is implemented. The theoretical economy is not discussed here. There are many striking business cycle fluctuation in the developing countries. The volatility of investment and government spending are higher than that of aggregate output. Consumption is more volatile than output. These variables in general are procyclical and highly persistent. However, net exports are highly volatile and countercyclical.

There are some crucial similarities between the cyclical fluctuation in developing countries and those of the developed country. Co-movement and persistence for consumption, investment and government spending are quite similar. Nonetheless, the business cycle features of the developing countries fluctuate more than those of the developed countries. The reasons could be the lack of well established financial markets, institution weakness and politics. The business cycle regularities in the Asian countries exhibit a high degree of co-movement. This chapter also finds that the business cycle fluctuations of output among developing countries are positively correlated.

Chapter 4

Methodology

4.1 Introduction

Economists attempt to explain the relationship among economic variables. They construct a model economy to help understand the real world and to analyze the relationships among variables. Usually, they consist of a number of equations that represents identities, behavioural relationships and conditions that convey equilibrium. These models are more likely to express as sets of relationships that take the mathematical form. In some senses, economists, like scientists, use the model economy as their laboratories for economic experiments, research and teaching. They use the model to analyze the characteristic of a set of variables through the characteristic of other variables. The economic model in the past few decades has undergone important change in order to study the time and time path of variables. Time has become an essential factor in studying economics. Economists recently built dynamic models focusing on the evolution of variables over time. Dynamic analysis plays a central role in many areas of economics, including real business cycles. The method used to study the business cycle changed in a crucial way by the breakthrough work of Lucas (1971) who began to study the business cycle using the neoclassical model with rational expectation to determine the characteristics of the equilibrium paths of the economy. Kydland and Prescott (1972) and Long and Plosser (1983) built on the work of Lucas by combining general equilibrium theory with a method to compute the equilibrium of the model economy and to investigate its empirical features. The general equilibrium models use specific restrictions on preferences, technologies and endowment patterns to study a particular economic issue. These works were an important development and pioneering in the real business cycle literature.

The main goal of this chapter is to illustrate the basic ideas and method of discounted dynamic programming with an application to the simple real business cycle model. It stochastically describes how to set up the functional or Bellman equation and several

approaches to solve it. Since the numerical method to solve the exact nonlinear solution is difficult to implement, this chapter describes how to solve the linear approximate equation. The methods of solving linear rational expectation model by Blanchard and Khan (1980) and King and Watson (1998) are introduced here. This chapter eventually explains some computational approaches for computing the equilibrium of the simple real business cycle model as an example. Two ways to solve the real business cycle model are described, the method of undetermined coefficients by Campbell (1992) and the method of finding a linear approximation to the first order conditions characterizing equilibrium by King, Plosser and Rebelo (1988).

4.2 Overview of methodology

To investigate a particular economic issue, a research question should include constructing an economic theory and confronting it with the facts to evaluate its validity. This chapter describes the tools to model, solve and estimate the economic model. In particular for the real business cycle model, the model economy is also filtered in order to have a cyclical component in a specific frequency range and so does the data.¹

Figure 4.1 summarizes the methodology and begins with an economic model. After setting up the model economy, dynamic programming is employed to do optimization, obtaining the first order necessary condition. Because it is difficult to analyze the non-linear equations, the next step is to log linearize the first order necessary conditions around the steady state.² The steady state is the starting point to analyze the model economy since it exhibits a sustained growth over time. The model under consideration might be difficult to solve analytically. It is sometime necessary to solve it numerically. This chapter also provides both analytical and numerical approaches to solving a simple real business cycle model.

¹ The filter issue is described in chapter 3 “Filter and Thai Facts”.

² Talmain (2002) suggests that because a second-order approximation to a function at any particular point corresponds to a first-order approximation of its derivation at that point, studying near the steady state can also be achieved by taking a quadratic approximation of the utility and production functions. Christiano (1990) studies the accuracy of two versions of Kydland and Prescott's (1980, 1982) procedure for approximating optimal decision rules. He finds that the Kydland-Prescott approximate decision rules that are very similar to those implied by value-function iteration. However, since this study solves the model along the same lines as in King *et al.* (1988), the necessary conditions and the resource constraints are transformed into linear equations through a log-linearization procedure.

Then one can derive the decision rules, namely, the optimal control variables are the function of state variables. From the decision rules, one can either calculate and plot the impulse response function or extend the model to the next level by filtering the model economy to evaluate its performance.

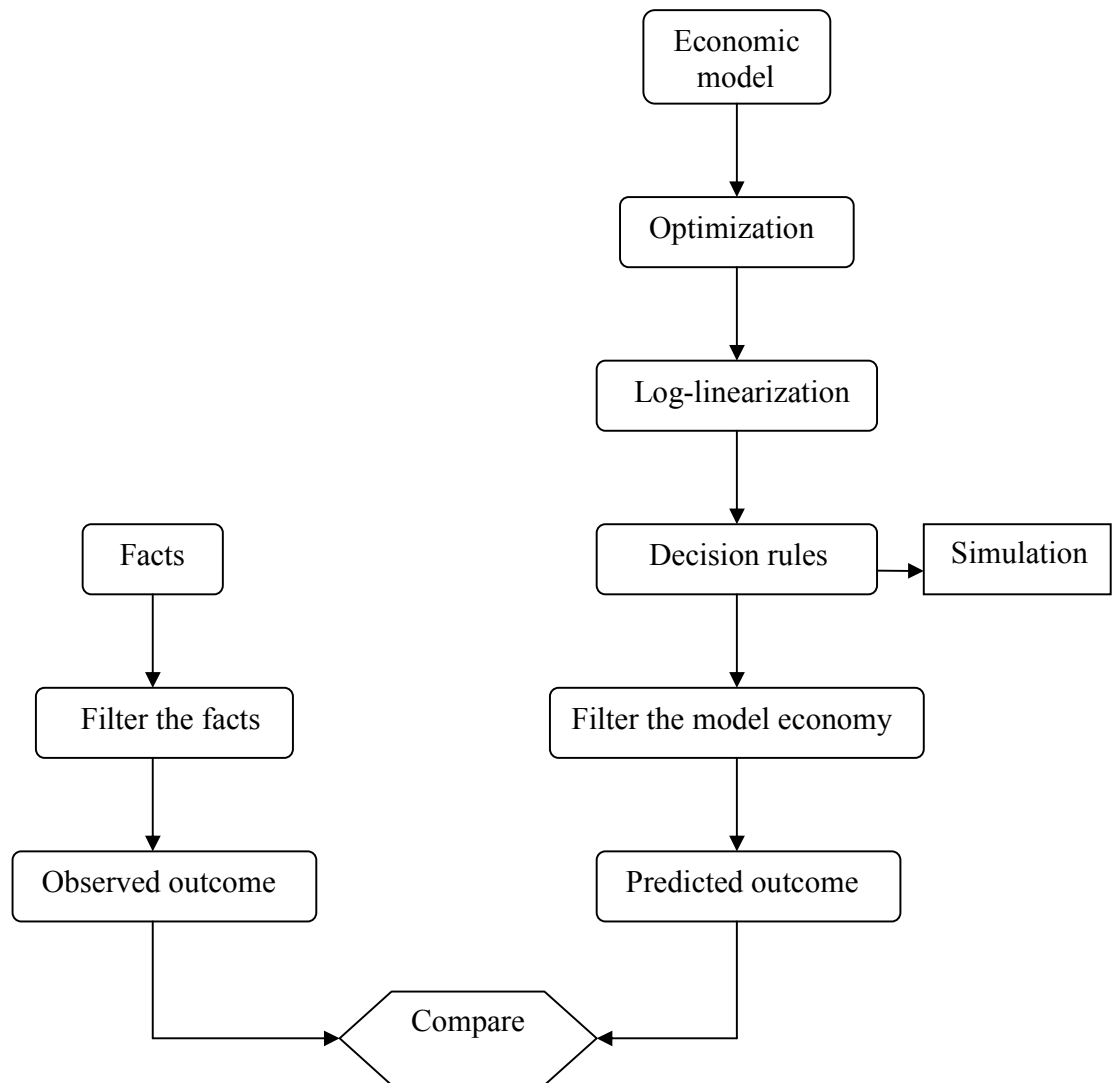


Figure 4.1 The road map of methodology

The approach to filter the model economy is described in chapter 3 in detail. It is important at this stage that the parameters can be estimated. The process of finding the model parameters is called “calibration”. Therefore the model economy can be calibrated for the

specific country. Note that for the calibration, the model does not have to be solved to choose the parameters. Instead, calibration is conducted by looking at evidence that is outside the model.³ Then one can generate the predicted outcome. For the real business cycle agenda, three main features of relevant time series, volatility, persistence and co-movement, are investigated. Because the main business cycle properties are calculated from the cyclical components, the data must also be filtered to extract its trend before computing the actual business cycle moments. Finally, to assess the model's validity, comparing the model's predictions against the fact's outcome is essential.

4.3 Dynamic programming

Since the recursive method concentrates on the tradeoff between the current and all future utility, this section reviews the basic idea of recursive methods, following the recursive macroeconomic theory of Sargent (2000), which are important to understand economic dynamic systems. Dynamic programming is the approach to formulate and solve general equilibrium models.⁴ The main objective for this section is to set out a recursive optimization problem. The stochastic or uncertainty problems are introduced here because most of the macroeconomics problems involve stochastic optimization. Consider the sequential problem, an infinite control $\{u_t\}_{t=0}^{\infty}$ is chosen to maximize the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t r(x_t, u_t), \quad \text{where } 0 < \beta < 1$$

$$\text{subject to } x_{t+1} = g(x_t, u_t, \varepsilon_{t+1}) \quad (4.1)$$

where x_0 is known.⁵ At time t , x_t is assumed to be known, however x_{t+j} , $j \geq 1$ is not known at t . That is, ε_{t+1} is realized at $t+1$ after u_t has been chosen at t . The solution takes the form of a policy function h mapping the state x_t to the control u_t as $u_t = h(x_t)$

³ The calibration procedure for closed economy is discussed empirically in chapter 5 and for opened economy in chapter 6.

⁴ For more complete treatment of this issue, see Bertsekas (1976), Sargent (1987), Stokey and Lucas (1989), and Cooper and Adda (2003).

⁵ Typically, the variables of an economic model are classified as either control or state variables. The evolution of state variables relates the consequences of these period events for all future periods.

and $x_{t+1} = g(x_t, u_t, \varepsilon_{t+1})$. The recursive structure of the solution is retained since controls dated t affect returns $r(x_s, u_s)$ for $s \geq t$ but not earlier. To find the decision rule, h , it is necessary to know another function $V_t(x)$ that represents the optimal value of the sequential problem. $V_t(x)$ is called the value function and is defined as:

$$V_t(x_0) = \max_u E_t \sum_{t=0}^{\infty} \beta^t r(x_t, u_t) \quad \text{subject to } x_{t+1} = g(x_t, u_t, \varepsilon_{t+1}) \quad (4.2)$$

If the $V_t(x_0)$ is known, then the decision rule could be computed by solving the Bellman equation:

$$V_t(x) = \max_u \{r(x, u) + \beta E[V(g(x, u, \varepsilon)) | x]\} \quad (4.3)$$

The sequential equation can be converted into a functional equation. Where $V(x)$ is the optimal value of the problem starting from x at $t=0$ and where $E\{V(g(x, u, \varepsilon)) | x\} = \int V(g(x, u, \varepsilon)) dF(\varepsilon)$. The first order equation of the right side of the Bellman equation is:

$$\frac{\partial r(x, u)}{\partial u} + \beta E \left[\frac{\partial g}{\partial u}(x, u, \varepsilon) V'(g(x, u, \varepsilon)) | x \right] = 0 \quad (4.4)$$

Since the shock process is stochastic the differentiation operator can pass through the expectation operator. Off corners or the interior solution, the value function satisfies:

$$V'_t(x) = \frac{\partial r}{\partial x}(x, h(x)) + \beta E \left\{ \frac{\partial g}{\partial x}(x, h(x), \varepsilon) V'(g(x, h(x), \varepsilon)) | x \right\} \quad (4.5)$$

In the special case in which $\partial g / \partial x \equiv 0$, the formula for $V'(x)$ becomes $V'(x) = \frac{\partial r}{\partial x}(x, h(x))$.

Substituting this formula into the first order necessary condition for the problem gives the stochastic Euler equation as below:

$$\frac{\partial r(x, u)}{\partial u} + \beta E \left[\frac{\partial g}{\partial u}(x, u, \varepsilon) \frac{\partial r}{\partial x}(\hat{x}, \hat{u}) | x \right] = 0 \quad (4.6)$$

where \hat{x} and \hat{u} denote next period value. Next section describes the sample application of dynamic programming.

4.4 Computational methods

This part introduces general ideas to set up a dynamic program and mainly focuses on several computational approaches for solving dynamic programs. Its ultimate goal is to have the Euler equation which is necessary for the optimal conditions. To see the practical application of dynamic programming principles to problems in economics, this section explains how to solve the dynamic program using the simple real business cycle model with fixed labour supply as an example.⁶

By way of illustration, consider the model economy of choosing a sequence of values for consumption $\{C_t\}_{t=0}^{\infty}$ to maximize:

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \quad \text{where } u(C_t) &= \frac{C_t^{1-\sigma}}{1-\sigma} \\ \text{subject to } Y_t &= F(A_t, K_t) = (A_t N_t)^\alpha K_t^{1-\alpha} \text{ and } N_t = 1 \\ K_{t+1} &= (1-\delta)K_t + I_t \\ Y_t &= C_t + I_t \\ A_{t+1} &= \rho A_t + \varepsilon_{t+1} \end{aligned} \quad (4.7)$$

⁶ This simple model is further modified to the more complicated model in chapter 5 for a closed economy model with general endogenous labour supply and exogenous government spending and in chapter 6 for the open economy.

Let C_t be the consumption in period t and let $u(C_t)$ represent the flow of utility from this consumption. Assume $u(\cdot)$ is real-valued, sufficiently differentiable, strictly increasing and concave and $u(\cdot)$ satisfies Inada condition ($u'(\cdot) > 0, u''(\cdot) < 0$ for all C and $\lim_{C \rightarrow 0} u_c(C_t) = \infty, \lim_{C \rightarrow \infty} u_c(C_t) = 0$). $u(\cdot)$ satisfies the constant coefficient of relative risk aversion and $1/\sigma$ is the intertemporal substitution of consumption. β is a discount factor where $\beta \in (0,1)$ and the discount rate is defined by β^{-1} . The production function is the Cobb-Douglas production function where K_t is the capital stock and N_t represents labour which is normalized to one because of the fixed labour. A_t is the total factor productivity and $1-\alpha$ is capital share. The capital stock accumulates according to $K_{t+1} = (1-\delta)K_t + I_t$ where I_t is investment and $\delta \in (0,1)$ is the rate of physical depreciation. According to the resource constraint, output is decomposed into consumption and investment. Finally, the technology driving process follows AR(1) where ρ measures the persistence of the shock.

4.4.1 General procedures for solving dynamic programming

This section describes the general principles for solving the dynamic programming problem using the simple real business cycle with fixed labour. The procedures are broken down into 4 steps. First of all, the constraints should be rearranged into the simple form. Then set the optimization problem up. Second, each variable is classified as either a state or control variables. Also, categorize each state variable further into controlled states and exogenous state variable. The third step is setting up the Bellman equation. The final step is to derive the Euler equation. Basically, there are three main approaches to achieving this objective, using the reduced form objective function, combining the Lagrange multiplier with the Bellman equation and using the Lagrangian techniques directly. To derive the Euler equation by the first two methods, the right hand side of the Bellman equation are differentiated with respect to control variables. Next it requires the envelope condition. To do so, we first differentiate both sides of the Bellman equation with respect to the controlled state variables and evaluate at the optimal levels of the control variables. Then we use the first order necessary conditions to simplify the equation. The outcome is the envelope condition. Finally, combine the first order necessary conditions and the one period updated

envelope condition to obtain an Euler equation. However, the third approach, the Lagrangian technique, is more simple. This approach does not require the Bellman equation. According to the real business cycle with fixed labour, the details for each step are the following,

Step 1. Write out an optimization problem

Rearrange constraints and rewrite the problem as:

$$\begin{aligned} \underset{\{C_t, K_{t+1}\}_{t=0}^{\infty}}{\text{Max}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \\ \text{subject to} \quad & F(A_t, K_t) = K_{t+1} - (1 - \delta)K_t + C_t \end{aligned} \quad (4.8)$$

Step 2. Categorize the variables in the model. In this simple model at time t , A_t and K_t are state variables. C_t and K_{t+1} are control variables. K_t is a controlled state variable and A_t is the exogenous state variable.

Step 3. State Bellman equation or functional equation for this problem:⁷

$$\begin{aligned} V_t(K_t, A_t) = \underset{C_t, K_{t+1}}{\text{Max}} \{ & u(C_t) + \beta E_t V(K_{t+1}, A_{t+1}) \\ \text{subject to} \quad & F(A_t, K_t) = K_{t+1} - (1 - \delta)K_t + C_t \end{aligned} \quad (4.9)$$

Step 4. Compute the Euler equation.

To compute the Euler equation we use the Lagrangian method. In addition to this method, two other approaches are the reduced form objective and a constraints optimization problem. All objectives are to solve the Bellman equation, yielding the same Euler equation. To compute the Euler equation, we compare three possible methods:

⁷ From the recursive method and the law of iterated expectation, $V_t(K_t, A_t) = E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j})$
 $= u(C_t) + E_t \sum_{j=0}^{\infty} \beta^{j+1} u(C_{t+1+j}) = u(C_t) + \beta E_t V(K_{t+1}, A_{t+1})$. So $V(K_t, A_t) = \underset{C_t, K_{t+1}}{\text{Max}} \{u(C_t) + \beta E_t V(K_{t+1}, A_{t+1})\}$.

Approach 1. Using the reduced form objective:

$$\begin{aligned} V_t(K_t, A_t) &= \underset{C_t, K_{t+1}}{\text{Max}} \{u(C_t) + \beta E_t V(K_{t+1}, A_{t+1})\} \\ &= \underset{K_{t+1}}{\text{Max}} \{u(F(A_t, K_t) - K_{t+1} + (1 - \delta)K_t) + \beta E_t V(K_{t+1}, A_{t+1})\} \end{aligned} \quad (4.10)$$

There is only one control variable, K_{t+1} , remain in the value function since the relationship governing C_t and K_{t+1} is integrated in the objective function. To achieve the first order necessary condition for K_{t+1} , differentiate the right hand side of the Bellman equation with respect to control variable remain. The first order condition for K_{t+1} is:

$$\frac{du(C_t)}{dC_t} \frac{\partial C_t}{\partial K_{t+1}}(K_t, A_t) + \beta E_t \frac{\partial V}{\partial K_{t+1}}(K_{t+1}, A_{t+1}) = 0 \quad (4.11)$$

Alternatively, $u_{C_t}(C_t) = \beta E_t V_{K_{t+1}}(K_{t+1}, A_{t+1})$. To obtain the envelope equation first differentiate both sides of the Bellman equation with respect to the controlled state variables, K_t , and use the first order necessary conditions to simplify the equation.⁸

$$\begin{aligned} V_{K_t}(K_t, A_t) &= \frac{du(C_t)}{dC_t} \frac{\partial C_t}{\partial K_t}(K_t, A_t) \\ &\quad - \frac{du(C_t)}{dC_t} \frac{\partial K_{t+1}}{\partial K_t}(K_t, A_t) + \beta E_t V_{K_{t+1}}(K_{t+1}, A_{t+1}) \frac{\partial K_{t+1}}{\partial K_t}(K_t, A_t) \end{aligned} \quad (4.12)$$

alternatively, $V_{K_t}(K_t, A_t) = \frac{du(C_t)}{dC_t} \frac{\partial C_t}{\partial K_t}(K_t, A_t) = u_{C_t}(C_t)[F_{K_t}(A_t, K_t) + (1 - \delta)]$

Defines the gross rate of return on a one period investment in capital as:

⁸ From the first order condition, $-\frac{du(C_t)}{dC_t} \frac{\partial K_{t+1}}{\partial K_t} + \beta E_t V_{K_{t+1}}(K_{t+1}, A_{t+1}) \frac{\partial K_{t+1}}{\partial K_t} = 0$

$$[F_{K_{t+1}}(A_{t+1}, K_{t+1}) + (1 - \delta)] = R_{t+1} \quad (4.13)$$

which is equal to the marginal product of capital plus depreciated capital. The envelope condition can be written as $V_{K_t}(K_t, A_t) = u_{C_t}(C_t)R_t$. Combine the first order necessary conditions and the one period updated envelope condition to obtain an Euler equation. With $V_{K_{t+1}}(K_{t+1}, A_{t+1}) = u_{C_{t+1}}(C_{t+1})R_{t+1}$, the Euler equation is:

$$u_{C_t}(C_t) = \beta E_t \{u_{C_{t+1}}(C_{t+1})R_{t+1}\}. \quad (4.14)$$

Approach 2. Treat the maximization problem as a constrained optimization problem and use Lagrangian techniques to put the constraints outside the utility function:

$$\begin{aligned} V_t(K_t, A_t) = \underset{C_t, K_{t+1}, \lambda_t}{\text{Max}} \{ & [u(C_t) + \beta E_t V(K_{t+1}, A_{t+1})] \\ & + \lambda_t [F(A_t, K_t) - K_{t+1} + (1 - \delta)K_t - C_t] \} \end{aligned} \quad (4.15)$$

In this case, there are two control variables and one Lagrange multiplier.

The first order necessary conditions with respect to C_t , K_{t+1} and λ_t are as below respectively:

$$u_{C_t}(C_t) = \lambda_t \quad (4.16)$$

$$\beta E_t V_{K_{t+1}}(K_{t+1}, A_{t+1}) = \lambda_t \quad (4.17)$$

$$F(A_t, K_t) - K_{t+1} + (1 - \delta)K_t - C_t = 0. \quad (4.18)$$

Differentiate both side of the Bellman equation with controlled state variable, the result is:

$$V_{K_t}(K_t, A_t) = \frac{du(C_t)}{dC_t} \frac{\partial C_t}{\partial K_t} + \beta E_t V_{K_{t+1}}(K_{t+1}, A_{t+1}) \frac{\partial K_{t+1}}{\partial K_t} + \lambda_t [F_{K_t}(A_t, K_t) + (1 - \delta)]$$

$$-\lambda_t \frac{\partial K_{t+1}}{\partial K_t} - \lambda_t \frac{\partial C_t}{\partial K_t} \quad (4.19)$$

Combine with the first order conditions to obtain the envelope condition:

$$V_{K_t}(K_t, A_t) = \lambda_t [F_{K_t}(A_t, K_t) + (1 - \delta)] = u_{C_t}(C_t) R_t \quad (4.20)$$

Update the envelope condition one period and combine with the first order condition to have the Euler equation, $u_{C_t}(C_t) = \beta E_t \{u_{C_{t+1}}(C_{t+1}) R_{t+1}\}$.

Approach 3. Instead of using the Bellman equation, this method forms the Lagrangian function and then maximizes it. Set up the Lagrangian function as:

$$L = \sum_{t=0}^{\infty} \beta^t u(C_t) + \sum_{t=0}^{\infty} \beta^t E_t \lambda_t [A_t^\alpha K_t^{1-\alpha} - K_{t+1} + (1 - \delta)K_t - C_t] \quad (4.21)$$

It is important to note that the constraints in this case are in the sequential form. Unlike the first order conditions for K_{t+1} of the second approach, the third approach requires the differentiation with respect to K_{t+1} of both the updated one period and the original Lagrangian function. Therefore, the first order necessary conditions with respect to C_t , K_{t+1} and λ_t are as below respectively:

$$u_{C_t}(C_t) = \lambda_t \quad (4.22)$$

$$\beta^t E_t \lambda_{t+1} [F_{K_{t+1}}(A_{t+1}, K_{t+1}) + (1 - \delta)] = \lambda_t \quad (4.23)$$

$$F(A_t, K_t) - K_{t+1} + (1 - \delta)K_t - C_t = 0 \quad (4.24)$$

The first equation of the necessary conditions interprets λ_t as the shadow price of wealth. The left hand side of the second represents the expected marginal gain from investing one unit in the current period. The term, $[F_{K_{t+1}}(A_{t+1}, K_{t+1}) + (1 - \delta)]$, represents the one period

return to capital. After removing λ_t in the first two equations of the first order conditions by substitution, the Euler equation is $u_{C_t}(C_t) = \beta E_t \{u_{C_{t+1}}(C_{t+1})R_{t+1}\}$.

The Euler equation is a necessary condition for optimality for any time period⁹. It essentially represents the intertemporal efficiency condition. The left hand side represents the marginal cost in terms of utility of investing in more capital. In other words, households reduce their consumption by one unit today, allowing for one more unit investment today and thus one more unit of expected capital tomorrow. This expectedly additional unit of capital yields expectedly additional production equal to the expectedly marginal product of capital and after production $1 - \delta$ unit of the expected capital still remains. Consequently, the right hand side of the Euler equation represents the expected marginal utility gain. If it is held, then it is impossible to increase utility by moving consumption across adjacent periods. At an optimum, these costs must be equal to benefits.

4.4.2 Making the model stationary

Since the real business cycle model is laid by the foundation of the neoclassical growth model, the goal now is to analyze the local dynamics around the steady state or balance growth path, where all variables in the model grow at a common rate of the exogenous growth rate of technology, $\gamma \approx (1 + g) = A_{t+1}/A_t$. The steady state value of any variable is denoted by dropping the time index. It is conventional to transform its variables in term of the variables that will be constant in the steady state. Rewrite the Euler equation as:

$$1 = \beta E_t \{ [u_{C_{t+1}}(C)/u_{C_t}(C)] R_{t+1} \} \quad (4.25)$$

With the period utility function, the Euler equation becomes:

$$1 = \beta E_t \{ [C_{t+1}/C_t]^{-\sigma} R_{t+1} \} \quad (4.26)$$

⁹ Baxter, Crucini and Rouwenhorst (1990) propose an Euler equation approach to solve the stochastic growth model. Baxter (1991) uses an Euler equation approach to solve suboptimal dynamic equilibria.

In the balance growth path, the gross rate of return is constant, the Euler equation eventually shows that $\gamma^\sigma = \beta R$. This condition involves the equilibrium growth rate of consumption to the real interest rate. In steady state, the faster technology growth implies the faster the consumption growth. Consumption growth is higher if the rate of return on capital is higher.

Since in steady state $R = (1 - \alpha)(A/K)^\alpha + (1 - \delta)$, it is possible to analyze the steady state representation of the equilibrium of this economy by the technology-capital ratio, A/K .

By substituting the steady state gross rate of return, R , in the Euler equation, $\gamma^\sigma = \beta R$, one can derive the technology-capital ratio as:¹⁰

$$\frac{A}{K} = \left[\frac{(r + \delta)}{(1 - \alpha)} \right]^{\frac{1}{\alpha}} \quad (4.27)$$

Therefore, the technology-capital ratio is constant at steady state. Given a level of technology, a higher rate of technology growth causes a lower capital stock. From the Cobb-Douglas production function, the output-capital ratio can be expressed in term of the technology-capital ratio:

$$\frac{Y}{K} = \left[\frac{A}{K} \right]^\alpha \quad (4.28)$$

The output-capital ratio is again also constant in the balance growth path. Given $F(A_t, K_t) = K_{t+1} - (1 - \delta)K_t + C_t$, dividing it by output and evaluating in steady state, the result is the consumption-output ratio¹¹. The consumption-output ratio is approximately

¹⁰ Since $\gamma^\sigma = \beta R$, $\gamma^\sigma = \beta[(1 - \alpha)(A/K)^\alpha + (1 - \delta)]$. Therefore, $\frac{A}{K} = \left[\frac{(\gamma^\sigma / \beta) - (1 - \delta)}{1 - \alpha} \right]^{1/\alpha}$ or $\left[\frac{R - (1 - \delta)}{1 - \alpha} \right]^{1/\alpha}$.

Let $R \approx 1 + r$, $\frac{A}{K} = \left[\frac{r + \delta}{1 - \alpha} \right]^{1/\alpha}$.

¹¹ From $Y_t = K_{t+1} - (1 - \delta)K_t + C_t$, $1 = \frac{K_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t} - (1 - \delta) \frac{K_t}{Y_t} + \frac{C_t}{Y_t}$. At the steady state,

$$\frac{C}{Y} = 1 - \left[\frac{(1-\alpha)(\delta+g)}{(r+\delta)} \right] \quad (4.29)$$

It is again constant at the steady state. The interesting thing about the real business cycle model is that the main ratios at the steady state can be expressed as the function of long run parameterized parameters such as β , g , r , α and δ .

The main objective is to solve for K_t and C_t as a function of the productivity shocks, A_t . Nevertheless, at the steady state there are unpleasantly nonlinear equation systems which are difficult to analyze. Therefore, the strategy is to linearly approximate the behaviour of the system in the steady state. The next section discusses the common approach to transform the model into a system of log-linear difference equations around the steady state.

4.4.3 Log-linearization of the model economy

Basically, the equilibrium equations resulting from the optimization problem are non-linear. We cannot easily analyze the model with non-linear equations because the non-linear system cannot be solved analytically. We want to approximate a solution by transforming the model in to a system of linear equations. By the model character, the economy is able to reach the steady state where all variables grow at the rate of technology change. Therefore, we transform the equilibrium linearly around the steady state and then solve the resulting linear system numerically. Without any disturbance, the strategy at this point is to do log-linear around the steady state and this method produces a good approximation near the stationary point for all variables. Taking a log-linear approximation to the first order conditions allows them to be solved analytically.

This section describes the procedure to approximate the nonlinear system of equations characterizing the equilibrium of this simple model economy with a system consisting of

$$\frac{C}{Y} = 1 - \gamma \frac{K}{Y} + (1-\delta) \frac{K}{Y} \text{ or } \frac{C}{Y} \approx (1-\delta) \left(\frac{1-\alpha}{r-\delta} \right) + 1 - (1+g) \left(\frac{1-\alpha}{r-\delta} \right). \text{ After simplifying, the consumption-output ratio at the steady state is approximately } 1 - [(1-\alpha)(\delta+g)/(r+\delta)].$$

log-linearization around the steady state.¹² We can approximate the deviation of a function around a stationary point by a linear function of the deviation of the variable from its steady state value. Technically speaking, we apply Taylor series approximation around the steady state. The resulting approximated equations are linear and they are easy to solve. According to the first order Taylor expansion, the nonlinear equation is linearized around its steady state, for a function of a single variable we use, where a is a steady state value for x_t :

$$f(x_t) \approx f(a) + f'(a)(x_t - a) \quad (4.30)$$

For a similar argument for a function of two variables:

$$f(x_t, y_t) \approx f(a, b) + f_x(a, b)(x_t - a) + f_y(a, b)(y_t - b) \quad (4.31)$$

For the simplicity to interpret, the deviation from the stationary point ($x_t - a$) is altered to be the percentage change deviation ($\hat{x} = (x_t - a)/a$). Therefore, we express each condition in terms of the percentage change from the steady state. We can also express the percentage change from the steady state as a log term:

$$d \ln x_t \approx \hat{x}_t = \frac{x_t - a}{a} \quad (4.32)$$

Since we represent the deviation from the steady state as the percentage deviation or log difference, this procedure is called log-linearization. A '^' denotes the logarithm of a given variable deviated from its steady state value. It is also important to know that $x_t - a = \frac{x_t - a}{a} a = a\hat{x}_t$. Applying this procedure to the first order equations, we can derive the log-linearized version of this model economy.

¹² The details of the log-linearization method can also be found in Klein (2002).

Let's start with the Euler equation (4.26). It can be written in term of log-linearization around the steady state as:

$$C(-\sigma C^{-\sigma-1})d \ln C_t = \beta E_t \{ C^{-\sigma} R d \ln R_{t+1} + RC(-\sigma C^{-\sigma-1})d \ln C_{t+1} \}$$

Making use of the fact that at the steady state $1 = \beta R$ associated with (4.32), we can write:

$$-\sigma C^{-\sigma} \hat{C}_t = C^{-\sigma} E_t \hat{R}_{t+1} - \sigma C^{-\sigma} E_t \hat{C}_{t+1}$$

which finally becomes:

$$\sigma E_t \hat{C}_{t+1} = \sigma \hat{C}_t + E_t \hat{R}_{t+1} \quad (4.33)$$

To obtain \hat{R}_{t+1} we need to log-linearize (4.13) around the steady state. We can write it as:¹³

$$R d \ln R_{t+1} = F_{KA}(A, K) A d \ln A_{t+1} + F_{KK}(A, K) K d \ln K_{t+1}$$

Making use of (4.27) and (4.32), we can rewrite as:

$$\hat{R}_{t+1} = \frac{\alpha(r+\delta)}{(1+r)} (\hat{A}_{t+1} - \hat{K}_{t+1}) \quad (4.34)$$

Substituting (4.34) in (4.33), we have the log-linearization of the Euler equation:

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\sigma} \frac{\alpha(r+\delta)}{(1+r)} E_t (\hat{A}_{t+1} - \hat{K}_{t+1}) \quad (4.35)$$

¹³ $F_{KA}(A, K) = \left(\frac{A}{K}\right)^\alpha \frac{\alpha(1-\alpha)}{A}$ and $F_{KK}(A, K) = -\left(\frac{A}{K}\right)^\alpha \frac{\alpha(1-\alpha)}{K}$.

The capital accumulation (4.24) is originally not log-linear. To log-linearize it, the first step is to transform it in term of variables which are constant at the steady state. We divide both sides of (4.24) by K_t :

$$\frac{K_{t+1}}{K_t} = (1 - \delta) + \left(\frac{A_t}{K_t} \right)^\alpha - \frac{C_t}{K_t} \quad (4.36)$$

By log-linearizing it around the steady state, leaving:

$$\begin{aligned} K_{t+1}K_t(-K_t^{-2})d\ln K_t + K_t^{-1}K_{t+1}d\ln K_{t+1} \\ = (1 - \delta) + [A^\alpha K(-\alpha K^{-\alpha-1})d\ln K_t + K^{-\alpha}A\alpha A^{\alpha-1}d\ln A_t] \\ - [CK(-K^{-2})d\ln K_t + K^{-1}Cd\ln C_t] \end{aligned}$$

Applying (4.32) and rearranging, we obtain:

$$\gamma(\hat{K}_{t+1} - \hat{K}_t) = \left[\frac{C}{K} - \alpha \left(\frac{A}{K} \right)^\alpha \right] \hat{K}_t + \alpha \left(\frac{A}{K} \right)^\alpha \hat{A}_t - \left(\frac{C}{K} \right) \hat{C}_t \quad (4.37)$$

At steady state, making use of (4.36) and the fact that $\gamma = \frac{K_{t+1}}{K_t}$, (4.37) can be written as:¹⁴

$$\gamma \hat{K}_{t+1} = (R - \gamma) \hat{K}_t + \gamma \hat{K}_t + \alpha \left(\frac{A}{K} \right)^\alpha \hat{A}_t - \left(\frac{C}{K} \right) \hat{C}_t \quad (4.38)$$

Applying (4.27) to (4.29) with (4.38), finally we have a linear equation linking next period's capital to today's capital, technology and consumption as below:

¹⁴ Since $\gamma = (1 - \delta) + \left(\frac{A}{K} \right)^\alpha - \frac{C}{K}$ at the steady state, $\frac{C}{K} - \alpha \left(\frac{A}{K} \right)^\alpha = (1 - \delta) + \left(\frac{A}{K} \right)^\alpha - \gamma - \alpha \left(\frac{A}{K} \right)^\alpha$. Therefore, it can be written as $\frac{C}{K} = R - \gamma$

$$\hat{K}_{t+1} = \left[\frac{1+r}{1+g} \right] \hat{K}_t + \left[\frac{\alpha(r+\delta)}{(1-\alpha)(1+g)} \right] \hat{A}_t + \left[\frac{(\delta+g)(1-\alpha)-(r+\delta)}{(1+g)(1-\alpha)} \right] \hat{C}_t \quad (4.39)$$

Since the real business cycle model requires the productivity shock to be a driving process of the model economy, it is necessary to specify the process of technology shock as log-linear around the steady state. Considering (4.7), the technology process is assumed to follow the first-order autoregression process AR(1) and it can be expressed log-linearly around the steady state as:

$$\hat{A}_{t+1} = \rho \hat{A}_t + \varepsilon_{t+1} \quad (4.40)$$

Equation (4.35), (4.39) and (4.40) form a system of log-linear expectation difference equations in capital, technology and consumption.

4.5 Solving linear rational expectation models

There are nowadays a number of algorithms for solving linear rational expectation model that we can choose. After the pioneering contribution of Blanchard and Khan (1980), several approaches of representing a system of linear rational expectation equations have been proposed. Several methods have built upon the original work to provide slightly different variations such as King and Watson (1998, 2002), Klein (2000), Christiano (2002) and Sims (2002). The Blanchard and Khan approach can work only on the non-singular system of linear rational expectation model, King and Watson proposed the method to solve the singular system. Christiano's method uses the higher order approximation, a second-order Taylor approximation of the log of the variables by first substituting all constraints into the objective function. Sims approach takes into account the expectational error terms and the system does not require explicitly which variables are predetermined and which are non-predetermined variables. The Blanchard and Khan work introduces the original concept to solve the linear rational expectation system; therefore, it is essential to understand the requirements to solve the system and the shortcomings of these approaches. Since King and Watson's method directly covers the drawback of the original work, this section reviews a

general representation of the linear rational expectation model and two methods of solving a system of linear rational expectation equation developed by Blanchard and Khan (1980) and King and Watson (1998)¹⁵.

4.5.1 The general representation

In general, a macroeconomic model for which the solution is derived can be represented in the linear rational expectation form:

$$AE_t y_{t+1} = B y_t + \sum_{j=0}^J C_j E_t x_{t+j} \quad (4.41)$$

where y_t and x_t is defined as a vector of endogenous and a vector of exogenous variables respectively. A , B and $C_0, C_1 \dots C_J$ are the coefficient matrices of the system. The vector of endogenous variables, y_t , are composed of a predetermined variable and non-predetermined variables. The value at $t+1$ of a predetermined variable is determined at period t . The driving process of the model is usually assumed to be the driving process of the exogenous variables, x_t , and it is represented in the form:

$$x_t = \omega \zeta_t \quad (4.42)$$

$$\zeta_t = \rho \zeta_{t-1} + G_t \varepsilon_t \quad (4.43)$$

where ω is a matrix relating the exogenous variables, x_t , to the exogenous state variables, ζ_t . ζ_t is governed by the AR(1) process and ρ is the coefficient matrix indicating how persistent ζ between the last and current period. G_t governs the response of the exogenous state variables to the innovation, ε_t . Next section gives the solution together with the necessary conditions for the existence of a stable and unique solution.

¹⁵ In details also see Blanchard and Khan (1980), King and Watson (1988). For details of the other methods of solving the linear rational expectation model, see also Taylor and Uhlig (1990), Judd (1992) Klien (2000), Christiano (2002), Sims (2002) and Grohe and Uribe (2004). All methods are mentioned in this footnote are able to solve for a singular system except for the one proposed by Blanchard and Khan.

4.5.2 Blanchard and Khan (1980) Method

Blanchard and Khan representation depends on the assumption that the endogenous variables can be easily separated into both the predetermined and non-predetermined variables. They proposed the eigenvalue decomposition method. This method basically decouples the system obtaining two independent difference equations. The eigenvalue or the root associated with the first equation is positive and stable; thus, this equation can be solved by backward recursive. On the other hand, the root of the second equation is unstable; therefore; this equation must be solved forward recursive. This approach is applied to a non-singular dynamic system. The necessary condition for a unique existence of solution is that the counting rule must be satisfied. Alternatively, the number of non-predetermined variables equals the number of unstable roots or the number of predetermined variables equal the number of stable roots.¹⁶

The Blanchard and Khan approach develops how to solve the rational expectation macroeconomics model with a mixture of predetermined and non-predetermined variable framework. Consider the endogenous matrix:

$$y_t = \begin{bmatrix} \Lambda_t \\ k_t \end{bmatrix} \quad (4.44)$$

where Λ_t is non-predetermined variable and k_t is predetermined variable in the sense that it does not respond to new information at period t , the macroeconomics model with A invertible or non-singular matrix can be written as:

$$AE_t y_{t+1} = By_t + CE_t x_t \quad (4.45)$$

and therefore, $E_t y_{t+1} = A^{-1}By_t + A^{-1}CE_t x_t$. For any variables, $E_t W_{t+s} = E_t(W_{t+s} | \Omega_t)$ where $\Omega_{t-1} \leq \Omega_t$ and $(y_t, x_t) \in \Omega_t$, where Ω_t represent the information available at t . Taking the

¹⁶ Firstly, the dynamic system is explosive when both eigenvalues are unstable. Secondly, the dynamics system has a large number of solutions if both roots are stable. Lastly, the dynamic model has a unique solution if one root is stable and another is unstable. For details, see also Blanchard and Khan (1980).

left eigenvectors of $W = A^{-1}B$ to be L ; it follows that $LW = JL$. Then take $T = LA^{-1}$ and $V = L$ we can write:

$$E_t y_{t+1} = W y_t + A^{-1} C E_t x_t \quad (4.46)$$

Multiply both sides by L , yielding $LE_t y_{t+1} = L W y_t + L A^{-1} C E_t x_t$. Therefore:

$$E_t L y_{t+1} = L W L^{-1} L y_t + T C E_t x_t, \text{ where } L W L^{-1} = J \quad (4.47)$$

alternatively, $E_t V y_{t+1} = J V y_t + T C E_t x_t$. Let $V y_t = y_t^*$, so we can write:

$$E_t y_{t+1}^* = J y_t^* + C^* E_t x_t \quad (4.48)$$

Since $y_t^* = \begin{bmatrix} u' & s' \end{bmatrix}' = \begin{bmatrix} u \\ s \end{bmatrix}$, it follows that:

$$E_t \begin{bmatrix} u_{t+1} \\ s_{t+1} \end{bmatrix} = \begin{bmatrix} J_u & 0 \\ 0 & J_s \end{bmatrix} \begin{bmatrix} u_t \\ s_t \end{bmatrix} + \begin{bmatrix} C_u^* \\ C_s^* \end{bmatrix} E_t x_t \quad (4.49)$$

The equation (4.49) can be considered to be two difference equations, one for u_t and another for s_t . Therefore, equation (4.49) is decoupled for u_t and s_t . The stability requirement implies that the equations for u_t , $E_t u_{t+1} = J_u u_t + C_u^* E_t x_t$, must be solved forward since the eigenvalue of J_u are greater than one in modulus, yielding the solution as follow:¹⁷

$$u_t = -\frac{1}{(J_u - FI)} C_u^* x_t = -(J_u - FI)^{-1} C_u^* x_t \quad (4.50)$$

Since $y^* = V y$, we can define the partitioned transformations as:

¹⁷ Since $E_t u_{t+1} = J_u u_t + C_u^* E_t x_t$, $u_t = \frac{1}{J_u} (E_t u_{t+1} - C_u^* E_t x_t)$. Solve for u_t by recursive forwards,

$$\begin{aligned} u_t &= \frac{1}{J_u} \left\{ E_t \left(\frac{1}{J_u} [E_{t+1} u_{t+2} - C_u^* E_{t+1} x_{t+1}] - C_u^* E_t x_t \right) \right\} = \frac{1}{J_u^2} E_t u_{t+2} - \frac{1}{J_u^2} C_u^* E_{t+1} x_{t+1} - \frac{1}{J_u} C_u^* E_t x_t = \dots = \\ u_t &= \frac{1}{J_u^k} E_t u_{t+k} - \frac{1}{J_u^k} C_u^* \sum_{j=0}^{k-1} E_t x_{t+j} = -\sum_{j=0}^{\infty} \frac{1}{J_u^{j+1}} C_u^* E_t x_{t+j} = -\sum_{j=0}^{\infty} \left(\frac{1}{J_u} \right)^j F^j \frac{1}{J_u} C_u^* x_t = -\frac{1}{\left(1 - \frac{1}{J_u} F\right) J_u} C_u^* x_t \end{aligned}$$

$$\begin{bmatrix} u \\ s \end{bmatrix} = \begin{bmatrix} V_{u\Lambda} & V_{uk} \\ V_{s\Lambda} & V_{sk} \end{bmatrix} \begin{bmatrix} \Lambda \\ k \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \Lambda \\ k \end{bmatrix} = \begin{bmatrix} R_{\Lambda u} & R_{\Lambda s} \\ R_{ku} & R_{ks} \end{bmatrix} \begin{bmatrix} u \\ s \end{bmatrix} \quad \text{with } R = V^{-1} \quad (4.51)$$

Therefore, $u_t = V_{u\Lambda}\Lambda_t + V_{uk}k_t$ and $\Lambda_t = V_{u\Lambda}^{-1}[u_t - V_{uk}k_t]$ and the unique solution requires that $V_{u\Lambda}$ is a non-singular matrix. To complete the solution of the model, since $k_t = R_{ku}u_t + R_{ks}s_t$, $k_{t+1} = R_{ku}u_{t+1} + R_{ks}s_{t+1}$ and $E_t s_{t+1} = J_s s_t + C_s^* E_t x_t$ also $s_t = V_{s\Lambda}\Lambda_t + V_{sk}k_t$ it follows that:¹⁸

$$k_{t+1} = R_{ku}E_t u_{t+1} + R_{ks}J_s V_{s\Lambda} V_{u\Lambda}^{-1}(u_t - V_{uk}k_t) + R_{ks}J_s R_{ks}^{-1}k_t + R_{ks}C_s^* x_t \quad (4.52)$$

Λ_t and k_{t+1} can be used recursively to construct $\{k_t, \Lambda_t, y_t\}_{t=1}^{\infty}$ given initial conditions for k_0 and x_0 together with the solution for u_t .

4.5.3 The King and Watson (1998) Method

Since many economic models do not fit directly into the Blanchard and Khan framework, The King and Watson approach introduces a way to solve the model with a singular system.¹⁹ King and Watson also employ the eigenvalue decomposition method. But they solve predetermined and non-predetermined variables separately. The early work of Blanchard and Khan is $AE_t y_{t+1} = By_t + CE_t x_t$, and the requirement is that the matrix A is non-singular or $|A| \neq 0$. However, it can be the case that in first-order form if A is permitted to be singular. The macroeconomics model (4.45) is $AE_t y_{t+1} = By_t + CE_t x_t$. Assume that last p elements of y_t are predetermined. Therefore, y_t follows the equation (4.44) where Λ_t is a matrix of non-predetermined variables. k_t is predetermined variables. The model can be rewritten as $AE_t y_{t+1} - BE_t y_t = CE_t x_t$ or alternatively as:

$$(AZ - B)E_t y_t = CE_t x_t \quad (4.53)$$

The requirement for the solvability of this model is that $|AZ - B| \neq 0$ and the counting rule of Blanchard and Khan (1980) must be held.

¹⁸ Since $k_{t+1} = R_{ku}E_t u_{t+1} + R_{ks}[J_s s_t + C_s^* E_t x_t]$ and after doing recursive forwards,

$$k_{t+1} = R_{ku}E_t u_{t+1} + R_{ks}J_s V_{s\Lambda} \{V_{u\Lambda}^{-1}[u_t - V_{uk}k_t]\} + R_{ks}J_s V_{sk}k_t + R_{ks}C_s^* x_t$$

¹⁹ Also see King and Watson (1980) in details.

4.6 Solving for decision rules of real business cycle model

Ultimately, we have the optimal paths of control variables as a function of the optimal paths of state variables or what we simply call a policy function or decision rule. Consequently, the model can be solved for a policy function at any given time period. However, according to section 4.4, the feasible solutions of the model can be broadly divided into three main groups. Firstly, the solutions are explosive or the model cannot be solved analytically. Secondly, there are a large number of solutions. Thirdly, the dynamic system has a unique solution. The type of plausible solutions of the dynamics system heavily depends upon the characteristic roots and vectors of one matrix in the fundamental equation. Two approaches to solve the decision rules are discussed in this section, the method of undertermined coefficients and linear approximation to the first order conditions.

4.6.1 The method of undertermined coefficients

To solve for the policy funtion, Campbell (1994) suggests the method of underternined coefficients.²⁰ The method of undetermined coefficients is an approach to finding the particular solution to linear constant-coefficient differential equations. The main idea is to make a guess with the undetermined coefficients to the form that the solution will take and to solve for any unknown coefficients. This approach is therefore called the method of undetermined coefficients. To see the application of this approach, the linear optimal equations of the real business cycle model mentioned in section 4.3 is used as an example.

The equation (4.35), (4.39) and (4.40) can respectively be rewritten of the form:

$$\begin{aligned}E_t \hat{C}_{t+1} &= \eta_{CC} \hat{C}_t + \eta_{CA} E_t \hat{A}_{t+1} + \eta_{CK} E_t \hat{K}_{t+1} \\ \hat{K}_{t+1} &= \eta_{KK} \hat{K}_t + \eta_{KA} \hat{A}_t + \eta_{KC} \hat{C}_t \\ \hat{A}_{t+1} &= \rho \hat{A}_t + \varepsilon_{t+1}\end{aligned}$$

²⁰ Zadrozy (1998) also suggest an eigenvalue method of undetermined coefficients for solving linear rational expectations models.

It is important to note that we can pass the expectation operator through by $E_t \hat{K}_{t+1} = \hat{K}_{t+1}$, since K_{t+1} is a choice variable at time t . Given $E_t(\varepsilon_t) = 0$, the expectation of (4.40) becomes $E_t A_{t+1} = \rho A_t$. The system of equations above can be transformed to the form where each variable at $t+1$ is expressed in term of the other variables at time t . The system simplifies to $E_t \hat{C}_{t+1} = \eta_{CC} \hat{C}_t + \eta_{CA} \rho \hat{A}_t + \eta_{CK} (\eta_{KK} \hat{K}_t + \eta_{KA} \hat{A}_t + \eta_{KC} \hat{C}_t)$. After grouping the similar terms, leaving:

$$E_t \hat{C}_{t+1} = \eta_{CK} \eta_{KK} \hat{K}_t + (\rho \eta_{CA} + \eta_{CK} \eta_{KA}) \hat{A}_t + \eta_{CK} \eta_{KC} \hat{C}_t \quad (4.54)$$

it is convenient to assign a new variable, ς , for the combination of η . The linear system of difference equations becomes:

$$E_t \hat{C}_{t+1} = \varsigma_{CK} \hat{K}_t + \varsigma_{CA} \hat{A}_t + \varsigma_{CC} \hat{C}_t \quad (4.55)$$

$$\hat{K}_{t+1} = \eta_{KK} \hat{K}_t + \eta_{KA} \hat{A}_t + \eta_{KC} \hat{C}_t \quad (4.56)$$

$$\hat{A}_{t+1} = \rho \hat{A}_t + \varepsilon_{t+1} \quad (4.57)$$

The solution of this linear system of the consumption policy function should represent consumption as a linear combination of the state variables. Since all equations are linear, conjecture a linear decision rule for C_t as a function of state variables K_t and A_t . Then:

$$C_t = \xi_{CK} \hat{K}_t + \xi_{CA} \hat{A}_t \quad (4.58)$$

Substituting the updating one period of the decision rule (4.58) in (4.55) for C_{t+1} and substituting the decision rule in (4.55) and (4.56):

$$E_t (\xi_{CK} \hat{K}_{t+1} + \xi_{CA} \hat{A}_{t+1}) = \varsigma_{CK} \hat{K}_t + \varsigma_{CA} \hat{A}_t + \varsigma_{CC} (\xi_{CK} \hat{K}_t + \xi_{CA} \hat{A}_t) \quad (4.59)$$

$$\hat{K}_{t+1} = \eta_{KK} \hat{K}_t + \eta_{KA} \hat{A}_t + \eta_{KC} (\xi_{CK} \hat{K}_t + \xi_{CA} \hat{A}_t) \quad (4.60)$$

$$\hat{A}_{t+1} = \rho \hat{A}_t + \varepsilon_{t+1} \quad (4.61)$$

Taking the expectation of (4.61) and substitute in (4.59) to get rid of $E_t A_{t+1}$:

$$\xi_{CK} \hat{K}_{t+1} = (\varsigma_{CK} + \varsigma_{CC} \xi_{CK}) \hat{K}_t + (\varsigma_{CA} + (\varsigma_{CC} - \rho) \xi_{CA}) \hat{A} \quad (4.62)$$

Making use of (4.60) to eliminate \hat{K}_{t+1} in (4.62) and grouping the similar terms:

$$\begin{aligned} & [\xi_{CK}(\eta_{KK} + \eta_{KC} \xi_{CK}) - (\varsigma_{CK} + \varsigma_{CC} \xi_{CK})] \hat{K}_t \\ & = [(\varsigma_{CA} + (\varsigma_{CC} - \rho) \xi_{CA}) - \xi_{CK}(\eta_{KA} + \eta_{KC} \xi_{CA})] \hat{A}_t \end{aligned} \quad (4.63)$$

We can solve for the first undertermined coefficient, ξ_{CK} , using (4.63). Each term in (4.63) must be separately zero. Using the coefficient of \hat{K}_t to solve for ξ_{CK} :

$$[\xi_{CK}(\eta_{KK} + \eta_{KC} \xi_{CK}) - (\varsigma_{CK} + \varsigma_{CC} \xi_{CK})] = 0$$

$$\text{Alternatively,} \quad \eta_{KC} \xi_{CK}^2 + (\eta_{KK} - \varsigma_{CC}) \xi_{CK} - \varsigma_{CK} = 0 \quad (4.64)$$

The stability of the system depends critically on the ξ_{CK} . It is necessary to have $|\xi_{CK}| < 1$ for an unexplosive solution. Solve (4.64) for ξ_{CK} and select the positive root or the one that leads to the stable solution.

Given ξ_{CK} from solving (4.64), ξ_{CA} can be found by setting the coefficient of \hat{A}_t from (4.63) equal zero:

$$[(\varsigma_{CA} + (\varsigma_{CC} - \rho) \xi_{CA}) - \xi_{CK}(\eta_{KA} + \eta_{KC} \xi_{CA})] = 0 \quad (4.65)$$

With the simulation of (4.60) and (4.61), we can find the value of capital and productivity at any time and in turn can find the decision rule for consumption by substituting ξ_{CK} from solving (4.64) and ξ_{CA} from solving (4.65) in (4.58). It is possible to solve for other flow variables such as output and investment. Output can also be expressed as a linear combination of capital and productivity at any time by log-linearize the production function. Applying log-linearization around the steady state of the production function, giving us $\hat{Y}_t = (1-\alpha)\hat{K}_t + \alpha\hat{A}_t$. To solve for investment, rearrange the resource constraint and log-linearize it around the steady state, having $\hat{I}_t = \hat{Y}_t - \hat{C}_t$.

4.6.2 The method of finding a linear approximation to the first order conditions

King, Plosser and Rebelo (1988a) provide an excellent way to solve for the decision rule. In general, the system dynamics of log-linearization can be separated into three parts: measurement equations, state equations and flow equations. The first vector equation is a measurement equation, containing the static equations of the system and linking the control variables to state and exogenous variables. The second vector equation denotes a state equations, explaining the dynamic relationship of state variables to state, control and exogenous variables. Basically, the state equations govern the dynamics of the model. The flow or the extra equations govern the particular interested variables and relate them to state and exogenous variables.

This section applies the KPR approach to solve the simple real business cycle with no government spending as an example. The key idea to solve the model is that we want to determine the fundamental of dynamic system which governs the optimal paths of state, k and co-state variables, λ . Given the optimal solutions of state and co-state variables, we solve for the optimal paths of the control variables. Therefore, we log-linearize (4.22) and (4.23) rather than the Euler equation.

Approximation of the (4.22) and (4.23) near the steady state yields the expressions:

$$-\sigma\hat{C}_t = \hat{\lambda}_t \quad (4.66)$$

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + \beta(1-\alpha)\alpha\left(\frac{A}{K}\right)^\alpha \hat{A}_{t+1} - \beta(1-\alpha)\alpha\left(\frac{A}{K}\right)^\alpha \hat{K}_{t+1} \quad (4.67)$$

Simplifying the resource constraint (4.38):

$$\gamma\hat{K}_{t+1} = R\hat{K}_t + \alpha\left(\frac{A}{K}\right)^\alpha \hat{A}_t - \left(\frac{C}{K}\right)\hat{C}_t \quad (4.68)$$

The linear system of difference equations consist of (4.66), (4.67) and (4.68). In this model economy, the control vector, the state vector and the exogenous vector are given by:

$$\mathbb{C}_{1 \times 1} = [\hat{c}], \quad \mathbb{S}_{2 \times 1} = \begin{bmatrix} \hat{k} \\ \hat{\lambda} \end{bmatrix} \quad \text{and} \quad \mathbb{Z}_{1 \times 1} = [\hat{A}_t].$$

Since equation (4.66) relates controls to state, this equation can be expressed as the measurement system equations below:

$$M_{CC} [\mathbb{C}]_{1 \times 1} = M_{CS} [\mathbb{S}]_{2 \times 1} + M_{Ce} [\mathbb{Z}]_{1 \times 1} \quad (4.69)$$

M_{CC} , M_{CS} and M_{Ce} are the matrix related control to control variables, the matrix related control to state variables and the matrix related control to exogenous variables respectively. Specifically, $M_{CC} = [-\sigma]_{1 \times 1}$, $M_{CS} = [0 \quad 1]_{1 \times 2}$, and $M_{Ce} = [0]_{1 \times 1}$. Equation (4.67) and (4.68) together can be expressed as a difference equation relating the state vector to variations in control and exogenous vectors. It follows:

$$M_{SS}(B) [\mathbb{S}_{t+1}]_{2 \times 1} = M_{SC}(B) [\mathbb{C}_{t+1}]_{1 \times 1} + M_{Se}(B) [\mathbb{Z}_{t+1}]_{1 \times 1} \quad (4.70)$$

where $M_{SS}(B)$, $M_{SC}(B)$ and $M_{Se}(B)$ are the matrix polynomials in the backshift operator B at most power 1. M_{SS} is the matrix relating state to state variables. M_{SC} is the matrix

related state to control variables. M_{se} is the matrix related state to exogenous variables. It equivalently expresses in more detail below:

$$\begin{aligned}
 & M_{ss0} [\mathbb{S}_{t+1}]_{2 \times 1} + M_{ss1} [\mathbb{S}_t]_{2 \times 1} \\
 & = M_{sc0} [\mathbb{C}_{t+1}]_{1 \times 1} + M_{sc1} [\mathbb{C}_t]_{1 \times 1} + M_{se0} [\mathbb{Z}_{t+1}]_{1 \times 1} + M_{se1} [\mathbb{Z}_t]_{1 \times 1} \quad (4.71)
 \end{aligned}$$

where $M_{ss0} = \begin{bmatrix} \beta(1-\alpha)\alpha(\frac{A}{K})^\alpha & -1 \\ \gamma & 0 \end{bmatrix}_{2 \times 2}$, $M_{ss1} = \begin{bmatrix} 0 & 1 \\ -R & 0 \end{bmatrix}_{2 \times 2}$, $M_{sc0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2 \times 1}$,
 $M_{sc1} = \begin{bmatrix} 0 \\ -\frac{C}{K} \end{bmatrix}_{2 \times 1}$, $M_{se0} = \begin{bmatrix} \beta(1-\alpha)\alpha(\frac{A}{K})^\alpha \\ 0 \end{bmatrix}_{2 \times 1}$, and $M_{se1} = \begin{bmatrix} 0 \\ \alpha(\frac{A}{K})^\alpha \end{bmatrix}_{2 \times 1}$

Next step is to derive the system of linear difference equations in state, and co state and exogenous variables. Given the fact that M_{cc} is invertible because of the strict concavity of utility function, we update (4.69) one period and yield:

$$[\mathbb{C}]_{1 \times 1} = M_{cc}^{-1} M_{cs} [\mathbb{S}]_{2 \times 1} + M_{cc}^{-1} M_{ce} [\mathbb{Z}]_{1 \times 1} \quad (4.72)$$

Replace (4.72) in (4.70), yield a system of linear difference equations in state, co-state and exogenous variables as:

$$(M_{ss}(B) - M_{sc}(B)M_{cc}^{-1}M_{cs})[\mathbb{S}_{t+1}]_{2 \times 1} = (M_{sc}(B)M_{cc}^{-1}M_{ce} + M_{se}(B))[\mathbb{Z}_{t+1}]_{1 \times 1} \quad (4.73)$$

or alternatively:

$$M_{ss}^*(B)[\mathbb{S}]_{2 \times 1} = M_{se}^*(B)[\mathbb{Z}]_{1 \times 1}$$

Therefore,

$$\begin{aligned}
 M_{ss0}^* &= M_{ss0} - M_{sc0}M_{cc}^{-1}M_{cs}, \quad M_{ss1}^* = M_{ss1} - M_{sc1}M_{cc}^{-1}M_{cs} \\
 M_{se0}^* &= M_{se0} + M_{sc0}M_{cc}^{-1}M_{ce} \quad \text{and} \quad M_{se1}^* = M_{se1} + M_{sc1}M_{cc}^{-1}M_{ce}
 \end{aligned}$$

To convert to a normal difference equation form, pre-multiply by the inverse of M_{ss0}^* , yielding the fundamental dynamic system of the neoclassical model:

$$M_{ss0}^{*-1} M_{ss}^*(B) [\mathbb{S}_{t+1}]_{2 \times 1} = M_{ss0}^{*-1} M_{se}^*(B) [\mathbb{Z}_{t+1}]_{1 \times 1}$$

or equivalently,

$$I_{2 \times 2} [\mathbb{S}_{t+1}]_{2 \times 1} = -M_{SS0}^{*-1} M_{SS1}^* [\mathbb{S}_t]_{2 \times 1} + M_{SS0}^{*-1} M_{Se0}^* [\mathbb{Z}_{t+1}]_{1 \times 1} + M_{SS0}^{*-1} M_{Se1}^* [\mathbb{Z}_t]_{1 \times 1} \quad (4.74)$$

Let $W_{2 \times 2} = -M_{SS0}^{*-1} M_{SS1}^*$, $R_{2 \times 1} = M_{SS0}^{*-1} M_{Se0}^*$, and $Q_{2 \times 1} = M_{SS0}^{*-1} M_{Se1}^*$

This method can also extend to the stochastic case by assuming the sequence of $\{\mathbb{Z}_t\}$ is stochastic as in Talmain (2002). The dynamics of the economy will be given by (4.74) when \mathbb{S}_{t+1} and \mathbb{Z}_{t+1} are replaced by the conditional expectation as:

$$E_t [\mathbb{S}_{t+1} | \mathbb{Z}_t]_{2 \times 1} = W_{2 \times 2} [\mathbb{S}_t]_{2 \times 1} + R_{2 \times 1} E_t [\mathbb{Z}_{t+1} | \mathbb{Z}_t]_{1 \times 1} + Q_{2 \times 1} [\mathbb{Z}_t]_{1 \times 1} \quad (4.75)$$

Taking the expectation conditional to \mathbb{Z}_{t-1} and using the law of iterated expectation to the (4.75), it can be shown that:²¹

$$\begin{aligned} E_{t-1} [\mathbb{S}_{t+1} | \mathbb{Z}_{t-1}]_{2 \times 1} \\ = W_{2 \times 2} E_{t-1} [\mathbb{S}_t | \mathbb{Z}_{t-1}]_{2 \times 1} + R_{2 \times 1} E_{t-1} [\mathbb{Z}_{t+1} | \mathbb{Z}_{t-1}]_{1 \times 1} + Q_{2 \times 1} E_{t-1} [\mathbb{Z}_t | \mathbb{Z}_{t-1}]_{1 \times 1} \end{aligned} \quad (4.76)$$

The dynamics of the stochastic economy at time t , by backward induction of the conditional expectation, is written as:

$$E_0 [\mathbb{S}_{t+1} | \mathbb{Z}_0]_{2 \times 1} = W_{2 \times 2} E_0 [\mathbb{S}_t | \mathbb{Z}_0]_{2 \times 1} + R_{2 \times 1} E_0 [\mathbb{Z}_{t+1} | \mathbb{Z}_0]_{1 \times 1} + Q_{2 \times 1} E_0 [\mathbb{Z}_t | \mathbb{Z}_0]_{1 \times 1} \quad (4.77)$$

Therefore, (4.74) also describes the dynamics of the stochastic economy for all t when \mathbb{S}_t and \mathbb{Z}_t are interpreted as conditional expectations with respect to \mathbb{Z}_0 .

The solutions of the dynamic system are governed by the characteristic roots and vectors of the matrix $W_{2 \times 2}$. The system in this basic model has one state or predetermined variable, k and one co-state variable, λ . There are three possible solutions depending on the magnitude of the characteristic roots associated with $W_{2 \times 2}$. Since W is 2×2 , there are two characteristic

²¹ According to the law of iterated expectation, $E_{t-1} [\mathbb{S}_{t+1} | \mathbb{Z}_{t-1}]_{2 \times 1} = E_{t-1} [E_t [\mathbb{S}_{t+1} | \mathbb{Z}_t]_{2 \times 1} | \mathbb{Z}_{t-1}]$. Apply this rule term by term for the equation (4.75).

roots associated with it. Let's denote them as μ_1 and μ_2 . To have a unique solution, Blanchard and Khan (1980) suggest the counting rule must hold.

Since there are more than one eigenvalues of matrix W corresponding to the fundamental difference equation, to solve for the optimal paths of state and co-state variables, the strategy is to decompose W as $W_{2 \times 2} = P\mu P^{-1}$ where P is the matrix of eigenvectors and μ is a diagonal matrix with the roots on the diagonal, ordered in ascending absolute value. Solve the equation by backward recursion yields:

$$[\mathbb{S}_t]_{2 \times 1} = W_{2 \times 2}^t [\mathbb{S}_0]_{2 \times 1} + \sum_{h=0}^t W_{2 \times 2}^h R_{2 \times 1} [\mathbb{Z}_{t-h+1}]_{1 \times 1} + \sum_{h=0}^t W_{2 \times 2}^h Q_{2 \times 1} [\mathbb{Z}_{t-h}]_{1 \times 1}, \quad (4.78)$$

or alternatively, since $W_{2 \times 2} = P\mu P^{-1}$, then $W_{2 \times 2}^t = P\mu^t P^{-1}$.

$$[\mathbb{S}_t]_{2 \times 1} = P\mu P^{-1} [\mathbb{S}_0]_{2 \times 1} + \sum_{h=0}^t P\mu^h P^{-1} \{ R_{2 \times 1} [\mathbb{Z}_{t-h+1}]_{1 \times 1} + Q_{2 \times 1} [\mathbb{Z}_{t-h}]_{1 \times 1} \} \quad (4.79)$$

It follows that this system has a unique solution because one eigenvalues becomes stable and one becomes unstable. There will be a unique choice of the initial value of the co-state variable, λ_0 , that will make the optimal paths of $\{\hat{\lambda}_t\}_{t=0}^{\infty}$ and $\{\hat{k}_t\}_{t=0}^{\infty}$ be consistent with the transversality condition, $\lim_{t \rightarrow \infty} \beta^{*t} \lambda_t k_{t+1} = 0$.

To pin down the initial value of the co-state variable, Blanchard and Khan (1980) with the work of Vaughan (1970), introduce the decouple approach. The basic idea is to separate the fundamental difference equation into the state difference equation and co-state difference equation and solve each one individually. We begin by partitioning the matrices P , μ , P^{-1} , R and Q as follows:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} p_{11}^* & p_{12}^* \\ p_{21}^* & p_{22}^* \end{bmatrix}, \quad R = \begin{bmatrix} R_{ke} \\ R_{\lambda e} \end{bmatrix}, \quad \text{and} \quad Q = \begin{bmatrix} Q_{ke} \\ Q_{\lambda e} \end{bmatrix}$$

Therefore, we can write:

$$P\mu^h P^{-1} = \begin{bmatrix} p_{11}\mu_1^h p_{11}^* + p_{11}\mu_2^h p_{21}^* & p_{11}\mu_1^h p_{12}^* + p_{12}\mu_2^h p_{22}^* \\ p_{21}\mu_1^h p_{11}^* + p_{22}\mu_2^h p_{21}^* & p_{21}\mu_1^h p_{12}^* + p_{22}\mu_2^h p_{22}^* \end{bmatrix}$$

We can see that if value of h becomes large, the second term in each summation will be unbounded and dominates the expression due to the fact that μ_2 is unstable. The suitable initial value of the co-state variable will make those second terms equal zero.

Next step is to decouple the fundamental difference system. We begin with the transformed system of Vaughan (1970). Let define a new vector of state and co-state variables, which are denoted \bar{k}_t and $\bar{\lambda}_t$:

$$\begin{bmatrix} \tilde{k}_t \\ \tilde{\lambda}_t \end{bmatrix} = P^{-1} \begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \tilde{S}_t \end{bmatrix}_{2 \times 1} = P^{-1} \begin{bmatrix} \hat{S}_t \end{bmatrix}_{2 \times 1} \quad (4.80)$$

Multiplying both sides of the fundamental dynamic equation by P^{-1} leads to:

$$\begin{aligned} \begin{bmatrix} \tilde{S}_{t+1} \end{bmatrix}_{2 \times 1} &= P^{-1} \begin{bmatrix} \hat{S}_{t+1} \end{bmatrix}_{2 \times 1} = P^{-1} W \begin{bmatrix} \hat{S}_t \end{bmatrix}_{2 \times 1} + P^{-1} R \begin{bmatrix} \hat{Z}_{t+1} \end{bmatrix}_{1 \times 1} + P^{-1} Q \begin{bmatrix} \hat{Z}_t \end{bmatrix}_{1 \times 1} \\ &= \mu \begin{bmatrix} \tilde{S}_t \end{bmatrix}_{2 \times 1} + P^{-1} R \begin{bmatrix} \hat{Z}_{t+1} \end{bmatrix}_{1 \times 1} + P^{-1} Q \begin{bmatrix} \hat{Z}_t \end{bmatrix}_{1 \times 1} \end{aligned}$$

The above equation is comprised of two decoupled difference equations, the transformed state and co-state component of this system.

The transformed state component of this system is given by:

$$\tilde{k}_{t+1} = \mu_1 \tilde{k}_t + [p_{11}^* R_{ke} + p_{12}^* R_{\lambda e}] \hat{Z}_{t+1} + [p_{11}^* Q_{ke} + p_{12}^* Q_{\lambda e}] \hat{Z}_t \quad (4.81)$$

Since the μ_1 elements are less than 1 in absolute value, this difference equation is stable.

The specification of the initial value \tilde{k}_0 fully determines the equation solution.

On the other hand, the second difference equation for the transformed co-state variable is unstable in the backward direction since the element of μ_2 exceeds 1 in absolute value. For this reason, as Vaughan points out, it is necessary to impose a transversality condition rather than an initial value of the transformed co-state. To do this, we pre-multiply the expression by μ_2^{-1} , yielding $\tilde{\lambda}_t = \mu_2^{-1} \tilde{\lambda}_{t+1} - \mu_2^{-1} [p_{21}^* R_{ke} + p_{22}^* R_{\lambda e}] \hat{\mathbb{Z}}_{t+1} - \mu_2^{-1} [p_{21}^* Q_{ke} + p_{22}^* Q_{\lambda e}] \hat{\mathbb{Z}}_t$.

We impose the terminal condition of $\tilde{\lambda}_t$ growing at a rate less than μ_2^{-1} (implied by the transversality condition) and solve it forward recursively, obtaining the solution:

$$\tilde{\lambda}_t = - \left\{ \sum_{j=0}^{\infty} \mu_2^{-j-1} (p_{21}^* R_{ke} + p_{22}^* R_{\lambda e}) \hat{\mathbb{Z}}_{t+j+1} + \sum_{j=0}^{\infty} \mu_2^{-j-1} (p_{21}^* Q_{ke} + p_{22}^* Q_{\lambda e}) \hat{\mathbb{Z}}_{t+j} \right\} \quad (4.82)$$

The two solutions for the transformed state and co-state variables, which may be written in matrix form as:

$$\begin{bmatrix} \tilde{\mathbb{S}}_{t+1} \end{bmatrix}_{2 \times 1} = \begin{bmatrix} \mu_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbb{S}}_t \end{bmatrix}_{2 \times 1} + \begin{bmatrix} p_{11}^* R_{ke} + p_{12}^* R_{\lambda e} \\ 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbb{Z}}_{t+1} \end{bmatrix}_{1 \times 1} + \begin{bmatrix} p_{11}^* Q_{ke} + p_{12}^* Q_{\lambda e} \\ 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbb{Z}}_t \end{bmatrix}_{1 \times 1} + \begin{bmatrix} 0 \\ I \end{bmatrix} f_{t+1}$$

where $f_t = - \left(\sum_{j=0}^{\infty} \mu_2^{-j-1} [p_{21}^* R_{ke} + p_{22}^* R_{\lambda e}] \hat{\mathbb{Z}}_{t+j+1} + \sum_{j=0}^{\infty} \mu_2^{-j-1} [p_{21}^* Q_{ke} + p_{22}^* Q_{\lambda e}] \hat{\mathbb{Z}}_{t+j} \right)$.

To return to the untransformed specification for the state and co-state variables, we rearrange (4.80), obtaining:

$$\begin{bmatrix} \hat{\mathbb{S}}_t \end{bmatrix}_{2 \times 1} = P \begin{bmatrix} \tilde{\mathbb{S}}_t \end{bmatrix}_{2 \times 1}$$

Thus, it follows that the solutions for the original variables take the forms:

$$\begin{aligned} \begin{bmatrix} \hat{\mathbb{S}}_{t+1} \end{bmatrix}_{2 \times 1} &= P \begin{bmatrix} \mu_1 & 0 \\ 0 & 0 \end{bmatrix} P^{-1} \begin{bmatrix} \hat{\mathbb{S}}_t \end{bmatrix}_{2 \times 1} + P \begin{bmatrix} p_{11}^* R_{ke} + p_{12}^* R_{\lambda e} \\ 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbb{Z}}_{t+1} \end{bmatrix}_{1 \times 1} \\ &\quad + P \begin{bmatrix} p_{11}^* Q_{ke} + p_{12}^* Q_{\lambda e} \\ 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbb{Z}}_t \end{bmatrix}_{1 \times 1} + P \begin{bmatrix} 0 \\ I \end{bmatrix} f_{t+1}, \text{ or alternatively} \\ \begin{bmatrix} \hat{k}_{t+1} \\ \hat{\lambda}_{t+1} \end{bmatrix} &= \begin{bmatrix} p_{11} \mu_1 p_{11}^* & p_{11} \mu_1 p_{12}^* \\ p_{21} \mu_1 p_{11}^* & p_{21} \mu_1 p_{12}^* \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{bmatrix} + \begin{bmatrix} p_{11} (p_{11}^* R_{ke} + p_{12}^* R_{\lambda e}) \\ p_{21} (p_{11}^* R_{ke} + p_{12}^* R_{\lambda e}) \end{bmatrix} \hat{\mathbb{Z}}_{t+1} \\ &\quad + \begin{bmatrix} p_{11} (p_{11}^* Q_{ke} + p_{12}^* Q_{\lambda e}) \\ p_{21} (p_{11}^* Q_{ke} + p_{12}^* Q_{\lambda e}) \end{bmatrix} \hat{\mathbb{Z}}_t + \begin{bmatrix} p_{12} \\ p_{22} \end{bmatrix} f_{t+1} \end{aligned} \quad (4.83)$$

The dynamic system is now stable and can be solved in the backward direction.

According to (4.80), we can write:

$$\tilde{k}_t = p_{11}^* \hat{k}_t + p_{12}^* \hat{\lambda}_t \quad \text{and} \quad \tilde{\lambda}_t = p_{21}^* \hat{k}_t + p_{22}^* \hat{\lambda}_t \quad (4.84)$$

The second of these conditions implies that $\hat{\lambda}_t = [p_{22}^*]^{-1} \tilde{\lambda}_t - [p_{22}^*]^{-1} p_{21}^* \hat{k}_t$ which implies an initial condition for the shadow price, given the initial capital stock and the initial condition on the transformed shadow price. Recall (4.82):

$$\tilde{\lambda}_0 = - \left\{ \sum_{j=0}^{\infty} \mu_2^{-j-1} (p_{21}^* R_{ke} + p_{22}^* R_{\lambda e}) \hat{\mathbb{Z}}_{j+1} + \sum_{j=0}^{\infty} \mu_2^{-j-1} (p_{21}^* Q_{ke} + p_{22}^* Q_{\lambda e}) \hat{\mathbb{Z}}_j \right\}$$

Therefore, similar to other parameters of the model, we have the initial value of co-state variable as a function of state and exogenous variable:

$$\hat{\lambda}_0 = [p_{22}^*]^{-1} \left\{ \sum_{j=0}^{\infty} \mu_2^{-j-1} (p_{21}^* R_{ke} + p_{22}^* R_{\lambda e}) \hat{\mathbb{Z}}_{j+1} + \sum_{j=0}^{\infty} \mu_2^{-j-1} (p_{21}^* Q_{ke} + p_{22}^* Q_{\lambda e}) \hat{\mathbb{Z}}_j \right\} - [p_{22}^*]^{-1} p_{21}^* \hat{k}_0$$

The combination of the relationship between (4.82) implies that:

$$\tilde{k}_t = [p_{11}^* - p_{12}^* [p_{22}^*]^{-1} p_{21}^*] \hat{k}_t + p_{21}^* [p_{22}^*]^{-1} \tilde{\lambda}_t$$

which implies that given the initial state variable and the initial co-state variable yield an initial condition of the transformed state variable. State differently, in the functional form:

$$\tilde{k}_0 = [p_{11}^* - p_{12}^* [p_{22}^*]^{-1} p_{21}^*] \hat{k}_0 + p_{21}^* [p_{22}^*]^{-1} \tilde{\lambda}_0$$

The Appendices, Section A illustrates the way to solve $\tilde{\lambda}_t$ in detail.

4.7 Simulation and the shocks off the steady state

After deriving the steady state solution and log-linearizing the first-order conditions around the steady state, the dynamic system ends up with a system of two coupled difference equations. This system is composed of the equation motion of the state variable, k_t , and of

the co-state variable, λ_t . The k_t equation can be solve backwards while the equation of λ_t can be solved forwards. Since the model is solved along the lines of King *et al.* (1988), a rational expectation solution for the state variables can then be written as:

$$\hat{S}_{t+1} = \begin{bmatrix} \hat{k}_{t+1} \\ \hat{A}_{t+1} \end{bmatrix} = \begin{bmatrix} M_{kk} & M_{kA} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{A}_t \end{bmatrix} = M\hat{S}_t + \varepsilon_{t+1} \quad (4.85)$$

The remainder of variables of interest will be a linear function of the state variable in the form:

$$\hat{Z}_t = \begin{bmatrix} \hat{\lambda}_t \\ \hat{\psi}_t \end{bmatrix} = \Pi_{4 \times 2} \hat{S}_t \quad (4.86)$$

where $\hat{\psi}_t = [\hat{c}_t \quad \hat{i}_t \quad \hat{y}_t]'$ and the sequential logic of the solution can be expressed by the figure 4.2 below:

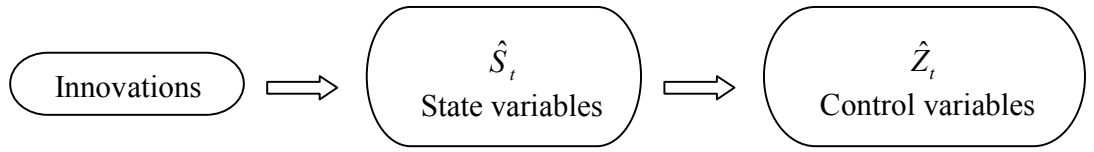


Figure 4.2 Logic of the solution

The equation (4.85) and (4.86) can be used to generate impulse response functions. For the model, it is necessary to initialize the stochastic processes, the exogenous shock and the state variables. The value of the state variables and the shock in the first period are used to determine the control variables in the first period. This process would be repeated over T periods to successively construct all the values for the control variables.

Namely, given the value of technology innovation, ε_t , the system equation of (4.85) is recursively utilized, obtaining the other or control variables of interest from (4.86). Recursive computation of the T period impulse simulation is straightforward.

It is also straightforward to conduct the shocks off steady state. In generally, the model economy is disturbed at the steady state at period one. The responses at period one of the endogenous variables (\hat{Z}_1) are calculated by the multiplication of a shock vector (\hat{S}_1) and the steady state condition (Π) as in equation (4.86). Next period at period two, the shock (\hat{S}_2) is weaker because it is not permanent. Because the steady state condition are the same over time, these weaker shocks time the steady state condition (Π) results in the responses of the endogenous variables at period two (\hat{Z}_2). In order to shock the economy off the steady state at period three, the new shock vector (\hat{S}_N) must be created. The shock vector at period three is basically the combination of the remaining shock from period two (\hat{S}_2) with the current shock at period three (\hat{S}_3). Put differently, the economy experiences the multiple shocks at period three. After that the recursive process would be repeated over a period of time.

Before estimating the model's moments, it is crucial to filter the model economy. The procedure to filter the model economy is explored in chapter 3. The model's moments can be calculated from the filtered model economy.

4.8 Conclusion

This chapter reviewed the concept to construct the business cycle model and to solve the decision rules. Particularly, it emphasizes the dynamic aspects of economic decisions for business cycle model. To help understand the application of model, the basic business cycle with fixed labour supply is used as an example. In this way, the chapter is able to present main tools such as dynamic programming and techniques to solve it. It has also provided the methods to solve linear rational expectation model. To solve for the decision rules of the business cycle model, both analytical and numerical approaches have been studied. The first approach follows an undetermined coefficient method by Campbell (1994) and the second approach is along the line with method of finding a linear approximation to the first order conditions by King *et al.*(1988). Next chapter discusses the real business cycle model with the application for Thai economy and heavily uses the key ideas reviewed in this chapter.

Chapter 5

A Closed Economy Model

5.1 Introduction

To explain the variation in aggregate macroeconomics variables, the real business cycle model uses a neoclassical growth model with the technology shock as a driving force. Agents adjust rationally their levels of consumption and labour supply in response to these fluctuations. The model views aggregate macroeconomic variables as the consequences of the decisions made by many individual consumers maximizing their utility subject to production possibilities and resource constraints. Agents own capital and rent it to firms. Firms buy capital and labour services from utility maximizing consumers and use them to produce a single output used as either consumption or investment. Uncertainty comes from a stochastic shock to the economy's production technology.

According to the two welfare theorems, the competitive equilibrium of the economy is Pareto efficient and any Pareto efficient allocation can be achieved as a competitive equilibrium.¹ We can also solve for the allocation of a competitive equilibrium by solving the maximization problem of a benevolent social planner as discussed by King and Rebello (1994). The social planner is benevolent and maximizes the representative agent's lifetime utility function. All agents will be allocated the same consumption and leisure. This chapter describes in detail that the solutions of the real business cycle models can be obtained by two different approaches, competitive equilibrium and the social planner problem. The economy should always be modelled as a dynamic general equilibrium with rational expectation. Then, it describes a specific real business cycle model calibrated and simulated for the Thai empirical data. Next section describes an

¹ As in Gravelle and Rees (2004), the first theorem of welfare economics states that if (a) there are markets for all commodities which enter into production and utility functions and (b) all markets are competitive, then the equilibrium of the economy is Pareto efficient. The second theorem of welfare economics states that, if all consumers have convex preferences and all firms have convex production possibility sets, any Pareto efficient allocation can be achieved as the equilibrium of a complete set of competitive markets after a suitable redistribution of initial endowments. These two results are very helpful, because they allow us to solve the much simpler social planner problem and ensure the existence of a competitive equilibrium automatically. A proof of the welfare theorem is not provided here. A proof can be obtained from Mas-Colell, Whinston and Green (1995) and Gravelle and Rees (2004).

economic environment both for competitive equilibrium and for the planner problem in details. The model economies are driven by two exogenous factors, total factor productivity and government spending variations. The forcing factors are governed by a first order autoregressive process. The Matlab codes following Burnside (1999) are exhibit in the Appendices.

5.2 Competitive Equilibrium

A typical time period is denoted by t . Time is discrete and the economy lasts forever. There are two types of identical economic actors: households and firms. Mathematically, the economy is identical to an economy where one price-taking consumer sells labour and capital services to one price-taking firm who then uses this capital to produce output for the final goods market. In general, households and firms maximize their objective functions subject to constraints and markets clear. This section follows King and Rebelo (1994).

5.2.1 Households

We can view households from the supply side of the economy in which they provide capital and labour to firms. Households live infinitely and value both consumption and leisure for their utilities. Generally, households make three decisions, how much labour to supply, how much capital to accumulate and how much to consume. All households are completely identical. Since there is a large number of household, we can treat households as behaving competitively. The actions of households do not affect market prices in the economy because their weight in population is negligibly small. Households make all time- t choices conditional on time t information. Since households face uncertainty over the future, the household's objective is to maximize expected life-time utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \quad (5.1)$$

where $\beta \in (0,1)$ is a time discount factor that households consider for future consumption and leisure. C_t and L_t are the amount of consumption and leisure at time period t . The time discount factor is less than 1 indicating that households are impatient. They yield

less utility from the same amount of consumption and leisure if those consumption and leisure occur later. The expectation symbol, E_t , is the conditional expectation of a future consumption and leisure in period t given the available information at t . The instantaneous utility function, U , takes the constant-relative-risk aversion form:²

$$U(C_t, L_t) = (1 - \sigma)^{-1} (C_t^\theta L_t^{1-\theta})^{1-\sigma} \quad (5.2)$$

where $\theta \in (0,1)$ is the consumption and leisure share parameter. σ is the measure of relative risk aversion or the inverse of the intertemporal elasticity of substitution. Stated differently, it is the household's attitude towards risk. High σ indicates high risk aversion and households are less willing to push their consumption in the future. When $\sigma \rightarrow 0$, utility is linear, when $\sigma \rightarrow \infty$, utility is Leontief, and when $\sigma \rightarrow 1$, utility is Cobb-Douglas or logarithmic utility. To ensure the feasibility of an optimal solution for consumption and leisure, we assume that $u(\cdot)$ is sufficiently differentiable, strictly concave and strictly increasing in consumption and leisure and satisfies the Inada condition. ($u'(\cdot) > 0, u''(\cdot) < 0$ for all C and L , and $\lim_{C \rightarrow 0} u_C(C_t, L_t) = \lim_{L \rightarrow 0} u_L(C_t, L_t) = \infty$, $\lim_{C \rightarrow \infty} u_C(C_t, L_t) = \lim_{L \rightarrow \infty} u_L(C_t, L_t) = 0$)

In each period, an individual agent faces a time constraint. Total amount of time allocated to work and leisure cannot exceed the endowment, which is normalized to 1 for simplicity:

$$N_t + L_t \leq 1 \quad (5.3)$$

where N_t is the amount of work at t . An individual agent also faces a budget constraint in each period. Households have a share of the profits of the representative firm. Labour and capital are paid their marginal products. Household's budget cannot exceed the amount of their consumption, investment and tax payment:

² Given a utility function $u(\cdot)$, the coefficient of relative risk aversion at x is $-xu''(x)/u'(x)$. Let notation $D_n f(\cdot)$ denote the partial derivative of f with its n th argument. Therefore, $D_1 u(\cdot) = u_C(\cdot)$. For this particular utility, it satisfies the constant coefficient of relative risk aversion (CRRA) form or $D_{11} u(C, L)C / D_1 u(C, L)$ is constant overtime. Labour supply responds to changes in the real wage. However, with the CRRA utility, labour supply does not alter when the real wage increase permanently. Stated differently, when the real wage grows constantly there is no trend growth in labour. Later on, this chapter we consider the steady state conditions where the real wage is growing constantly.

$$w_t N_t + R_t K_t + \Pi_t \leq C_t + I_t + T_t \quad (5.4)$$

where at period t , w_t is the real wage, R_t is the rental price of capital, K_t is the capital, Π_t is the household's share of profit of the firms, I_t is a gross investment and T_t denotes lump-sum taxes

5.2.2 Firms

Like households, competitive firms are assumed to be identical and so small that their hiring decisions do not affect the real wage rate and the rental price of capital. Firms produce one homogenous final good which is consumed by households. The quantity of output is influenced by productivity disturbances, capital, the level of technology progress and labour. The production technology is described through a Cobb-Douglas production function which it exhibits a constant return to scale:³

$$Y_t = A_t F(K_t, X_t N_t) \quad (5.5)$$

where A_t is total factor productivity or a technology disturbance, X_t is the labour augmenting technical change and is assumed to grow at a constant rate, γ , where $\gamma \geq 1$ ($X_{t+1} = \gamma X_t$). In functional form, we can write the production technology as $Y_t = A_t K_t^{1-\alpha} (N_t X_t)^\alpha$ where α denotes the labour share. The parameter α measures the importance of the labour input in the production function. The economy has an initial value of capital $K_0 > 0$, an initial total factor productivity $A_0 > 0$, and an initial level of labour augmenting technical change $X_0 > 0$. The production function is also twice differentiable, strictly concave and strictly increasing in capital and labour and satisfies Inada condition.⁴

At time t firms hire workers at real wage w_t and rent the capital from households at rate R_t . Because part of the capital disappears (δ) in the production process and it is not

³ The details of constant return to scale can be found in Barro and Sala-I-Martin (1995).

⁴ $F'(\cdot) > 0, F''(\cdot) < 0$ for all K_t, N_t and $\lim_{K \rightarrow 0} F_K(\cdot) = \lim_{N \rightarrow 0} F_N(\cdot) = \infty$, $\lim_{K \rightarrow \infty} F_K(\cdot) = \lim_{N \rightarrow \infty} F_N(\cdot) = 0$.

returned to households, households receive only $R_t - \delta$ as net rental payment when they rent their capital to firms. Therefore, the real interest rate can be defined as the rental price of capital net of depreciation. Let r_t be real interest rate, the rental price of capital and the real interest rate relates according to $r_t = R_t - \delta$.

The firms in the economy decide how much labour and capital to hire and take wages and the rental price of capital as given and the firms maximize their profit period by period according to:⁵

$$\Pi_t = Y_t - w_t N_t - R_t K_t \quad (5.6)$$

The capital stock accumulates according to:

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (5.7)$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital. In period t a representative agent maximizes its expected life-time utility function subject to the sequence of time, budget constraints and the capital accumulation process.

5.2.3 Government

In each period, the government spends an exogenous flow of government spending, G_t which is financed by lump-sum taxes, T_t . The government has a balanced budget and is given by:

$$G_t = T_t \quad (5.8)$$

It is interesting to study the effect of taxes and government spending according to the public finance rule. The flow of government spending G_t has the relationship: as $(1 - \tau)Y_t = G_t + TR_t$. Government collects taxes from a representative agent's income by

⁵ It does not matter to include the profits in the budget constraint since profits are zero in equilibrium. The zero profit result does not happen only for the Cobb-Douglas production function. Any constant return to scale production function, together with price taking behaviour by firms will yield this consequence.

rate τ and has a transfer payment of TR_t .⁶ However, this chapter only demonstrates the equilibrium optimization for the simple government spending rule where government is financed by lump-sum taxes, T_t .

5.2.4 Market clearing conditions

The market in this economy consists of a market for capital, labour and goods. For competitive equilibrium to exist, the three markets have to clear. For the capital market, since capital is the only asset in this economy, the assets held by households have to be equal to the capital required by firms. Therefore, the capital market clears ($K^S = K^D = K$). For the labour market, households supply labour from which they generate income to purchase a bundle of consumer goods. The demand for labour by firms equals the supply of labour by households and the market clears ($N^S = N^D = N$). Finally, for the goods market, the supply of goods by firms equals its output. Demand in the goods market stems from consumption, investment and taxes payment from households. For clearing in the goods market, output has to be equal to consumption, investment and government spending ($A_t F(K_t, X_t N_t) = C_t + I_t + G_t$).

5.2.5 Equilibrium

In equilibrium, the real wage and interest rate are adjusted such that all three markets clear. Households take w_t and R_t as given and solve their problems according to:

$$\begin{aligned} & \underset{C_t, N_t, I_t, K_{t+1}}{\text{Max}} \quad E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \\ & \text{subject to} \\ & \quad N_t + L_t = 1 \\ & \quad w_t N_t + R_t K_t + \Pi_t = C_t + I_t + T_t \\ & \quad K_{t+1} = (1 - \delta) K_t + I_t \\ & \quad G_t = T_t \end{aligned}$$

The competitive firms take w_t and R_t as given and solve their problems, according to:⁷

⁶ Many applications with public finance and monetary economics, the competitive equilibrium need not be Pareto optimal.

⁷ Firm's optimization problem is static since they take capital and labour from households and solve the problem period by period. Firms rent all inputs period by period and sell output period by period.

$$\underset{K_t, N_t}{Max} \Pi_t = A_t F(K_t, X_t N_t) - w_t N_t - R_t K_t$$

For all these markets clear, it is necessary to have the following conditions:

$$\begin{aligned} K^S &= K^D = K \\ N^S &= N^D = N \\ A_t F(K_t, X_t N_t) &= C_t + I_t + G_t \end{aligned}$$

5.3 The social planner problem

Given the same functional form chosen to represent household preferences and to represent technology as a competitive equilibrium, another way to set up the basic real business cycle model is to solve for the possible allocation of the social planner problem. This section again follows King and Rebelo (1994). Because markets are competitive, and there is no externality, the equilibrium of the model must be equivalent to the Pareto efficiency. The social planner can decide how much the households consume, work and how much firms produce. The social planner is benevolent and looks for the best possible allocation.⁸ A benevolent social planner maximizes the agent well being or life-time expected utility function. The social planner is one solving a Pareto efficient problem. The planner faces the resource constraints of the economy:

$$\begin{aligned} \underset{C_t, N_t, K_{t+1}}{Max} \quad & E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \\ \text{subject to} \quad & \\ & N_t + L_t = 1 \\ & A_t F(K_t, X_t N_t) = C_t + I_t + G_t \\ & K_{t+1} = (1 - \delta)K_t + I_t \end{aligned}$$

The solution of this problem would be the optimal sequences of consumption, labour and capital accumulation decision.

⁸ Social planner does not follow price. The price does not exist in the budget constraints.

5.4 Productivity disturbances

The uncertainty of the economy in the standard real business cycle model is driven from the firm in the production process. The economy experience technology shocks and these shocks result in a dynamic response by household that involves variation in output, investment, consumption and employment. Essentially, the economy expands at a time of positive technology shock or a high productivity. Firms take the benefits of a positive technology shocks by producing more output, hiring more workers and investing in more new capital. Firms are willing to pay high wages because of high productivity and accordingly households are willing to work harder. On the other hand, economic recession is the time of following negative technology shocks or technological regress. During recessions, output and employment fall because of negative technology shocks. It is not a good time to produce, and firms are not willing to pay high wages, to invest and to hire more workers. Households simply have a substantially lower incentive to work.

Since the source of business cycle fluctuations is technology disturbances, the exogenous total factor productivity, A_t , is introduced to a firm production process. Therefore, if A_t is high, output will be high even though production inputs are unchanged. It is also necessary to study the effects of fiscal policy on business cycle fluctuation. Another type of disturbance in this chapter is a government spending shock. These two types of shock exhibit real disturbances because technology fluctuation alter the level of production from given quantity of inputs and government spending shock alter goods, in the economy, available for households.

The process for total factor productivity and government spending are set by defining \hat{A}_t and \hat{G}_t as the log-deviation of the actual productivity and government spending level from its average, the process of technology and government spending disturbances are respectively given by $A_t = Ae^{\hat{A}_t}$ and $G_t = Ge^{\hat{G}_t}$, where in detail:

$$\hat{A}_t = \log A_t - \log A \quad (5.9.1)$$

$$\hat{G}_t = \log G_t - \log G \quad (5.9.2)$$

5.5 The steady state and the stationary economy

The steady state describes the long-run features of the economy. According to the neoclassical model, all components of the national income identity at steady state grow overtime at the growth rate of labour augmenting technology, γ which is constant, either for a competitive equilibrium or a social planner problem. For instance, at steady state consumption grows at a constant rate, i.e., $c = c_t = c_{t+1} = c_{t+2}$ at steady state. If the economy starts with capital at steady state level, it will never leave that steady state. If the economy starts with capital higher or below its steady state value, it may overtime approach the steady state value. The steady state is a starting point in studying the dynamic properties of the model. It is important to determine the characteristic of the model at the steady state.

However, to ensure that the feasible steady states exist, the restrictions on the production function and preferences must be imposed. Swan (1963) and Phelps (1966) suggest that the labour augmenting variable must be included in the Cobb-Douglas production function, rationalizing the production function in this chapter:⁹

$$Y_t = A_t F(K_t, X_t N_t) = A_t K_t^{1-\alpha} (N_t X_t)^\alpha \quad (5.10)$$

where $X_t N_t$ represents the effective labour units and it grows at rate γ . In order for preferences to be consistent with steady state growth, King et. al (1988) show that two restrictions must be imposed on preferences. One is that the intertemporal elasticity of substitution in consumption must be invariant to the scale of consumption. Another one is that the income and substitution effect associated with sustained growth in labour productivity must not change with labour supply.¹⁰ These two conditions imply the period utility function has the forms:

⁹ The details of the restriction can be found in the Appendices, Section A.

¹⁰ Two restrictions are imposed on preferences to make sure that the model is consistent with the steady state growth. Firstly, from the first order condition, $D_1 u(C_t, L_t) = \lambda_t$ and $\beta^t [r+1] = \lambda_t / \lambda_{t+1}$ where λ_t is shadow price of consumption, β^t is discount factor and r is the real interest rate. Therefore, $D_1 u(C_t, L_t) / D_1 u(C_{t+1}, L_{t+1}) = \beta^t (1+r)$ and it is constant at the steady state since r is constant. Because consumption is growing at constant and the marginal utility is to grow at constant, it must be the case that the intertemporal elasticity of substitution of consumption, $D_{11} u(C, L) C / D_1 u(C, L)$, is constant and independent of the level of consumption. Secondly, the real wage is growing at the steady state, but the

$$\begin{aligned}
u(C_t, L_t) &= \frac{1}{(1-\sigma)} C_t^{1-\sigma} v(L_t) & \text{if } 0 < \sigma < 1 \text{ and } \sigma > 1 \\
u(C_t, L_t) &= \log(C_t) + v(L_t) & \text{if } \sigma = 1
\end{aligned} \tag{5.11}$$

The second form of the momentary utility functions above is the special case of the first when we take the limit to the first equation as $\sigma \rightarrow 1$.

Since time allocated to work is restricted by the endowment which is normalized to 1, at the steady state, growth rate of labour is zero because it cannot grow. Thus, the constant growth rate of labour is zero. The Cobb-Douglas production function with the capital accumulation implying that output, consumption, capital and investment per capita all have the same steady state growth rate and are equal to the growth rate of labour augmenting technology, $\gamma = \gamma_Y = \gamma_C = \gamma_K = \gamma_I$ where $\gamma_N = 1$.¹¹

Since there is a sustained growth in the model, to get rid of non-stationary and to generate the stationary economy, we deflate all variables by the growth component, X . Let a lower case letter represent the detrended variable such that $i = I/X, k = K/X, y = Y/X, c = C/X$.¹² After we specify the value of deflated variables, we will estimate the solution to the system about them. As a result, the effective rate of time preference is changed to β^{*t} where $\beta^{*t} = \beta^t \gamma^{1-\sigma}$. β^{*t} is a modified discount factor or we can say that it is a discount factor at steady state. It satisfies $\beta^* \in (0,1)$ to ensure finites of lifetime utility. Output, national income identity and capital accumulation are changed to be $y_t = A_t F(k_t, N_t)$, $y_t = c_t + i_t + g_t$ and $\gamma k_{t+1} = (1-\delta)k_t + i_t$ respectively.

hours worked cannot grow. Consequently, households supply a constant number of hours when the real wage rate grows at a constant rate. It must be the case that there are offsetting income and substitution effect of the change in real wage. For the additional restrictions that ensure that consumption and leisure are goods and the utility is concave, see also King, Plosser and Rebelo (1988) and Barro and King (1984).

¹¹ The feasible steady state condition and steady state marginal product can be found in details in the Appendices, Section A.5.

¹² The details of the transformed economy are illustrated in Appendices, Section A.10.

5.5.1 Necessary conditions

This section also follows King and Rebelo (1994). For competitive equilibrium, households optimize their problem subject to their constraints as below:¹³

$$L = \sum_{t=0}^{\infty} \beta^* u(c_t, L_t) + \sum_{t=0}^{\infty} \beta^* \Phi_t (w_t N_t + R_t k_t + \Pi_t - c_t - i_t - g_t) + \sum_{t=0}^{\infty} \beta^* \Psi_t (i_t - \gamma k_{t+1} + (1-\delta)k_t)$$

First order conditions, for $\forall t = 0, 1, 2, \dots, \infty$

$$\begin{aligned} D_1 u(c_t, 1 - N_t) - \Phi_t &= 0 \\ D_2 u(c_t, 1 - N_t) - \Phi_t w_t &= 0 \\ \Phi_t - \Psi_t &= 0 \\ \beta^* (\Phi_{t+1} R_{t+1} + (1-\delta)\Psi_{t+1}) - \gamma \Psi_t &= 0 \\ \lim_{t \rightarrow \infty} \beta^* \lambda_t k_{t+1} &= 0 \end{aligned} \tag{5.12}$$

Let $\Phi_t = \Psi_t = \lambda_t$ for simplicity. We also impose the transversality condition to guarantee that at the end households use all valuable capital. The current value of the terminal capital must get close to zero. If this condition fails, households hold capital forever. They not really optimize their problems.

Firms solve their static problems as:

$$\max_{n_t, k_t} [A_t F(k_t, N_t) - w_t N_t - R_t k_t]$$

First order conditions:

$$\begin{aligned} A_t D_2 F(k_t, N_t) &= w_t \\ A_t D_1 F(k_t, N_t) &= R_t \end{aligned}$$

Market clearing condition:

$$A_t F(k_t, N_t) = c_t + i_t + g_t$$

For the social planner problem, the benevolent planer optimize a representative agent's expected lift-time utility subject to the resource constraints of the economy as below:

¹³ The optimization can also be achieved by using another dynamic programming approach, the respective Bellman equation as $V(k_t, A_t, g_t) = \max_{c_t, N_t, k_{t+1}} \{u(c_t, N_t) + \beta EV(k_{t+1}, A_{t+1}, g_{t+1})\}$ subject to $AF(k, N) = c + \gamma k_{t+1} - (1-\delta)k_t + g_t$, where the variable A_t, g_t are observed at the beginning of the period and follows a first-order linear Markov process.

$$L = \sum_{t=0}^{\infty} \beta^{*t} u(c_t, L_t) + \sum_{t=0}^{\infty} \beta^{*t} \lambda_t (A_t F(k_t, N_t) - \gamma k_{t+1} - c_t + (1-\delta)k_t + g_t) + \sum_{t=0}^{\infty} \beta^{*t} w_t (1 - L_t - N_t)$$

First order conditions, for $\forall t = 0, 1, 2, \dots, \infty$.

$$\begin{aligned} D_1 u(c_t, L_t) - \lambda_t &= 0 \\ D_2 u(c_t, L_t) - w_t &= 0 \\ \lambda_t A_t D_2 F(k_t, N_t) - w_t &= 0 \\ \beta^{*t} \lambda_{t+1} [A_{t+1} D_1 F(k_{t+1}, N_{t+1}) + (1-\delta)] - \gamma \lambda_t &= 0 \\ A_t F(k_t, N_t) - \gamma k_{t+1} - c_t + (1-\delta)k_t + g_t &= 0 \\ 1 - L_t - N_t &= 0 \\ \lim_{t \rightarrow \infty} \beta^{*t} \lambda_t k_{t+1} &= 0 \end{aligned} \tag{5.13}$$

We also have to impose the transversality condition for this problem.

The two problems lead to the same necessary conditions. Since the first order conditions of the competitive equilibrium and those of social planner problems are equivalent, we can simply demonstrate the relationship between the first and the second welfare theorems. The similarity between the necessary conditions of the two methods supports the two welfare theorems that can apply to this economy.

The optimal paths of consumption $\{c_t\}_{t=0}^{\infty}$, leisure $\{L_t\}_{t=0}^{\infty}$ and capital $\{k_t\}_{t=0}^{\infty}$ satisfy the necessary conditions, the original constraints, the initial condition of capital and transversality condition. We can essentially interpret the necessary conditions as one main property, equalizing cost to benefit. These necessary conditions can be divided into three features. First, the marginal utility of consumption ($D_1 u(c_t, L_t)$) equals its shadow price (λ_t). Second, the utility value of goods produced with a marginal unit of work ($\lambda_t A_t D_2 F(k_t, N_t)$) equals its utility cost ($D_2 u(c_t, L_t)$). Put differently, the marginal benefit of working in goods units equals the marginal cost of leisure in goods units. Third, the current utility cost ($\gamma \lambda_t$) equals to the present utility value of the future product of capital ($\beta^{*t} \lambda_{t+1} [A_{t+1} D_1 F(k_{t+1}, N_{t+1}) + (1-\delta)]$). For the rest of this chapter, the model will be solved using the social planner problem.

5.5.2 Steady state ratio

It is important to determine the value of variables in steady state in order to use the steady state equations to solve for the model's parameter values.¹⁴ This section again follows King and Rebelo (1994). There are several ways to estimate the steady state ratios. However, each method begins with $\beta^* \lambda_{t+1} [A_{t+1} D_1 F(k_{t+1}, N_{t+1}) + (1 - \delta)] = \gamma \lambda_t$ because the production side of the model determines almost everything about the steady state. One can derive the ratio by replacing the interest rate on the requirement above. Another can yield the ratio by substituting the rental price of capital on the same equation. This chapter estimates the steady state main ratio using the last approach.

To derive capital-output ratio, $\beta^* \lambda_{t+1} [A_{t+1} D_1 F(k_{t+1}, N_{t+1}) + (1 - \delta)] = \gamma \lambda_t$ can be interpreted as $\beta^* [(1 - \alpha)y]/k + (1 - \delta) = \gamma$ at steady state and therefore, $(k/y) = [(1 - \alpha)\beta^*]/[\gamma - \beta^*(1 - \delta)]$. Therefore, technology and preference determine the capital-output ratio at the steady state. For the steady state interest rate (r_{ss}), we use the facts that $r_t = R_t - \delta$ and $\beta^* [AD_1 F(k, N) + (1 - \delta)] = \gamma$. The interest rate at steady state is derived as $AD_1 F(k, N) = (\gamma / \beta^*) - 1 + \delta$, $r_{ss} = \frac{\gamma}{\beta^*} - 1$. To derive the investment-capital ratio, we begin with the detrended capital accumulation equation at steady state. $\gamma k = (1 - \delta)k + i$ and $(i/k) = \gamma - 1 + \delta$. For the investment-output ratio (s_i), we use the relationship of investment-capital ratio and capital-output ratio according to $s_i = (i/y) = (i/k).(k/y)$. Since the government-output ratio (s_g) is given to the model and along with resource constraint, we can simply define the steady-state consumption-ratio as $s_c = 1 - s_i - s_g$.

5.5.3 The local elasticities

Two of the main structures of the model are preferences and technology. A representative agent has preferences for both consumption and leisure. We can estimate the elasticity of marginal utility near steady state. With the constant returns to scale production function, technology also has implications for the elasticities of marginal

¹⁴ The details are exhibit in the Appendices, Section A.9.

products near the steady state. This section and the next follow the methodology from King, Plosser and Rebello (1988).

For the elasticity of marginal utility, denote ξ_{xy} as the elasticity of marginal utility of x with respect to y . So for example, ξ_{cc} is the elasticity of marginal utility of consumption with respect to consumption ξ_{cL} is the marginal utility of consumption with respect to leisure. Similarly, ξ_{LL} is the elasticity of marginal utility of labour with respect to labour and that with respect to consumption is ξ_{Lc} . In functional forms, we obtain

$$\begin{aligned} \frac{cD_{11}u(c, 1-N)}{D_1u(c, 1-N)} &= \xi_{cc} \quad , \quad \frac{(1-N)D_{12}(c, 1-N)}{D_1u(c, 1-N)} = \xi_{cL} \\ \frac{cD_{21}u(c, 1-N)}{D_2u(c, 1-N)} &= \xi_{Lc} \quad , \quad \frac{(1-N)D_{22}(c, 1-N)}{D_2u(c, 1-N)} = \xi_{LL} \end{aligned}$$

where $D_j f$ is the partial derivative of the function f with respect to its j argument. When the period utility function is additively separable, $u(C_t, L_t) = \log(C_t) + v(L)$, it follows that $\xi_{cc} = -1$, ξ_{cL} and $\xi_{Lc} = 0$, $\xi_{LL} = LD^2(v)/D(v)$. However, when the period utility function is multiplicatively separable, $u(C_t, L_t) = \frac{1}{(1-\sigma)} C^{1-\sigma} v(L)$, the following results hold; $\xi_{cc} = -\sigma$, $\xi_{Lc} = 1-\sigma$, $\xi_{cL} = LDv(L)/v(L)$ and $\xi_{LL} = LD^2(v)/D(v)$. For elasticity of marginal product, let ξ_{kk} denote the elasticity of marginal product of capital with respect to capital and that with respect to labour is ξ_{kN} . In the same way, denote ξ_{Nk} and ξ_{NN} as the elasticity of marginal product of labour with respect to capital and labour respectively. In functional forms, we have:

$$\begin{aligned} \frac{kD_{11}F(k, N)}{D_1F(k, N)} &= \xi_{kk} \quad , \quad \frac{ND_{12}F(k, N)}{D_1F(k, N)} = \xi_{kN} \\ \frac{ND_{22}F(k, N)}{D_2F(k, N)} &= \xi_{NN} \quad , \quad \frac{kD_{21}F(k, N)}{D_2F(k, N)} = \xi_{Nk} \end{aligned}$$

For the Cobb-Douglas production function, $\xi_{kk} = -\alpha$, $\xi_{kN} = \alpha$, $\xi_{NN} = \alpha - 1$ and $\xi_{Nk} = 1 - \alpha$. It is clearly to see that for Cobb-Douglas technology, $\xi_{kk} = -\xi_{kN}$ and $\xi_{NN} = -\xi_{Nk}$.¹⁵

5.5.4 Log-linearization around the steady state

We apply the log-linearization procedure for all equations in the first order conditions (5.13). This section follows the methodology from King, Plosser and Rebello (1988).

We omit the time index for the steady state value of each variable. The log-linear equations are described below, From the necessary conditions for consumption, $D_1 u(c_t, L_t) - \lambda_t = 0$, we log-linearize and obtain the simple form of linear approximation by replacing ξ_{cc} and ξ_{cL} in the equation above:

$$\xi_{cc} \hat{c}_t - \xi_{cL} \left(\frac{N}{1-N} \right) \hat{N}_t = \hat{\lambda}_t \quad (5.14.1)$$

By combining the leisure and the labour necessary conditions, we obtain $D_2 u(c_t, L_t) - \lambda_t A_t D_2 F(k_t, N_t) = 0$. With the endowment, $1 - L_t - N_t = 0$ and for simplicity, we substitute ξ_{NN} and ξ_{Nk} . We obtain the simple form of linear approximation as follow:

$$\xi_{Lc} \hat{c}_t - \left\{ \xi_{LL} \left(\frac{N}{1-N} \right) + \xi_{NN} \right\} \hat{N}_t = \xi_{Nk} \hat{k}_t + \hat{\lambda}_t + \hat{A}_t \quad (5.14.2)$$

The leisure necessary conditions can be interpreted as labour supply and it demonstrates the influence of real wage and shadow price on the quantity of labour supplied. The technology shows that the labour necessary condition exhibits an equivalence between marginal product of labour and the real wage. It implies that real wage depends on the technology shock, capital and labour.¹⁶ It also can be interpreted as a labour demand such that the amount of labour is related to technology shock, real wage and capital.

¹⁵ The derivation in details is shown in the Appendices, Section A.7.

¹⁶ The real wage is raised by productivity and by increase in the capital-labour ratio since the real wage equals $\alpha y/N$.

From the fourth necessary condition, $\beta^{*t} \lambda_{t+1} [A_{t+1} D_1 F(k_{t+1}, N_{t+1}) + (1 - \delta)] - \gamma \lambda_t = 0$ and for the Cobb-Douglas production function assumption, denote η_x as the elasticity of the gross marginal product of capital with respect to x when evaluated at steady state. Therefore, $\eta_k = k \beta^* A D_{11} F(k, N) / \gamma$, $\eta_N = N \beta^* A D_{12} F(k, N) / \gamma$ and $\eta_A = \beta^* A D_1 F(k, N) / \gamma$. We log-linearize and which yields:

$$\eta_k \hat{k}_{t+1} + \eta_N \hat{N}_{t+1} + \eta_A \hat{A}_{t+1} + \hat{\lambda}_{t+1} = \hat{\lambda}_t \quad (5.14.3)$$

From the fifth necessary condition $A_t F(k_t, N_t) - \gamma k_{t+1} - c_t + (1 - \delta)k_t + g_t = 0$ and for simplicity, denote $\phi = \gamma / (\gamma - (1 - \delta))$. Since $\delta < 1$ and $\phi > 1$, it yields

$$\hat{y}_t = s_i \phi \hat{k}_{t+1} + s_c \hat{c}_t + s_g \hat{g}_t - s_i (\phi - 1) \hat{k}_t \quad (5.14.4)$$

As a result, the dynamic linear system is obtained in a simple form and can be solved numerically. The model solution is discussed on chapter 4, methodology.

Before proceeding to the next topic, we define the arguments for technology and government spending shocks:

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_{A,t} \quad (5.15.1)$$

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \varepsilon_{G,t} \quad (5.15.2)$$

The parameter ρ measures how persistent each shock is. In other words, how important that last period shock to determine how large it is this period. The variables $\varepsilon_{A,t}$ and $\varepsilon_{G,t}$ are innovations to technology and random shocks to government expenditures respectively and they are normally distributed and serially uncorrelated.

5.6 Model calibration for The Thai economy

Since the model has been constructed, the next step is to measure how accurate the model is in explaining the aggregate fluctuation of the economy. Nonetheless, before

evaluating the performance of the model, it is necessary for us to make certain assumptions about certain parameters. To achieve this we calibrate the model. Kydland and Prescott (1982) evaluated their real business cycle model by using microeconomic evidence and long-run characteristics to select the possible model parameters and choose the model parameters consistent with empirical measures. They concluded that the data are used to calibrate the model economy in order to mimics the real world as closed as possible.

The idea is that we want to construct the model's prediction to capture the properties of the data. The characteristics of the calibration are that the model parameters are not estimated but rather determined by the long-run properties of the economy and that the moments of the simulated economy are comparable with those of the actual economy.¹⁷ We choose model parameters such that the long-run behaviour of the model matches the long-run observations from the data. Apparently, the long-run facts are different from country to country. Even though there is the evidence that the long-run observation from the USA and most industrial counties grow continually overtime and the model generates the good measurement of certain empirical facts, it is very interesting to explore the real business cycle of the emerging economies to see how well the model fits the property of business fluctuation in those countries facts.¹⁸ This chapter presents the parameter values calibrated for Thai data and evaluate the model's prediction for Thai economy.

To obtain some calibration values, for the parameter values, we use data for Thailand over the period 1976 to 2005 using 1988 as a base year. The Thai data series are from 1976-2004 and are obtained from the World Development Indicators provided by the World Bank. For the year 2005 with base year 1988, the data are obtained from the Bank of Thailand and the Office of National Economic and Social Development Board of Thailand (NESDB) for consumption, investment, government spending and capital. There are many unknown parameters in the description of the model as the following table 5.1 summarises.

¹⁷ The broader definition can be found in Cooley (1997).

¹⁸ Solow (1970), Plosser (1989) and Cooley and Prescott (1995) observes the evidence of the per capita value of the industrial country and the United States long-run features grow continually overtime.

We can broadly group these variables into three parts, preference, technology and shock parameters. For Thai economy, we firstly describe how we obtain the technology parameters, and how we calibrate the preference and finally driving process parameters.

Table 5.1 The list of parameters

The time discount factor at steady state	β^*
The real interest rate at steady state	r_{ss}
The relative risk aversion	σ
The consumption share parameter	θ
The labour share parameter	α
The capital depreciation	δ
The growth rate of technological changes	γ
The persistence of technology	ρ_A
The persistence of government spending	ρ_g
The random shock of technology parameters	ε_A
The random shock of government spending parameters	ε_g

5.6.1 Technology parameters

The growth rate of technological change is chosen to equate to the growth rate of real output per capita in Thailand during the last 30 years which is 4.69% annually or 1.173% quarterly. Given the value of the growth rate of technological change, the aggregate depreciation rate (δ) of this economy depends on the long-run averages for the investment-output ratio, capital-output ratio from the observation period. For Thailand over the sample period, the depreciation rate turns out to be a quarterly rate of 0.99%.¹⁹

Next, we turn to the evidence on the number of hours worked. The labour parameter is determined by the fraction of time allocated to work in a week multiplied by the fraction of Thai population that works²⁰. The labour parameter is approximately 0.192 for Thai economy over the sample period. The division of national income between capital and

¹⁹ The depreciation rate is derived from the capital accumulation equation $k_{t+1} = (1-\delta)k_t + i_t$. It follows $(k_{t+1}/y_{t+1})(y_{t+1}/y_t) = (1-\delta)(k_t/y_t) + (i_t/y_t)$ then $k\gamma = (1-\delta)k + i$. Eventually, $\delta = (i/y/k/y) + 1 - \gamma$. In Thailand over the sample period, investment-output ratio is 29.98% approximately, and the capital-output ratio is 3.46 on an annual basis (due to the short of the capital stock series, this capital output ratio is based on the gross capital stock series provided by NESDB from 1982-2005). It is important to note that the capital stock is a state variable and refers to a variable at a point in time whereas investment and output are flow variables and refer to a period of time. Therefore, investment and output depend on the period length.

²⁰ The average hours worked in a week is 40 hours (8 hours a day and 5 days a week), divided by total time available 112 hours (16 hours a day and 7 days a week), multiplied by the fraction of population that works, 0.54 (the average labour force divided by population over the sample period for Thailand). Ghez and Becker (1975), Juster and Stafford (1991) find that households allocate about one-third of their time to market activities.

labour is quite constant over a long period of times for many countries. For this study, the share parameters in the Cobb-Douglas production function can be obtained by the average capital and labour shares of total output over the sample period for Thai data set²¹. It follows that the labour share is 0.64 and the capital share 0.36.

5.6.2 Preference parameters

For Thailand, the lending interest rate adjusted for inflation is used for the real interest rate.²² Since the World Bank provides the data of the real interest rate until 2004, the real interest rate for 2005 is derived from the calculation²³. The average real interest rate over the sample period is 7.12% annually or 1.61% quarterly. The time discount factor at steady state is chosen such that the steady state real interest rate matches its long run average in the data. Given the steady state real interest rate and the real growth rate, we obtain the quarterly value for β^* of 0.9957 according to the equation $r_{ss} = \gamma/\beta^* - 1$. Consumption share parameter is chosen from the necessary condition which is 0.188 in our observation.²⁴ The parameter σ is difficult to obtain from long-run observations and many previous literatures always choose it independently. Following many literatures, this chapter sets the curvature of utility function, $\sigma = 2$ for Thai preference as well.

5.6.3 The shock parameter

To complete the calibration of this model economy, we are required to estimate the parameters that generate exogenous shock of the driving process. Prescott (1986) measures technology disturbance using the Solow residual which is defined as changes in output less the sum of the changes in capital's input times capital share and the changes in labour's input times labour share.²⁵ Prescott claims that the Solow residual can be

²¹ Capital share $(1-\alpha)$ comes from the first order condition $AD_1F(k, N) = r_{ss} + \delta$ and labour share is α . Plabo and Perri (2005) also suggest that labour share $= \alpha/1 + (r_{ss} - 1)\theta$ since households have to pay interest rate.

²² Inflation is calculated using GDP deflator.

²³ The real interest rate for 2005 follows the Fisher's equation $(1 + n_r) = (1 + \pi)(1 + r_r)$ where n_r is the lending rate, π is inflation as measured by the GDP deflator or, another word, it is percent change of GDP deflator and r_r is the real interest rate for 2005

²⁴ From $D_1u(c, L)AD_2F(k, N) = D_2u(c, L)$, we obtain $(c/1 - \bar{N})(1 - \theta/\theta) = \alpha y/\bar{N}$. Let s_c is the consumption-output ratio, $\theta = s_c/((\alpha(1 - \bar{N})/\bar{N}) + s_c)$.

²⁵ There are several procedures used to estimate the exogeneous process for technology shock. Backus Kehoe and Kydland (1992) construct the residuals using the output and aggregate employment. Capital did not account for the Solow residuals. So their Solow residuals were given by $\log A_t = \log Y_t - \alpha \log N_t$. Glick and Rogoff (1995) also estimate the Solow residuals without capital. Some of the reasons are that

used to explain US post-war output and suggests that technology shock plays an important role as a source of economic fluctuations. Based on the Cobb-Douglas technology, the Solow residual evolves according to:

$$\log A_t = \log y_t - (1 - \alpha) \log k_t + \alpha \log N_t \quad (5.16.1)$$

$$\log A_{t+1} = \log y_{t+1} - (1 - \alpha) \log k_{t+1} + \alpha \log N_{t+1} \quad (5.16.2)$$

Using the calibration of α , we can generate a series for A_t and their difference. Given that A_t is governed by the first order autoregressive process or $AR(1)$, it follows $\log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t}$. The value of A_t relies on the value in the past A_{t-1} and a random shock to innovation, $\varepsilon_{A,t}$. The random shocks are independently and normally distributed, with zero mean and constant variance. By using time series analysis, we can estimate the persistence parameter (ρ_A) and the standard deviation of the shock to innovation.²⁶ They are 0.930 and 0.0156 over the period of observation for the persistence and the standard deviation respectively. The same argument with the government spending shock, the shock is also assumed to follow the $AR(1)$ or $\log G_t = \rho_G \log G_{t-1} + \varepsilon_{G,t}$. Eventually, the government spending persistence parameter (ρ_g) is 0.944. This shock has a standard deviation of about 0.0184.

5.7 Model simulation for The Thai economy

The following numerical values are assigned to the structural parameters for Thai economy based on quarterly from 1976-2005. The calibrated parameters are summarized in table 5.1

data on capital stocks are not available or have to be constructed using investment series. Zimmerman used several methods to estimate the Solow's residuals. See also Zimmerman (1994). The distinctions of each alternative are also made because data is not available or only for short sample. Bergoing, Kehoe and Soto (2002) measure the total factor productivity by incorporating the unobserved capacity utilization of capital and the unobserved labour effort in the production function. Greenwood, Hercowitz and Kruseoll (2000) incorporates technological change specific to new investment goods and calibrate the model to U.S. data.

²⁶ Based on the regression analysis, the random shock to innovation is governed by $\hat{\varepsilon}_{A,t} = \log A_t - \hat{\rho} \log A_{t-1}$. Therefore, $\sigma_{A,\varepsilon_t}^2 = \frac{1}{T} \sum_{t=0}^T \hat{\varepsilon}_t^2$. This method is also applied for the government spending shock.

Table 5.2 Calibrated parameters for Thailand 1976-2005 quarterly

Technology				Preferences			Driving process			
γ	δ	α	\bar{N}	β	σ	θ	ρ_A	σ_{ε_A}	ρ_g	σ_{ε_g}
0.01173	0.0099	0.6396	0.192	0.9957	2	0.1880	0.930	0.0156	0.944	0.0184

Basically, there are two ways to study the predictions of the model. One way is to summarize its dynamic response to innovations in the driving variables. Another is to summarize the statistics from the mode's prediction. In this section, the first approach is investigated and the later approach will be discussed in the model evaluation section. The simulation or impulse response functions allow us for a deeper analysis of the dynamic reactions to a one percent change in the capital stock at the beginning of the period, k_t , the current technology shock, A_t and the current government spending shock, g_t .²⁷

5.7.1 Capital stock shocks

This section shows how a shock to the current level of the capital affects the optimal choices of the social planner. The experiment is that the economy is at the deterministic steady state and suddenly the capital increases by 1 percent in period 0. The impulse response traces out how flow and endogenous variables response to the capital shock.

Figure 5.1 A below depicts the response of capital stock, labour and total factor productivity to a 1 percent increase in the current capital stock. By the construction of the model, in period 0, only capital stock increase by 1 percent. After the shock, the capital stock fades away to its normal level. Since the economy experiences an unusually high capital, it is wealthier. In response, labour supply decreases by roughly 0.2 percent or leisure increases. Labour supply is not only lower in the period right after the shock, but also persistently lower and slowly converges to its normal level. The total factor productivity does not alter after the increase in capital.

²⁷ Romer (2001), Heijdra and Van der plog (2002) consider the basic neoclassical model's response to two kinds of forcing variables, technology and government spending.

Figure 5.1 Capital stock shocks

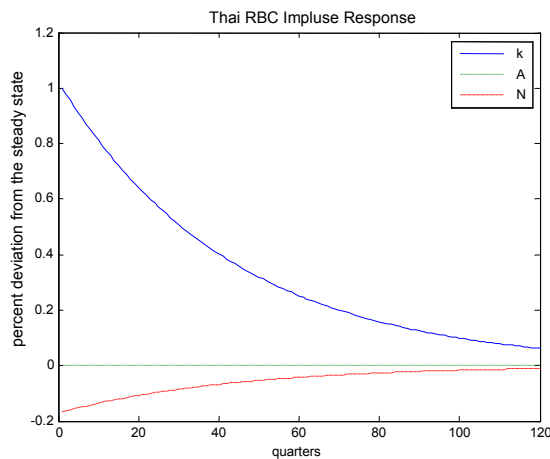


Figure 5.1 A

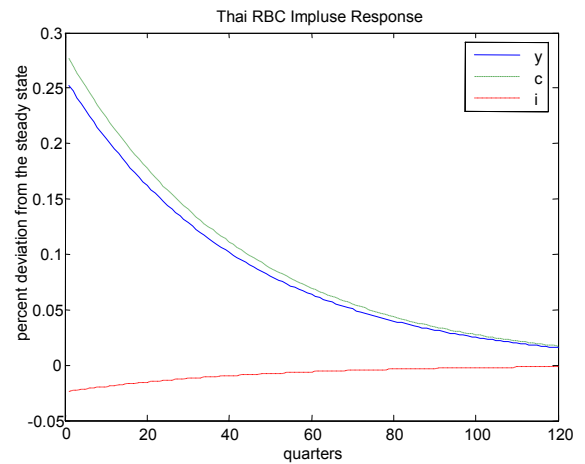


Figure 5.1 B

Because the capital stock is higher than its optimal level, investment is significantly declined as in figure 5.1 B. The Thai investment decreases to 0.025 percent right after the shock. The gradually transitional increase in investment leads to a reduction of the capital stock back to the steady state level. Since the economy is wealthier, the current consumption dramatically increases. As illustrated in the graph, Thai consumption increases to 0.2 percent. Households prefer not to consume entirely additional output in the current period. Instead, they smooth the consumption overtime. Therefore, consumption is consistently higher and gradually comes back to its steady state. Output increases by 0.25 percent because the capital rises up. Nonetheless, the increase in output is mitigated by the decline in labour supply. The overall responses of output increased mainly because of the increase in capital.

Figure 5.1 C plots the responses of the real wage and the rental price of capital to a 1 standard deviation shock to capital stock. The wage rises approximately by 0.4 percent because the output increases and labour supply declines. Overtime, it declines back slowly to its normal level since output and labour supply move back to their steady state levels. Since the increase in capital is obviously higher than that of output, the rental price of capital significantly declines right after the shock.

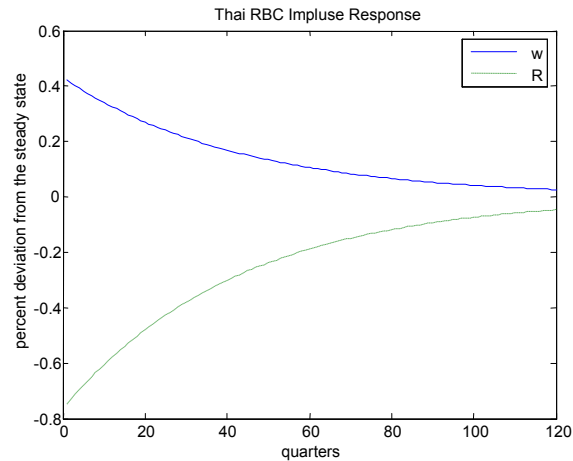


Figure 5.1 C

It decreases by 0.75 percent. Overtime, the rental price of capital increases to its initial status as both output and capital stock decline.

5.7.2 Technology shocks

The impulse responses are dominated by the fact that the productivity shock raises the desirability of work effort, production, investment and consumption; however, the latter part of impulse responses is governed by the reduction of capital back toward its stationary level. Since the persistence of technology is 0.930, the economy experiences a temporary technology shock but with a high degree of persistence. Put differently, the economy faces an extended interval in which productivity is above normal level. To capture the effects of a technology shock, the productivity is increased by 1 percent in the initial period. An increase in productivity causes an increase in the marginal product of labour. As a result in this initial period, household faces a significantly high opportunity cost of leisure. In response to a temporary productivity increase, work effort has to rise. Since the current wage is higher than the expected future wages, there is a high incentive to substitute inter-temporally labour supply. The impact effect on output is approximately 1.5 percent. Part of this response stems from an increase in labour and part of this response is due to the direct effect of the shock.

Given the fact that the marginal utility of consumption is decreasing, households prefer to smooth consumption paths. Households would not consume it all in one period even though it is possible to do. Only a small part of output will be consumed, but the rest of it will be invested. Therefore, Thai investment rises by nearly 1.2 percent in response to

a 1.5 percent increase in output. High productivity is associated with high interest rate, since interest rate is the differentiation of output with respect to capital subtracting depreciation. Later on the rate of return is below the normal level because the capital stock has been accumulated while productivity shock and labour supply has been declined. Therefore, the consumption path is initially high and then declines back towards the steady state level. Later as productivity converges gradually to its steady state level, work effort falls below its normal level as the economy enters the phase of reducing capital. Eventually, investment also fades away to its normal level as the economy runs down the capital.

Figure 5.2 A, shows how the effects on the level of technology die away gradually. Slowly, capital is built up and then returns to normal. The highest effect is 0.75 percent after 25 quarters. Work effort increases roughly by 0.7 percent in the period of the shock and then declines relatively rapidly, falling below normal after 20 quarters. Later it gradually moves up toward its steady state level. Figure 5.2 B shows that output increases in the period of shock and then slowly returns to normal level. Since output increases, so do consumption and investment. However, consumption response is less than that of output. Consumption rises by 0.35 percent in response to a 1.5 percent increase in output.

Figure 5.2 Technology shocks

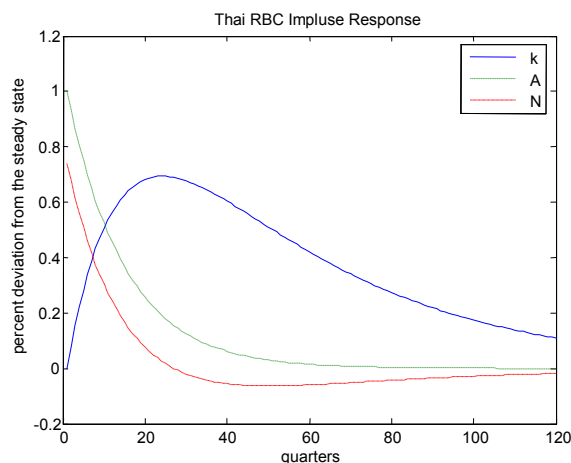


Figure 5.2 A

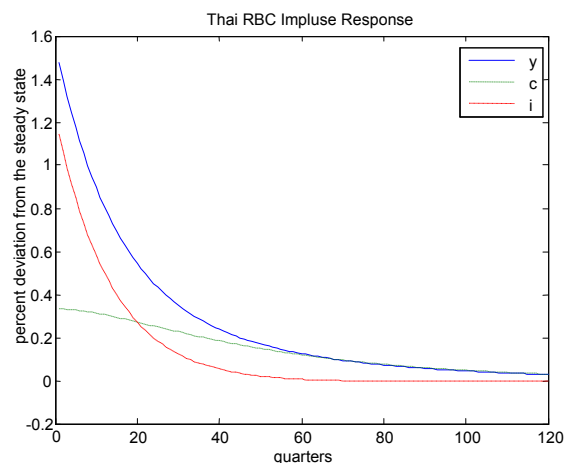


Figure 5.2 B

Figure 5.2 C demonstrates that the wage rises about 0.7 percent and then returns very slowly to its steady state. The interest rate increases roughly by 1.5 percent in the period

of the shock and then back toward normal fairly quickly. Since the capital stock moves more slowly than labour supply, the interest rate falls below normal after 18 quarters. The movements in the interest rate are the main source of the movements in labour supply.

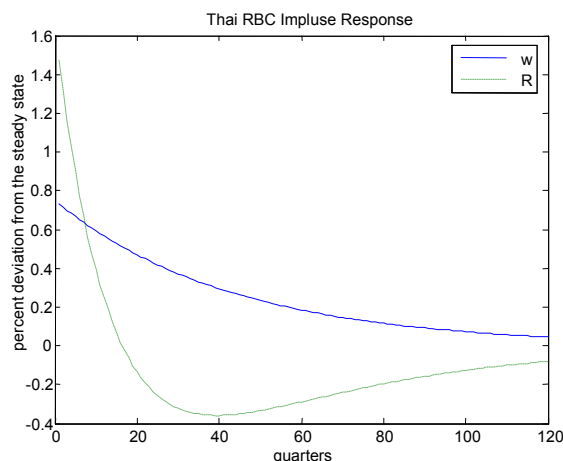


Figure 5.2 C

Technology disturbance cause the interest to rise and the wage to be temporarily high induces individual to allocate more of their labour forces. When the individuals are well reward, they are willing to supply their labour forces. This is a good time for them to work, not to enjoy their leisure. The willing to reallocate labour supply overtime is called inter-temporal substitution of labour. The increase in the interest rate is an important role of in increase in labour supply in real business cycle theory.²⁸

5.7.3 Government spending shocks

The model is constructed in such a way that it allows a fluctuation in fiscal policy.²⁹ Therefore, this section studies the effects of fiscal policy in an optimizing equilibrium framework. Similar to the technology shock, the government spending shock shows how the optimal choices of the social planner would response to the shock. The experiment is that the economy again is at the deterministic steady state and the government spending increases suddenly by 1 percent in period 0.

²⁸ Since the government spending financing by lump sum taxes causes a negative wealth effect, consumption and leisure fall. However, labour supply and output increase. Therefore, consumption performs counter cyclical.

²⁹ Bergoeing and Soto (2005), Baxter and King (1993) study the dynamic response to the government spending by incorporating the government expenditure in the utility function which this chapter does not pursue. Baxter and King (1993) study the effect of the US fiscal policy to the general equilibrium. They assume the war expenditure as the government spending and conclude that duration of fiscal policy has an influence on the time path of model variables.

By using the same calibrated parameters for Thailand, we consider an unanticipated increase in government spending of 1 percent of good unit. As shown in the figure 5.3, the government spending has a small effect on the dynamic evolution of the model variables. Figure 5.3 depict the dynamics response to government spending shocks. Since individuals experience a reduction in income after an increase in government spending, generally, an increase in government spending causes a negative wealth effect to representative agents. The negative wealth effect reduces private individuals' income and in turn individuals responded by cutting their level of consumption and leisure as depicted in figure 5.3 A and B. Therefore, labour supply increases by 0.045 percent and output increases by 0.03 percent in response to a shift in government spending. The labour supply increases and then gradually returns to steady state level. Even though output increases, its multiplier is less than 1. Since the persistence of the government spending shock is 0.944, the shock is not permanent and households respond by reducing their capital holding. The maximum impact is a decline of 0.045 percent after 30 quarters. The change in output is small and captures the movements of labour supply and capital because technology is unchanged.

Figure 5.3 Government spending shocks

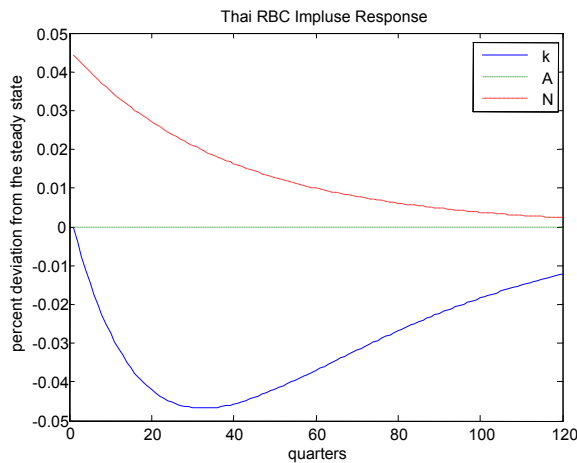


Figure 5.3 A

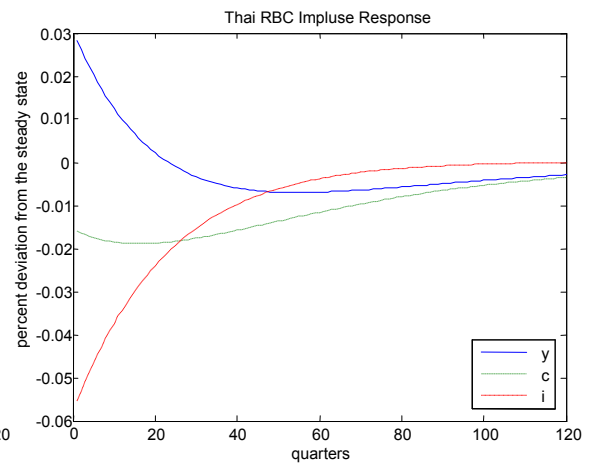


Figure 5.3 B

After 30 quarters, output declines to below the steady state level. Figure 5.3 C trace out the affect of the real wage and the rental price of capital. The real wage declines significantly in response to an increase in labour supply. The maximum impact is a decline of 0.02 percent after 20 quarters. In contrast to the real wage, the rental price of capital increases immediately right after the shock and it reaches as expected the highest

level of 0.045 percent after 20 quarters. The reason is that there is an increase in the level of labour supply along with a decline in the capital stock.

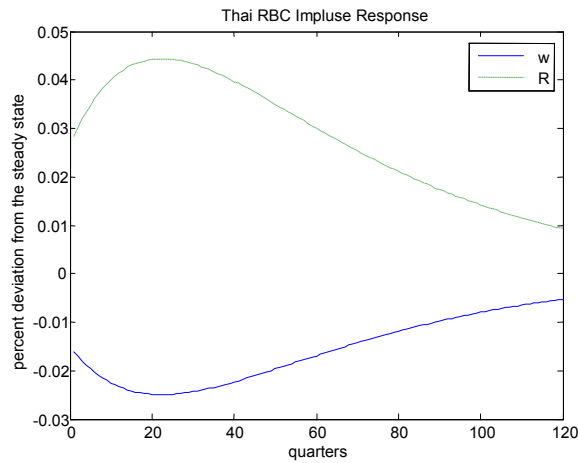


Figure 5.3 C

5.7.4 Multiple shocks

This section proceeds and studies the reaction of the model economy to multiple shocks at the same time. The analysis of multiple shocks on the dynamic general equilibrium is important because in reality, the Thai economy may not only face one shock at a time. It could be the case that the economy experiences more than one shock simultaneously. The effects of the multiple shocks are explored by performing a technology, government spending and capital shocks simultaneously. Firstly, the model is simulated by 1 percent increase in technology and 1 percent decrease in government spending shock at period 0. The economic responses of the 1 percent negative government spending shock exhibit the mirror reverse of the figure 5.3. The simulation illustrates that the 1 percent increase in technology shock dominates the 1 percent decrease in government spending. Therefore, the simulation is very much identical to the simulation shocked by only technology. Unlike the government spending shock, technology shock is the heart of the real business cycle model and it has the direct impact to output. In order to cancel out the effect of an increase in technology shock to output, the government spending shock needs to decrease massively. In this case, to force output unchanged at the impact, the government spending shock must decrease roughly by 50 percent. Even though the artificial economy has a 1 percent increase both technology and government spending shocks, the effects are very similar to the results driving by the technology shock only. Secondly, the model is simulated by a 1 percent increase in capital stock and technology

shocks. The effects are again similar to the outcome shocking by the technology. However, the amplitudes of output and consumption are higher to approximately 1.8 and 0.6 percent. Although the model depends on technology shocks which primarily affect the fluctuation, the sizes of the other shocks are very crucial as well.

5.7.5 The shocks off the steady state

Even though it is relatively easy to analyse the economy at the steady state, the economy is sometimes off the steady state such as in the rapid expansion and contraction periods. Since the economy is off the steady state after the shock at the steady state, the model is able to mimic the off steady state condition by simulating the first shock. Another way to study the effect on the main macro-economic variables of fiscal policy such as an increase in government spending can be conducted by simulating one shock after another or ultimately a series of shocks. To construct the model economy so as to simulate the shock off the steady state, it is necessary to apply the idea of the multiple shocks. In generally, the model economy is disturbed at the steady state at period one. The responses at period one of the endogenous variables are calculated by the multiplication of a shock vector and the steady state condition. Next period at period two, the shock is weaker because it is not permanent.

Given the fact that the steady state condition are the same over time, these weaker shocks times the steady state condition results in the responses of the endogenous variables at period two. In order to shock the economy off the steady state at period three, the new shock vector must be created. The shock vector at period three is basically the combination of the remaining shock from period two with the current shock at period three. Stated differently, the economy experiences the multiple shocks at period three.

Figure 5.4 A depicts a 1% negative technology shock followed in the following period by 1% positive government spending. Figure 5.4 B illustrates a 1% negative government spending shock followed in the following period by 1% positive technology shock. By comparing to the technology shock, the government spending shock in the RBC model has a smaller effect on the variables. A 1% change in technology typically causes 1.47% deviation of output from its trend at the impact while a 1% change in government spending leads to 0.025% output deviation from its trend at the impact.

Figure 5.4 The shock off the steady state

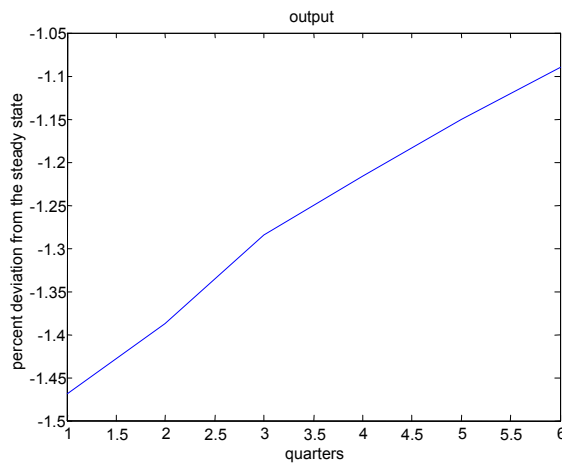


Figure 5.4 A

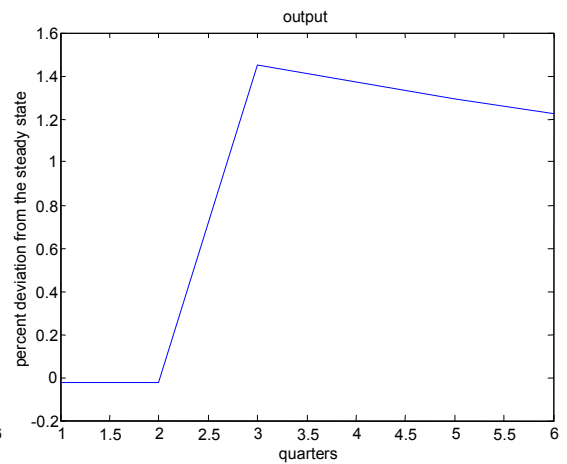


Figure 5.4 B

Given the fact that the productivity shock is able to generate the fluctuation in output, it is straightforward to replicate the Thai economic expansion and contraction periods. The experiment in this section is to replicate the output fluctuation in Thailand during the Asian crisis 1997 and determine how big the government spending shock should be to recover the economy.

Figure 5.5 The shock off the steady state (replicating the 1997 crisis)

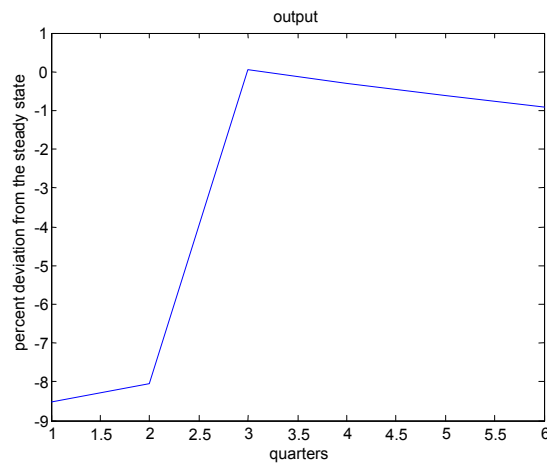


Figure 5.5 A

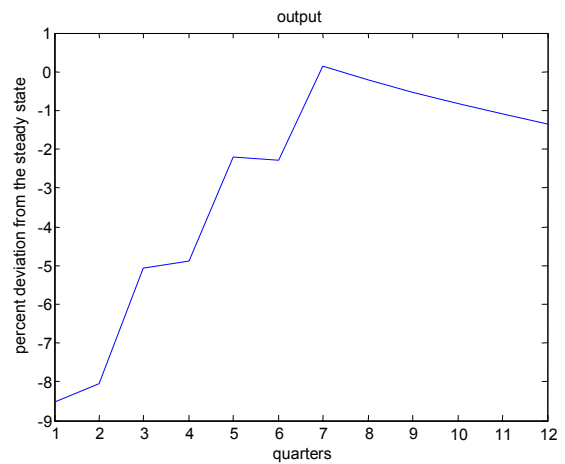


Figure 5.5 B

In 1997 quarter 2, the percent deviation of output from its trend was 6.00% and it declined to 3.86% the quarter after. After that it declined abruptly and hit the lowest point in 1998 quarter 3 at -8.55%. To replicate this fact, the model economy is simulated by the 5.8% negative productivity shock. The 300% positive shock in government

spending can minimize the cycle and return the output to 0 percent deviation from its trend as shown in figure 5.5 A. A series of a 100% increase in government spending at quarter 3, 5 and 7 are necessary in order to turn the economy to the steady state is exhibited in figure 5.5 B.³⁰ In Thailand, the long rung government spending was about 69,184 million baths per quarter during 1993-2006.

The capital stock is one of the main components of output. A technology shock and the capital shock in the real business cycle model are essentially the supply side shocks. The negative technology shock reduces potential output and the recovery process could be driven by an increase in another supply side shock, capital stock. The next experiment is to return the output back from the crisis to its steady state using an increase in capital stock. Figure 5.5 C depicts the increase in 30% of capital stocks at period 3. Only the 30% increase in capital stock can return output from the crisis to the steady state where as the government spending requires up to 300% shock.

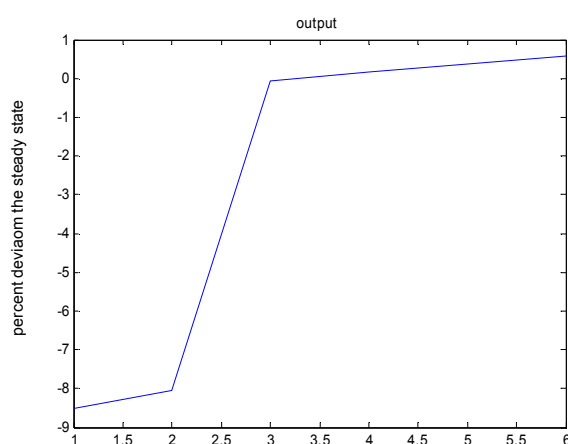


Figure 5. 5 C

A government spending shock has a comparatively small effect on macroeconomic variables to the technology shock and capital stock shock. The reasons are that the government spending is a demand side shock and it does not have a direct effect to production function. The technology shock has more direct effect to the production process. Therefore, the role of technology shock mainly causes the aggregate fluctuation.

³⁰ Figure 5.5 B illustrates the implication of a series of shocks using a government spending. The experiment is that the government could only increase its spending 100% each quarter. It will take three times of a 100% increase in government spending to turn the economy to the steady state.

In the presence of a supply side shock, the demand side policies seem to have a limited applicability.

5.8 Model simulation with different parameter values

To understand the business cycle properties of the model, we now turn, to study its dynamic evolution with respect to different important parameter values. The persistence of the shocks has a crucial influence on the driving process and in turn on the response of the model variables. The parameter that the model economy is most sensitive to is the persistence of the shocks. The closer the persistence of the shocks to one, the higher is the degree of persistence is. In general, when technology shocks are highly persistent, the wealth effect of a shock is higher (because its impact is longer-lived), and its inter-temporal substitution effect is smaller.³¹ Since the inter-temporal elasticity of substitution of consumption and labour have an important influence on the variance of the model variables, it is necessary to simulate the model with different values of the inter-temporal substitutions as well. Basically, when the inter-temporal elasticity of substitution of consumption and labour is high, the representative households are more willing to substitute consumption and labour overtime. Therefore, it is essential to simulate the model with different values of persistence of the shocks and with different values of the inter-temporal elasticity of substitution of consumption and labour.³² In this sense, the sensitivity analysis is conducted.

5.8.1 A pure temporary productivity shock

The pure temporary productivity shock means that the productivity has no serial correlation, $\rho_A = 0$. The economy, in this case, experiences the one percent increase in productivity shock only at time $t = 1$.

³¹The wealth effects can be estimated by Hicksian decomposition. It is the constant increments to the consumption and leisure paths that result in the same utility change. Intuitively, the wealth effect induces the representative household to consume more and work less. See also King (1991).

³²There are some literatures documenting the temporary and permanent technology shock for developed countries such as Rave (1997) for the UK and King and Rebelo (1998) for the US. Povoledo (2001) simulates the model for Italy following different value of the intertemporal elasticity of substitution of consumption and labour.

Figure 5.6 A pure temporary productivity shock

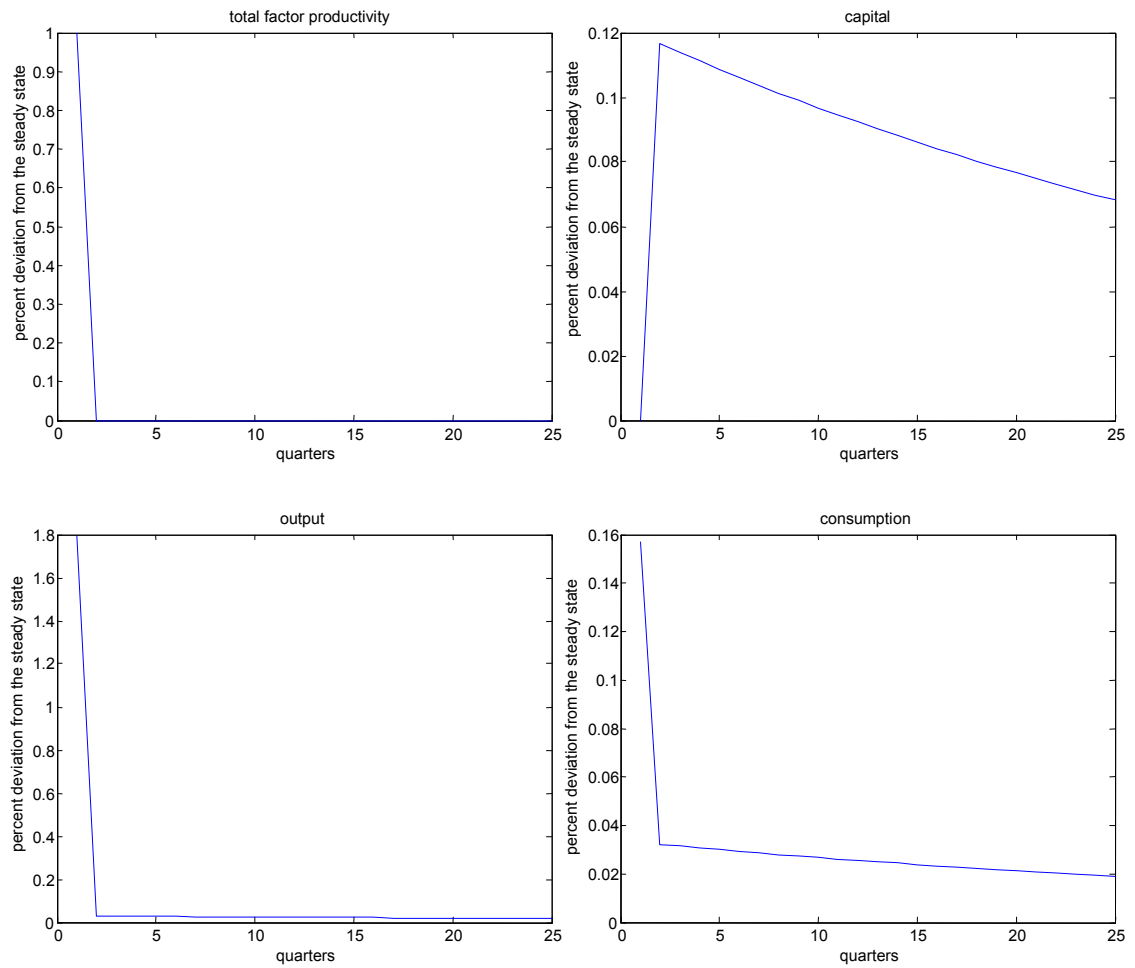


Figure 5.6 A

When the shock is pure temporary, typically there is a very small wealth effect that depresses labour supply but temporarily causes significantly higher wages and induces the representative households to work more hour immediately. The pure temporary productivity shock effects are shown in figure 5.6 A for technology, capital, output and consumption and in figure 5.6 B for investment, labour, real wage and rental price of capital.

At time $t = 2$ or one period after the shock, productivity is back to its normal level as in figure 5.4 A. Capital rises at the impact period to 0.12 percent on the Thai model economy. The impact effect on output is an increase of approximately 1.8 percent with the pure temporary productivity shock. The capital fades away overtime, since the capital next period is the function of the capital this period and the productivity.

Consumption increases to 0.16 percent at the impact period and one period after the shock consumption level is at 0.03 percent and then slowly declines overtime.

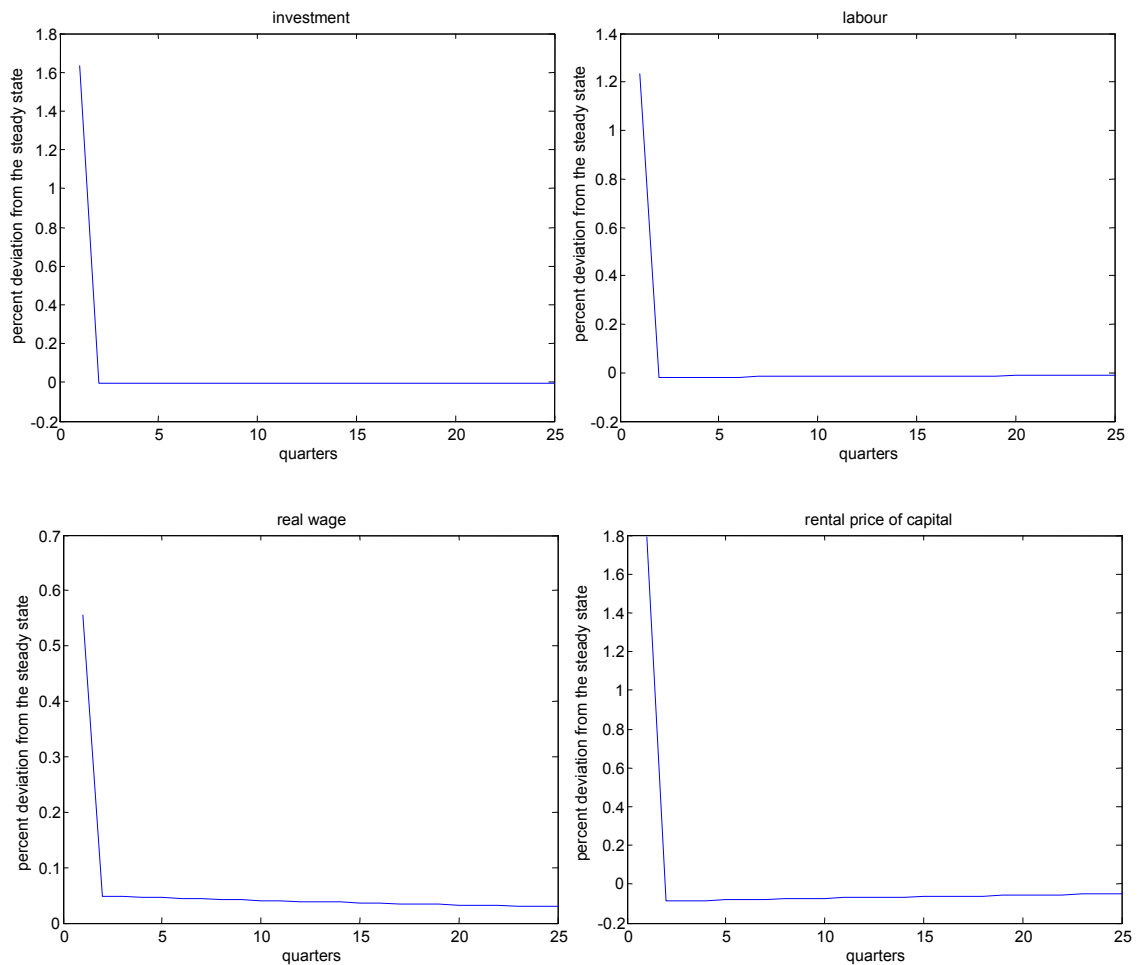


Figure 5.6 B

Figure 5.6 B shows that the investment also jumps at the impact period to 1.6 percent and falls to its normal level during the transition. Since the wealth effect is significantly small and the wage effect is quite large as a result of the pure temporary technology shock, the agents understand that there is a higher amount of real wage and they supply their labour. Therefore, in the Thai labour market, labour supply is much higher than labour demand and the real wage is unambiguously positive and rises about 0.55 immediately. The rental price of capital increases right away since output increases and capital remain unchanged at the impact.

5.8.2 A permanent productivity shock

This section explains the dynamic responses of the model variables resulting from a permanent productivity shock using the same calibrated parameters for Thai economy. In this particular case, the model economy faces a random walk technology process or the technology features a unit root ($\rho_A=1$).³³ When the shock is permanent, there are much larger wealth effects and the pattern of inter-temporal substitution in response to wages is reversed since future wages are high relative to current wages. The permanent temporary productivity shock effects are shown in figure 5.7 A for technology, capital, output and consumption and in figure 5.7 B for investment, labour, real wage and rental price of capital.

Figure 5.7 A permanent productivity shock

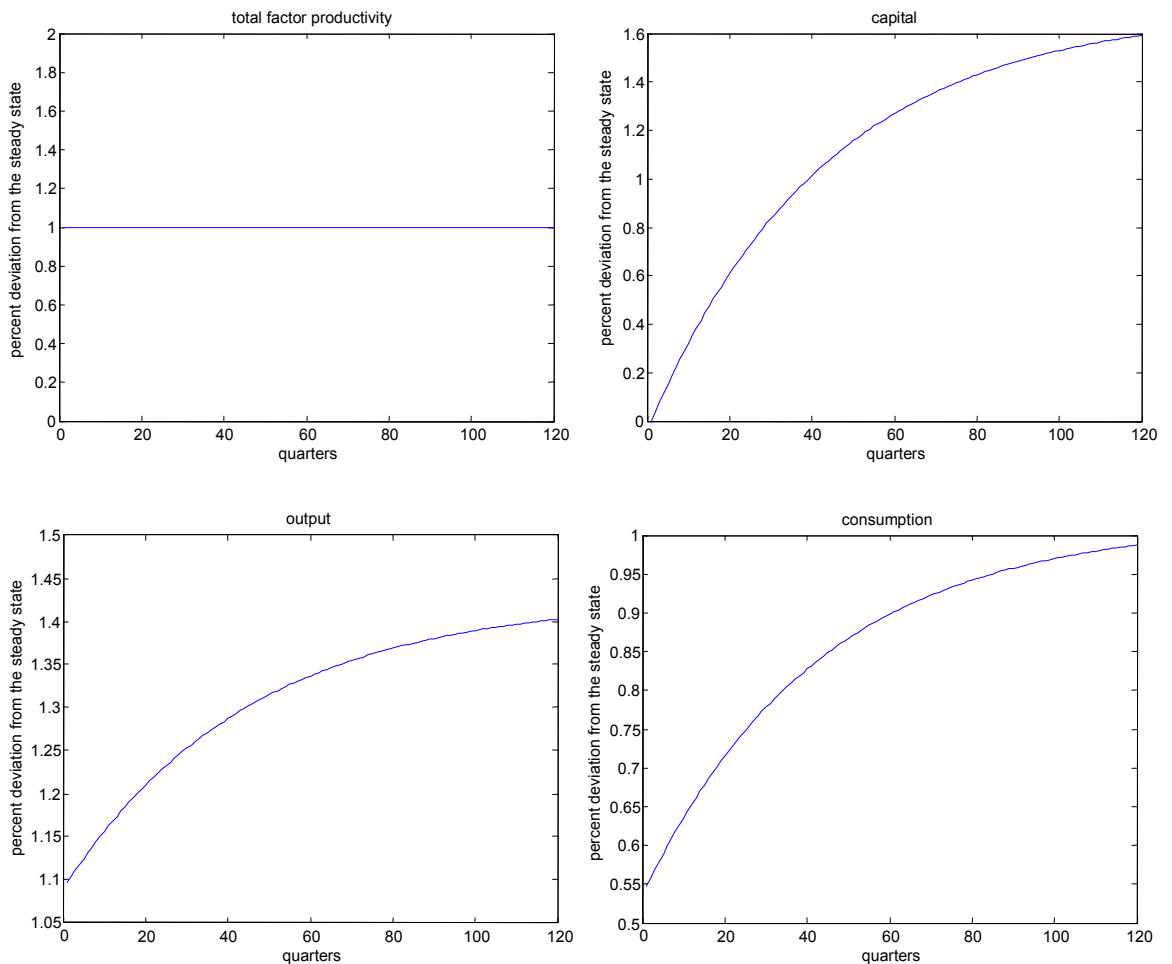


Figure 5.7 A

³³ Nelson and Plosser (1982) give an empirical support for the idea of the technology process behaves a random walk.

A one percent permanent shock in technology causes output to rise by 1.1 percent immediately. It obviously is lower than that of purely temporary shock and it is slightly lower than that of a realistic temporary shock ($\rho_A=0.930$). Consumption increases immediately by 0.55 percent because the households are wealthier. Clearly an increase in consumption in this special case is larger than that of purely temporary and realistic temporary shock.

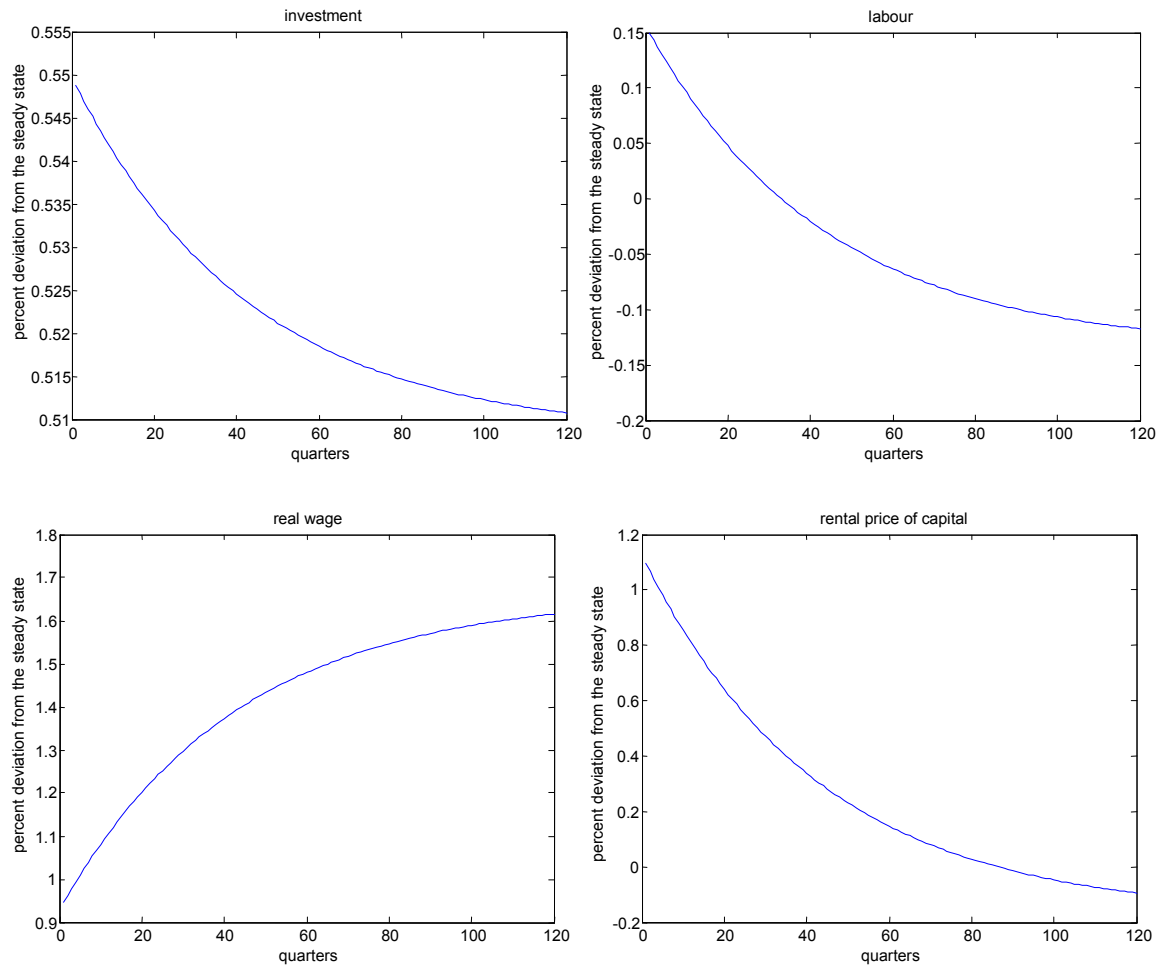


Figure 5.7 B

The permanent shock in technology lowers the impact response on investment from about 1.6 percent in the pure temporary shock and about 1.2 in the realistic temporary shock to 0.55 in the permanent case. The comparison of the initial response of labour supply documents that the higher initial impact on labour supply is the pure, realistic and permanent temporary shock respectively. Labour supply rises to 0.15 percent in this extreme case of the Thai economy. The real wage increases about 0.95 percent and the rental price of capital rises to 1.1 percent for the initial response. The rental price of

capital jumps up and then gradually declines back to its normal level. The time path of the rental price of capital explains the response of consumption as well.

5.8.3 Pure temporary government spending shock³⁴

Using the same argument as the pure temporary productivity shock, the pure temporary government spending shock ($\rho_g = 0$) allows the model economy calibrated for the Thai economy to experience a one standard deviation shock in government spending at time $t = 1$ only. Even though the shock is quite short-lived, households adjust their decisions accordingly.

Figure 5.8 A and B plots the responses of model variables, A for technology, capital, output and consumption and B for investment, labour, real wage and rental price of capital. Immediately, output slightly rises about 0.003 percent and then decline while consumption and investment fall to -0.0016 percent and -0.1 respectively response to the shock. Labour supply rises at the impact to 0.0045 percent and it explains why output increases. The initial impact on real wages is a fall of -0.016 percent whereas the rental price of capital rises to 0.0028 percent. At period two, output and consumption have a negative value and gradually inclines back to its normal level.

Investment increases to its steady state level when there is no government spending shock. After the pure temporary shock in government spending, labour supply response more at the beginning than it does during the transition. Obviously, the real wage and the rental price of capital move in the opposite direction because of capital and labour supply movement.

³⁴The effect of the permanent government spending shock generally results in a much longer negative wealth effect. Individuals consume less and work harder for a longer periods. Individuals cut their consumptions right after the shock and keep reducing it. They raise their labour at the impact and keep increasing it. Output increases on the impact and decline toward its steady state. The responses of variables to the permanent government spending shock do not plot here. Additionally, based on the previous RBC literatures, persistence technology shock can capture the stylized fact fluctuation, not government spending. There are a numbers of literature study the effects of fiscal policy such as Christiano and Eichenbaum (1992a), Braun (1994), Baxter and King (1993), McGratten (1994), Chari, Christiano and Kehoe (1994), Cooley and Ohanian (1997).

Figure 5.8 A pure temporary government spending shock

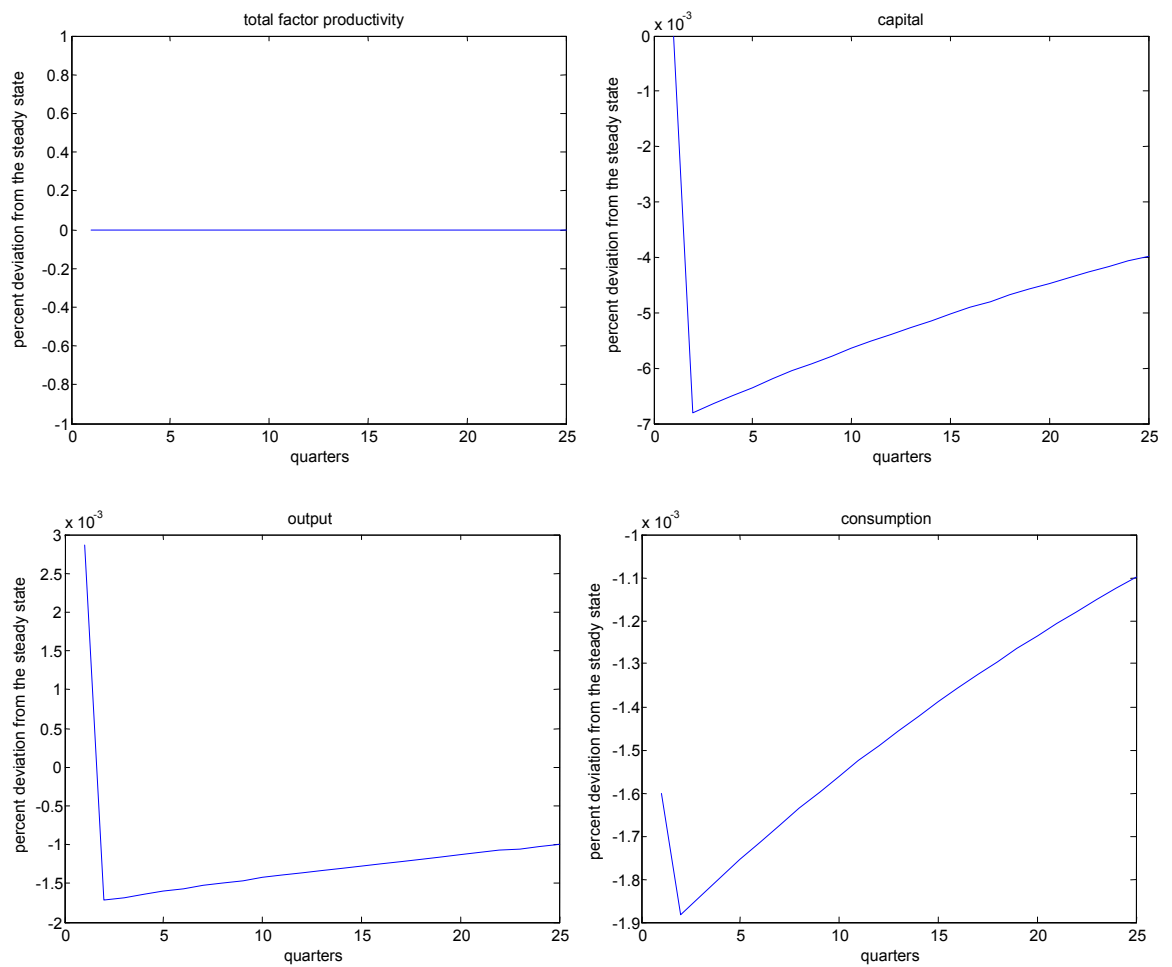
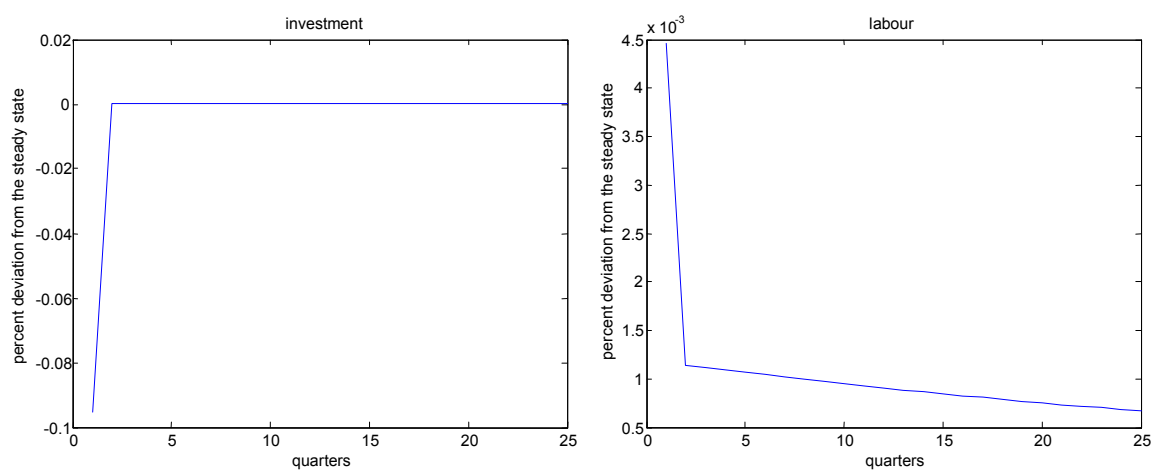


Figure 5.8 A



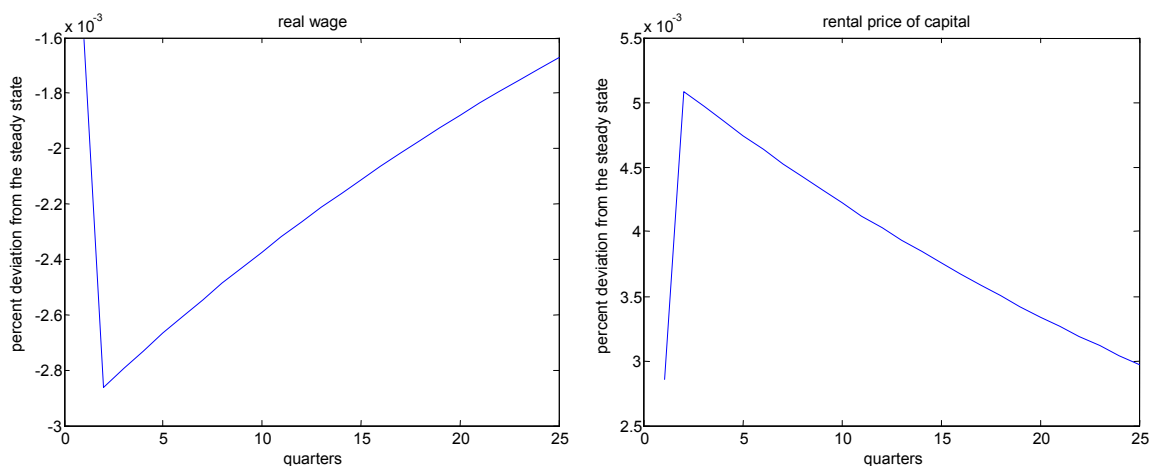


Figure 5.8 B

5.8.4 High and low substitution for the Thai economy

It is crucial in the RBC framework to study the effect of the elasticity on the model economy's response to the shock. σ is the measure of the inverse of the inter-temporal elasticity of substitution. When σ increases or the inter-temporal substitution of consumption and labour decrease, the representative households are less willing to substitute consumption and in labour through time. As a result the volatility or the fluctuation should decrease response to the shocks. Since the government spending shock cannot produce the realistic business cycle fluctuation, this section focuses on the dynamic effects with one type of shock, one percent standard deviation increase in productivity shock. Since the realistic σ is 2, a grid is chosen, reflecting some parameter choices.³⁵ The low substitution economy represents by $\sigma=10$ and $\sigma=0.4$ corresponds to the high substitution economy.³⁶ Figure 5.9 A and B depicts the response of consumption, labour, output and investment to the low and high substitution economy responsively.

The initial response of consumption of the high substitution economy is slightly lower than that of the low substitution economy because the consumption share parameter for the Thai economy is only 0.188. Labour, investment and output of the high substitution

³⁵ It is set to 2 following Backus, Kehoe and Kydland (1995) which is commonly used in many literatures for the multiplicative instantaneous utility function. Ortega, Estudios, Espana (1998), Ravn (1997), Baxter and Crucini (1993, 1995) also set the parameter to 2.

³⁶ To determine the low and high elasticity of the economy, the value of realistic σ is multiplied by 5 for low elasticity economy and is divided by 5 for high elasticity economy.

economy have a higher positive value than that of the low substitution economy immediately after a one percent increase in productivity shock.

Figure 5.9 The low and high substitution economy

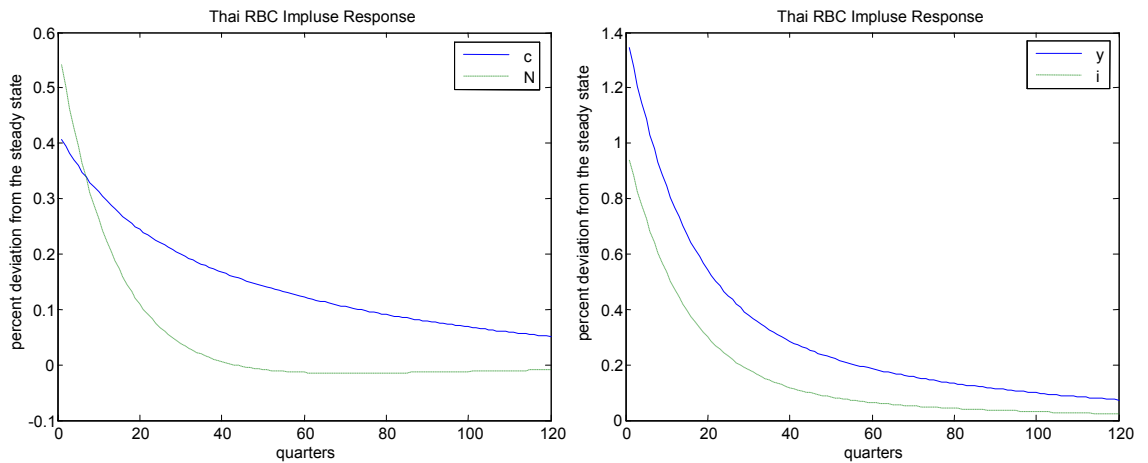


Figure 5.9 A. The low substitution economy

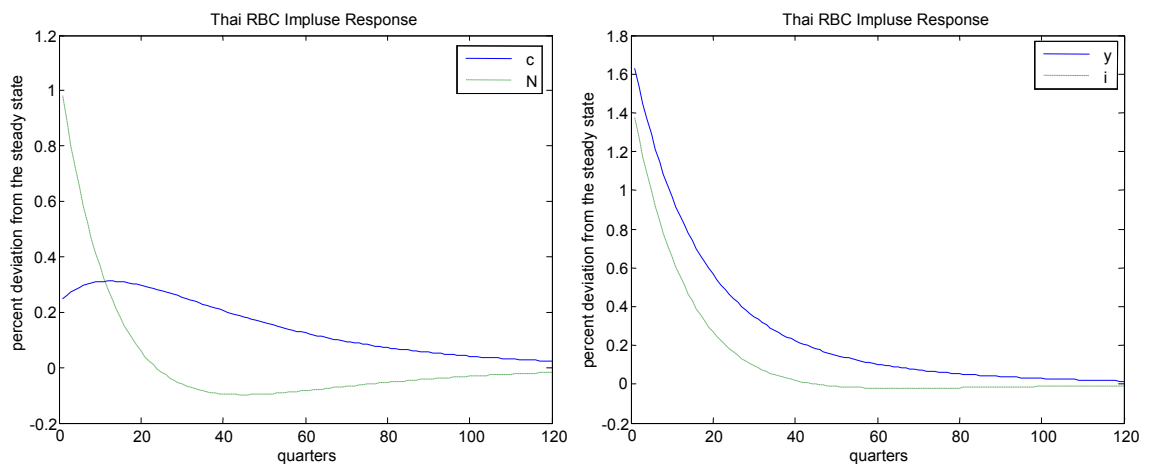


Figure 5.9 B. The high substitution economy

Because the impulse response yields the rough idea of the dynamic response, it is easier to understand the high and low substitution model economy by using the population moments. This chapter explores the influence of the inter-temporal substitution on the dynamic response in detail next section.

5.9 Model performance

This study has documented so far the model simulations or impulse responses calibrated for the Thai economy in response to different kinds of shock. Since the business cycle is about the fluctuation of aggregate variables, the salient features of the business cycle still have not been discussed. This section attempts to evaluate how well the business cycle models with associated shocks can capture some main cycle properties of Thailand. Specifically, this part focuses on three main features of time series, volatility, persistence and co-movement of each variable. Volatility is measured by a standard deviation and it provides the information of the magnitude of fluctuation. Persistence is measured by the auto-correlations. It indicates the inertia of the cycle or determines the degree of the last period value influencing a current one. Persistence is commonly measured by the first order autoregressive process. Lastly, co-movement measures the degree of movement of two variables. Co-movement determines the counter-cyclical and pro-cyclical characteristics of a time series and it is measured by correlations with output.

5.9.1 Model performance with different frequency

The model economy was filtered by using both the band pass filter by Baxter and King and the common HP filter just as the original Thai data to avoid the conclusions that depend too much upon the filter in use.³⁷ The rest of this chapter uses the band pass filter by Baxter and King. Since the quarterly Thai macroeconomics time series are available from 1993 to 2006, this section also investigates the model performance on major macroeconomics variables with difference data frequency by using quarterly data from 1993Q1 to 2006Q3 and annual data from 1976 to 2005. For annual frequency, the data are obtained from World Bank (World Development Indicator) except for wages obtained from National Economic and Social Development Board of Thailand (NESDB). The quarterly data set is from NESDB except for real interest rate and labour. The real interest rate is obtained from IMF (International Finance Statistic). Due to the short and limited data for Thailand, the labour input for quarterly frequency provided by National Statistical Office Thailand ranges from 1998Q1 to 2006Q3.

Tables 5.3 and 5.4 present business cycle properties filtered by the band pass and the HP filters respectively for RBC model of Thailand driven by an increase in 1 percent

³⁷ The filters are discussed in more depth in chapter 8.

productivity shock using the parameter values discussed on Table 5.1 in term of annual frequency.

Table 5.3

Cyclical behavior of the Baxter and King band pass filtered model economy from 1976 to 2005 annually³⁸

Table 5.3 A; Volatility and Persistence

	<i>Std</i>	<i>Std/y</i>	Auto-correlation with:		
			<i>x</i> (+1)	<i>x</i> (+2)	<i>x</i> (+3)
Output	3.41	1.000	0.903	0.654	0.350
Consumption	2.54	0.744	0.913	0.685	0.403
Investment	6.54	1.915	0.898	0.635	0.319
Labour	3.95	1.158	0.897	0.633	0.316
Wages	3.33	0.975	0.907	0.668	0.373
Rental rate	6.97	2.043	0.895	0.627	0.306

Table 5.3 A presents the percent standard deviation of the key variables and the percent standard deviation of each variable with respect to output from the model economy. In addition, it shows the persistence of the model economy by measuring the auto-correlation. The table shows that investment in the model economy fluctuates more than does output. Consumption in the model economy fluctuates less than does output. The reason consumption is less volatile than output is the temporary shock process. The temporary technology shock generates an increase in the current level of output following by a decline back to its normal level. Households expect the fall in output in the future, they are more likely to save and to smooth their consumption. Therefore, consumption increases by less than output does. The volatility ratio of investment to output is 1.915 while that of consumption to output is 0.744. In term of persistence, since one measure of the persistence is the first order auto-correlation, the model economy generally performs a high persistence.

³⁸ For the band pass filter, the frequencies associated with periods in the range from 2 to 8 years with 3 year centered moving average.

Table 5.3 B; Co-movement

	Cross-Correlation of Output with:								
	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$
Output	0.087	0.350	0.654	0.903	1.000	0.903	0.654	0.350	0.087
Consumption	-0.023	0.239	0.551	0.827	0.971	0.935	0.746	0.487	0.244
Investment	0.173	0.425	0.707	0.923	0.977	0.835	0.549	0.224	-0.043
Labour	0.179	0.386	0.614	0.782	0.810	0.674	0.420	0.140	-0.086
Wages	0.033	0.291	0.594	0.852	0.970	0.905	0.689	0.411	0.162
Rental rate	0.254	0.469	0.701	0.858	0.858	0.679	0.379	0.061	-0.186

Table 5.3 B provides the cross-correlation of each of the variables with respect to output. A positive sign represents that the variable is pro-cyclical (positive correlated) while a negative sign indicates that the variable is countercyclical (negative correlated). A zero value indicates that the variable shows no correlation to output at all. The table, furthermore, documents the cross-correlation with output at different leads and lags. The cyclical component of the variable lags the cycle by j year if the highest correlation is reached at $t + j$ while it leads the cycle j year if the highest correlation is reached at $t - j$. The strong correlation means that the correlation is larger than or equal to 0.5. As seen from the table, the outstanding property of the model economy is that the cyclical component of all variables is strongly contemporaneously correlated with output. Labour also has a strong contemporaneous correlation to output, 0.810, but the degree of correlation to output is less than other cyclical component series.

Table 5.4
Cyclical behavior of the HP filtered model economy from 1976 to 2005 annually³⁹

Table 5.4 A; Volatility and Persistence

	<i>Std</i>	<i>Std/y</i>	Auto-correlation with:		
			$x(+1)$	$x(+2)$	$x(+3)$
Output	3.00	1.000	0.611	0.312	0.095
Consumption	2.16	0.722	0.656	0.378	0.163
Investment	5.84	1.948	0.587	0.278	0.058
Labour	3.53	1.180	0.585	0.274	0.055
Wages	2.88	0.962	0.631	0.340	0.124
Rental rate	6.27	1.948	0.577	0.263	0.043

Table 5.4 A shows that, with the HP filter, the model still exhibits the same crucial properties of business cycle. Consumption and output in the model economy fluctuates less than does investment. The investment and consumption volatility relative to output

³⁹ For the HP filter, the parameter λ was set at 1600 for this experiment.

is 1.948 and 0.722 respectively. The volatility of the HP filtered series are quite close, slightly lower, to the one of the band pass filter. There is the evidence that the cyclical components have a low persistence, about 0.6 for the first auto-correlation.

Table 5.4 B; Co-movement

	Cross-Correlation of Output with:								
	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$
Output	-0.053	0.095	0.312	0.611	1.000	0.611	0.312	0.095	-0.053
Consumption	-0.108	0.037	0.257	0.565	0.974	0.664	0.407	0.205	0.056
Investment	-0.008	0.136	0.344	0.623	0.980	0.547	0.228	0.007	-0.134
Labour	0.012	0.132	0.303	0.530	0.818	0.437	0.161	-0.026	-0.143
Wages	-0.078	0.066	0.281	0.579	0.970	0.626	0.352	0.146	0.000
Rental rate	0.043	0.172	0.350	0.585	0.877	0.431	0.117	-0.090	-0.211

Table 5.4 B suggests, for the model economy filtered by the HP filter, the cyclical component of all variables are strongly contemporaneous correlated with output. Unlike the model economy filtered by the band pass filter, the cross correlation of each cyclical component relative to output significantly declines with leads and with lags. Labour also displays a strong contemporaneous correlation to output, 0.818, but the degree of correlation to output is less than other cyclical component series.

To evaluate the model performance, the features of the Thai business cycle are compared to those of the model. The main business cycle statistics for Thailand by the band pass and the HP filter respectively during 1976 to 2005 annually are shown in table 5.5 and 5.6.

Table 5.5 documents that in general the Thai cyclical component conforms with the facts described by Kydland and Prescott (1990).⁴⁰ The striking feature of the Thai cycle is that consumption volatility is higher than output volatility. The volatility of consumption is 2.58 while that of output is 2.39. It suggests that unlike the consumption in developed countries, the consumption in Thailand is less smooth than output. People in developed countries consider their future income to make a decision how much to consume in the current period. They tend to minimize the fluctuation in consumption. Compared to

⁴⁰ Kydland and Prescott (1990) document that consumption, investment and output are positively correlated and show a high degree of persistence. All variables are procyclical, except of trade balance. Investment is three times more volatile than output, whereas consumption is slightly less than output.

people in developed countries, Thai people minimize the variation in consumption in a less degree.

Table 5.5

The Baxter and king band pass filtered cyclical statistics of the Thai key variables from 1976 to 2005 annually⁴¹

Table 5.5 A; Volatility and Persistence

	<i>Std</i>	<i>Std/y</i>	Auto-correlation with:		
			<i>x</i> (+1)	<i>x</i> (+2)	<i>x</i> (+3)
Output	2.39	1.000	0.406	-0.195	-0.251
Consumption	2.58	1.079	0.345	-0.246	-0.236
Investment	9.09	3.803	0.398	-0.252	-0.299
Labour	1.00	0.418	0.143	-0.464	-0.256
Wages	2.46	1.029	0.589	0.195	-0.131
Rental rate	1.80	0.753	-0.173	-0.122	-0.091

Output-real GDP per capita, Consumption-household final consumption expenditure per capita, Investment-gross fixed capital formation per capita, Labour-aggregate employment, Wages-compensation to employment per capita, and Rental rate-real interest rate. All variables, seasonally adjusted and 1988 local price, are in logarithms (except for rental rate of capital) and are removed the trend by the band pass filter. Sources: World Bank (World Development Indicator) and National Economic and Social Development Board of Thailand

The volatility of investment with respect to output is approximately 3.8. The volatility of output is 3.41 in the artificial economy but 2.39 in the Thai economy. Consumption, in this model economy, fluctuates less than output does; however, the Thai economy shows the opposite direction. The volatility of consumption is 2.54 in the model whereas it is 2.58 in the data. Since the investment fluctuations in the model and data are, 6.54 in the model and 9.09 in the data, the volatilities relative to output are not close to each other. The model predicts high volatility of labour input, real wages and rental rate while the facts exhibit another way. The model economy suggests a high degree of the first order auto-correlation while the facts display a weaker auto-correlation. The model does not have a good match for either volatility or persistence.

For the contemporaneous correlation with output, table 5.5 B reports that even though all variables have pro-cyclical behaviours, only consumption and investment are strongly pro-cyclical. The rental rate of capital has the weakest positive correlation with output at only 0.180.

⁴¹ Export and import cyclical component are discussed on chapter 6 “Open economy”. Since government spending is exogenous variable by the assumption of the model, this section does not discuss the comparison of the government spending.

Table 5.5 B; Co-movement

	Cross-Correlation of Output with:								
	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$
Output	-0.139	-0.251	-0.195	0.406	1.000	0.406	-0.195	-0.251	-0.139
Consumption	-0.097	-0.196	-0.209	0.333	0.936	0.423	-0.167	-0.321	-0.169
Investment	-0.071	-0.218	-0.255	0.328	0.964	0.468	-0.168	-0.337	-0.209
Labour	0.126	-0.175	-0.339	-0.098	0.557	0.469	-0.157	-0.422	-0.280
Wages	-0.296	-0.475	-0.246	0.242	0.696	0.629	0.183	0.012	0.025
Rental rate	0.177	-0.192	-0.435	-0.121	0.180	-0.208	0.239	0.231	-0.075

Comparing with the model performance from table 5.3 B, predicting all variables are strongly pro-cyclical, the model can mimic the Thai facts for consumption and investment. On the other hand, the model cannot account for the actual cross-correlation at all. The actual economy shows that the series are countercyclical from year 2 onward, but this fact could not be found in the model economy. In addition, the model displays stronger degree of cross correlation in year 1 than that of the actual economy.

Table 5.6

The HP filtered cyclical statistics of the Thai key variables from 1976 to 2005 annually

Table 5.6 A; Volatility and Persistence

	<i>Std</i>	<i>Std/y</i>	Auto-correlation with:		
			$x(+1)$	$x(+2)$	$x(+3)$
Output	5.09	1.000	0.758	0.356	0.035
Consumption	5.15	1.012	0.732	0.327	0.007
Investment	18.72	3.678	0.722	0.289	-0.040
Labour	1.35	0.265	0.435	-0.117	-0.248
Wages	6.81	1.338	0.847	0.519	0.135
Rental rate	2.69	0.528	0.314	0.087	-0.151

Output-real GDP per capita, Consumption-household final consumption expenditure per capita, Investment-gross fixed capital formation per capita, Labour-aggregate employment, Wages-compensation to employment per capita, and Rental rate-real interest rate. All variables, seasonally adjusted and 1988 local price, are in logarithms (except for rental rate of capital) and are removed the trend by the band pass filter. Sources: World Bank (World Development Indicator) and National Economic and Social Development Board of Thailand

Considering the Thai facts filtered by the HP filter from table 5.6 A, one can find the same important business cycle features as the Thai facts filtered by the HP filter. Nonetheless, the volatility and the first order auto-correlation of the series filtered by the HP are larger than that of the series filtered by the band pass. The model economy, from table 5.4 A, predicts a significantly low both volatility and volatility relative to output of investment. The simulated series cannot capture the actual one.

Table 5.6 B suggests that the correlation of output with consumption is exactly the same with that from the model and the correlation of output with investment from the actual economy are slightly lower than the artificial one. Again, the model economy predicts a high correlation with output for labour, wages and rental rate of capital.

Table 5.6 B; Co-movement

	Cross-Correlation of Output with:								
	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$
Output	-0.200	0.035	0.356	0.758	1.000	0.758	0.356	0.035	-0.200
Consumption	-0.223	0.026	0.337	0.727	0.974	0.755	0.362	0.010	-0.215
Investment	-0.149	0.062	0.339	0.724	0.969	0.726	0.295	-0.075	-0.315
Labour	0.293	0.157	0.107	0.207	0.396	0.244	-0.189	-0.479	-0.526
Wages	-0.623	-0.468	-0.124	0.305	0.676	0.842	0.768	0.570	0.317
Rental rate	-0.125	-0.368	-0.501	-0.414	-0.231	-0.138	0.195	0.330	0.302

The cross correlation of output with consumption and investment are slightly higher than the model economy. Even though it seems that the model provides the good match for correlation with output, the number of literature documents another way.⁴² These facts are inconsistent with the facts filtered by the band pass.

Although Cooley and Prescott (1995) claim that the HP filter is the standard filter used in real business cycle literatures, many researchers find a problem when using it. Baxter and King (1999), Stock and Watson (1999) suggest that the HP filter may not be able to perform well. Roberto, Victor and Adolfo (2002) documents that the potential problems with the HP filter are more evident with annual data; therefore, they study the Brazilian business cycle and use both the HP and the band pass filter for annual data. It is important to study the business cycle regularities with different frequencies. In a later chapter we compare in considerable details the properties of the different filters. The next section investigates the Thai business cycle with quarterly data.

Since the macroeconomic time series available for Thailand are too short to study 1976 to 2005 quarterly, this chapter investigates the Thai business cycle during 1993q1 to 2006q3.

⁴² Cooley (1995) studies the US. cyclical behaviour from 1954 to 1991 quarterly and finds that consumption and investment are counter cyclical at $x(+5)$ or 1 year. Kydland and Prescott (1990) report the same result for the US during 1954 to 1989, quarterly. Povoledo (2000) finds the same results for Italy and the US from 1970 to 1998 quarterly.

Using the same calibration procedure as in section 5.7 and applying for quarterly data from 1993q1 to 2006q3, the real interest rate is 1.173 % and table 5.7 presents the parameterized variables for Thai economy

Table 5.7 Calibrated parameters for Thailand 1993q1-2006q3

Technology				Preferences			Driving process	
γ	δ	α	\bar{N}	β	σ	θ	ρ_A	σ_{ε_A}
0.00808	0.0108	0.6652	0.1931	0.9965	2	0.1641	0.9058	0.0192

Growth rate per capita presented during 1993-2006, 0.808 percent, is lower than that of during 1976-2005, 1.173 percent. The economics crisis on 1997 could play an important role to account for the average low growth in the last decade. Since the affect of capital and output ratio last decade was at 3.746, the depreciation per quarter is 1.08% and the labour share is 0.665. The depreciation rate and labour share during 1931-2006 are slightly higher than those during 1976-2005. The discounted value equal to 0.9965 is quite common to other business cycle literatures and is about the same as the period of 1976-2005. While the technology persistence during 1993-2006 is lower than that from 1976-2005, the variance of technology innovation shows another way around. Table 5.8 presents business cycle properties filtered by the band pass and table 5.9 reports the businesses cyclical regularities by using the HP filters for the RBC model of Thailand. The RBC model using the calibrated parameters for Thailand discussed on table 5.7 is driven by an increase in 1 percent productivity to produce the fluctuation around a trend of the main Thai variables.

Table 5.8
The Baxter and King band pass filtered cyclical statistics of the model economy from 1993q1 to 2006q3

Table 5.8 A; Volatility and Persistence

	Std	Std/y	Auto-correlation with:		
			$x(+1)$	$x(+2)$	$x(+3)$
Output	4.69	1.000	0.897	0.632	0.314
Consumption	1.96	0.418	0.900	0.644	0.333
Investment	12.64	2.70	0.896	0.630	0.310
Labour	4.97	1.059	0.898	0.636	0.320
Wages	3.12	0.665	0.899	0.639	0.326
Rental rate	8.14	1.735	0.898	0.635	0.319

As in table 5.8 A, the model economy once again displays business cycle phenomena. Consumption fluctuates less than output does and investment is the most volatile series. The volatility of consumption, 1.96, is about half that of output, 4.69. Investment is about three times more volatile than output. The volatility of interest rate, 8.14, is the second highest among the volatilities of all variables. The model shows the high degree of persistence for all variables. The first order auto correlation coefficient for all variables is approximately 0.90. The model economy suggests that all variables are strongly pro-cyclical. The investment series performs the strongest contemporaneous correlation behaviour at 0.994. All variables are counter cyclical at 5 quarters after the technology shock.

Table 5.8 B; Co-movement

	Cross-Correlation of Output with:										
	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
Output	-0.126	0.046	0.314	0.632	0.897	1.000	0.897	0.632	0.314	0.046	-0.126
Consumption	-0.210	-0.047	0.216	0.540	0.819	0.957	0.899	0.681	0.398	0.150	-0.017
Investment	-0.094	0.077	0.341	0.652	0.905	0.994	0.877	0.601	0.277	0.008	-0.162
Labour	-0.070	0.085	0.323	0.599	0.822	0.898	0.789	0.539	0.246	0.002	-0.152
Wages	-0.159	0.003	0.259	0.566	0.828	0.943	0.865	0.633	0.344	0.095	-0.069
Rental rate	-0.039	0.119	0.359	0.634	0.851	0.914	0.788	0.522	0.216	-0.035	-0.191

Table 5.9

The HP filtered cyclical statistics of the model economy from 1993q1 to 2006q3

Table 5.9 A; Volatility and Persistence

	<i>Std</i>	<i>Std/y</i>	Auto-correlation with:		
			$x(+1)$	$x(+2)$	$x(+3)$
Output	5.73	1.000	0.763	0.558	0.384
Consumption	2.47	0.431	0.788	0.601	0.438
Investment	15.34	2.678	0.757	0.548	0.371
Labour	6.10	1.065	0.769	0.568	0.396
Wages	3.88	0.677	0.778	0.583	0.415
Rental rate	9.97	1.740	0.767	0.565	0.392

The model economy filtered by the HP filter display a high volatility compare to the one filtered by the band pass for all series. Investment is about 2.7 times as volatile as output. The investment and the rental rate are more fluctuate than the other cyclical series. The model performs a strongly first order auto correlation; however, the persistence of the HP filtered model is lower than the one of the band pass filtered.

Table 5.9 B shows that all variables are strongly pro-cyclical, particularly investment. The cross correlations one period after the shock are lower than the model filtered by the band pass. At 5 quarters after the shock, all variables are still pro-cyclical but the degree of correlation is quite weak.

Table 5.9 B; Co-movement

	Cross-Correlation of Output with:										
	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
Output	0.118	0.238	0.384	0.558	0.763	1.000	0.763	0.558	0.384	0.238	0.118
Consumption	0.037	0.158	0.306	0.485	0.699	0.949	0.769	0.607	0.464	0.339	0.232
Investment	0.144	0.261	0.403	0.570	0.766	0.993	0.741	0.526	0.345	0.195	0.073
Labour	0.153	0.259	0.384	0.531	0.702	0.897	0.671	0.478	0.314	0.179	0.067
Wages	0.082	0.198	0.339	0.509	0.709	0.942	0.742	0.566	0.414	0.283	0.173
Rental rate	0.184	0.289	0.412	0.551	0.720	0.908	0.664	0.456	0.283	0.140	0.025

To investigate how well the model mimics some main business cycle regularity of Thailand for quarterly data during 1993q1 to 2006q3, the features of the Thai business cycle are compared to those of the model. The main business cycle statistics for Thailand by the band pass and the HP filter respectively are shown in table 5.10 and 5.11.

Table 5.10
The band pass filtered cyclical statistics of the Thai key variables from 1993q1 to 2006q3⁴³

Table 5.10 A Volatility and Persistence

	<i>Std</i>	<i>Std/y</i>	Auto-correlation with:		
			$x(+1)$	$x(+2)$	$x(+3)$
Output	3.13	1.000	0.908	0.709	0.442
Consumption	3.52	1.125	0.921	0.726	0.455
Investment	10.45	3.339	0.937	0.792	0.587
Labour	0.03	0.01	0.661	0.266	0.077
Wages	1.22	0.390	0.879	0.589	0.239
Rental rate	2.19	0.700	0.917	0.702	0.415

Output-real GDP per capita, Consumption-household final consumption expenditure per capita, Investment-gross fixed capital formation per capita, Labour-aggregate employment, Wages-compensation to employment per capita, and Rental rate-real interest rate. All variables, seasonally adjusted and 1988 local price, are in logarithms (except for rental rate of capital) and are removed the trend by the band pass filter. Sources: National Economic and Social Development Board of Thailand, IMF (International Finance Statistic), National Statistical Office Thailand.

The Thai stylized facts display that investment fluctuates about 3.34 times as high as output while the model predicts only 2.70 times. The volatility of labour input in

⁴³ Since the export and import data are not available for seasonally adjusted series, this table does not contain the cyclical property of export and import. In addition, this chapter focuses on closed economy model, import and export cyclical properties are discussed in detail on the open economy.

Thailand is substantially low at 0.03. The rental rate does not exhibit a substantially high volatility as in the model does. The persistences are about 0.90 for output, consumption, investment, wages and rental rate of both model and actual economy. Labour in Thailand displays a lower persistence, 0.661, while the model predicts a value of 0.898. Even though the model economy does not provide the perfect match for all variables in the Thai economy, the model economy does perform the business cycle properties of output, consumption and investment.

Table 5.10 B; Co-movement

	Cross-Correlation of Output with:										
	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(+1)$	$x(+2)$	$x(+3)$	$x(+4)$	$x(+5)$
Output	-0.092	0.160	0.442	0.709	0.908	1.000	0.908	0.709	0.442	0.160	-0.092
Consumption	-0.124	0.124	0.407	0.679	0.886	0.989	0.926	0.746	0.484	0.193	-0.075
Investment	-0.081	0.157	0.411	0.645	0.830	0.946	0.927	0.813	0.622	0.386	0.147
Labour	0.461	0.510	0.486	0.364	0.163	-0.087	-0.442	-0.644	-0.679	-0.623	-0.526
Wages	-0.015	-0.130	-0.286	-0.463	-0.625	-0.722	-0.704	-0.544	-0.280	0.025	0.309
Rental rate	-0.417	-0.524	-0.555	-0.485	-0.306	-0.039	0.294	0.602	0.819	0.911	0.879

The model economy can account for a contemporaneously strong correlation to output for some variables. Table 5.10 B provides the facts that output, consumption and investment are contemporaneously strongly pro-cyclical as the model economy reflects. In contrast, labour, wages and rental rate are contemporaneously countercyclical. The Thai wages are strongly countercyclical at -0.722. Christiano and Eichenbaum (1992) suggest that the weakly contemporaneous correlation of real wages to real output are considered as poorly explained by macroeconomic theories. There are also negative correlations of labour and rental rate with real output contemporaneously, but they are weak at -0.087 and -0.039. In fact, the negative correlation of rental rate and output is even stronger with lags. Therefore, there is the evidence that the interest rate has a negative leading indicator for real output in Thailand. King and Watson (1996) also find the same results for the US economy and they concluded that many macroeconomic models, including RBC model, are unable to capture this facts. By looking at different leads and lags, it can be inferred that labour leads real output by 4 quarters. The cross correlation suggests that real wages and rental rate lags real output by approximately one year. One year after the shock the variables perform a countercyclical cross correlation as in model and actual economy for output, consumption and investment.

Table 5.11 A documents the fact that for quarterly data the HP filtered cyclical statistics of the Thai economy are similar to those of the band pass filter. The model economy predicts quite high volatility for all variables except for consumption as in table 5.9.

Table 5.11
The HP filtered cyclical statistics of the Thai key variables from 1993q1 to 2006q3

Table 5.11 A; Volatility and Persistence

	<i>Std</i>	<i>Std/y</i>	Auto-correlation with:		
			<i>x</i> (+1)	<i>x</i> (+2)	<i>x</i> (+3)
Output	3.77	1.000	0.859	0.656	0.467
Consumption	3.98	1.056	0.897	0.725	0.512
Investment	12.32	3.268	0.866	0.752	0.628
Labour	2.77	0.735	-0.068	-0.754	-0.020
Wages	2.99	0.793	-0.115	0.093	0.009
Rental rate	2.69	0.714	0.468	0.271	0.281

Output-real GDP per capita, Consumption-household final consumption expenditure per capita, Investment-gross fixed capital formation per capita, Labour-aggregate employment, Wages-compensation to employment per capita, and Rental rate-real interest rate. All variables, seasonally adjusted and 1988 local price, are in logarithms (except for rental rate of capital) and are removed the trend by the band pass filter. Sources: National Economic and Social Development Board of Thailand, IMF (International Finance Statistic), National Statistical Office Thailand.

Even though the model shows a lower value of the first order auto correlation, it is able to account for the persistence for output, consumption and investment. It is particularly interesting that HP filter exhibits a weakly negative first order auto-correlation for labour and wages. Labour, wages and rental rate are less persistence in the Thai data than in the model.

Table 5.11 B; Co-movement

	Cross-Correlation of Output with:										
	<i>x</i> (-5)	<i>x</i> (-4)	<i>x</i> (-3)	<i>x</i> (-2)	<i>x</i> (-1)	<i>x</i>	<i>x</i> (+1)	<i>x</i> (+2)	<i>x</i> (+3)	<i>x</i> (+4)	<i>x</i> (+5)
Output	0.135	0.298	0.467	0.656	0.859	1.000	0.859	0.656	0.467	0.298	0.135
Consumption	0.055	0.254	0.463	0.660	0.842	0.954	0.895	0.728	0.528	0.323	0.110
Investment	0.054	0.239	0.448	0.622	0.794	0.895	0.869	0.803	0.669	0.498	0.303
Labour	0.110	0.185	-0.030	0.033	0.112	-0.069	-0.345	0.212	0.128	-0.095	-0.269
Wages	-0.006	-0.103	-0.163	-0.279	-0.250	-0.275	-0.259	-0.231	-0.176	0.042	0.104
Rental rate	-0.305	-0.333	-0.309	-0.266	-0.187	0.017	0.256	0.434	0.570	0.600	0.559

The model economy from table 5.9 B can account for a strong correlation of consumption and investment to output. Similar to the band pass filter, table 5.11 B gives the facts that output, consumption and investment are strongly pro-cyclical as the model economy does. The labour, wages are counter cyclical and the rental rate is noticeably

weak pro-cyclical. These results are consistent with the actual data filtered by the Baxter and King band pass.

The model both filtered by Baxter and King band pass and HP filter fail to fully explain the striking features of Thai business cycle. First, Thai consumption more fluctuate than output does. Second, Thai labour, wages and interest rate have essentially no contemporaneous co-movement with real output. However, the band pass filtered model economy provides the better match to the facts filtered by the band pass for the persistence. The band pass filtered model also exhibits a better explanation for the cross correlation as in table 5.8 B and 5.10 B for output, consumption and investment.

5.9.2 Model performance with different parameters

Since the calibration of the model allow us to evaluate the model with different crucial parameters, it is important to assign different parameters to the model in order to evaluate the quantitative implications of this basic theoretical economy. This section explores the influence the inter-temporal substitution on the model performance. The low and high substitution economies are represented by $\sigma=10$ and $\sigma=0.4$ respectively. Generally, the high substitution economy has a large macroeconomic impact with only small variation in productivity. Because the construction of the model leaves σ free to be set in different value, this section examines the model with $\sigma=1$ or the log utility case as well.⁴⁴

Table 5.12 The Baxter and King band pass filtered cyclical statistics of the Thai key variables 1993q1 to 2006q3

Thai economy	Volatility		Cross correlation			Persistence
	<i>Std</i>	<i>Std/y</i>	$x(-1)$	x	$x(+1)$	
Output	3.13	1.000	0.908	1.000	0.908	0.908
Consumption	3.52	1.125	0.886	0.989	0.926	0.921
Investment	10.45	3.339	0.830	0.946	0.927	0.937
Labour	0.03	0.01	0.163	-0.087	-0.442	0.661
Wages	1.22	0.390	-0.625	-0.722	-0.704	0.879
Rental rate	2.19	0.700	-0.306	-0.039	0.294	0.917

⁴⁴ When the intertemporal substitution equals to 1, the original utility becomes the log utility and it is an additive utility function, $u(c, L) = \theta \ln c + (1 - \theta) \ln L$.

Since the model economy is able to capture the facts better with the band pass filter for quarterly data, this section investigates the model performance for 1 percent increase in technology with low and high substitution economy applying for Thai economy during 1993q1 to 2006Q3. The Thai business cycle features are summarized again in table 5.12 for the convenience

When different values of inter-temporal substitution are assigned to the model economy calibrated to Thai economy, all models exhibit in some respects the business cycle regularities. The high substitution economy has striking features that substantially amplify technology fluctuations and cause strong co-movements of output to the other variables. As in table 5.13, all variables are strongly pro-cyclical for $\sigma=0.4$.

Model economy with variable labour		Volatility		Cross correlation			Persistence
		<i>Std</i>	<i>Std/y</i>	$x(-1)$	x	$x(+1)$	
$\sigma = 10$	Output	3.88	1.000	0.896	1.000	0.896	0.896
	Consumption	2.71	0.698	0.817	0.921	0.836	0.898
	Investment	8.61	2.220	0.853	0.944	0.838	0.895
	Labour	3.61	0.932	0.774	0.852	0.755	0.898
	wages	3.56	0.918	0.809	0.907	0.818	0.898
	Rental rate	7.21	1.858	0.811	0.880	0.768	0.898
$\sigma = 2$	Output	4.69	1.000	0.897	1.000	0.897	0.897
	Consumption	1.96	0.418	0.819	0.957	0.899	0.900
	Investment	12.64	0.270	0.905	0.994	0.877	0.896
	Labour	4.97	1.059	0.822	0.898	0.789	0.898
	wages	3.12	0.665	0.828	0.943	0.865	0.899
	Rental rate	8.14	1.735	0.851	0.914	0.788	0.898
$\sigma = 1$	Output	5.11	1.000	0.897	1.000	0.897	0.898
	Consumption	1.61	0.315	0.788	0.950	0.920	0.903
	Investment	14.29	2.92	0.909	0.996	0.877	0.896
	Labour	5.62	1.100	0.840	0.915	0.802	0.898
	wages	2.91	0.570	0.824	0.948	0.879	0.900
	Rental rate	8.59	1.681	0.863	0.924	0.795	0.897
$\sigma = 0.4$	Output	5.54	1.000	0.897	1.000	0.897	0.897
	Consumption	1.28	0.231	0.715	0.913	0.928	0.898
	Investment	17.29	3.120	0.912	0.997	0.876	0.896
	Labour	6.28	1.133	0.854	0.929	0.812	0.898
	wages	2.70	0.488	0.813	0.948	0.890	0.901
	Rental rate	9.04	1.631	0.873	0.932	0.798	0.897

Table 5.13 The model with different values of intertemporal substitution

The low substitution economy or $\sigma=10$ displays a strong contemporaneous correlation with out put as well. The high and low substitution economy performs a high persistence

as well. With log utility function, the dynamics responses to the productivity shock have a high correlation with output and strong persistence as well. The volatility of all variables in the high substitution economy is higher than that of the low, standard and log utility economy, except for consumption.

It is interesting to note that, for this model economy, the volatility of consumption decreases as the value of inter-temporal substitution increases. Or alternatively, the high substitution economy leads to the low volatility of consumption and the volatility of consumption is high in the low substitution economy. One explanation of this discrepancy is that the structure of the utility function allows the consumption and leisure smoothing parameters dependent on only one value, σ . When σ increases, the volatility of consumption and leisure should decline because the representative agents are less willing to smooth their consumption and labour supply overtime. Nevertheless, only labour supply fluctuates less in the low substitution economy. The reason could come from the fact that the consumption share, θ , in the utility function is quite low, only 0.16 and in turn the leisure share parameter is high, 0.84. All of these reasons account for the low volatility of consumption in the high substitution economy and high volatility of consumption in the low substitution economy.⁴⁵

When σ decreases, the volatility of labour supply relative to output increases because the representative agents are more eager to substitute hours work over time in response to fluctuation in productivity, and in this model, they become relatively less willing to substitute consumption, as a result of the volatility of consumption decreases. Therefore, the volatility of real wages decreases since, in the neoclassical model, wages are perfectly correlated with the capital-labour ratio. Alternatively, a reduction in σ induces an increase in consumption smoothing and a decline in labour smoothing. If the representative agents are more willing to smooth their consumption, they will invest more after the shock. This could increase the correlation between capital and hours, inducing low volatility of wages.

⁴⁵ It is also possible to set different smoothing parameters to consumption and leisure in the utility function. Generally, the utility is additively separable. To explores the labour fluctuation, many labour economists follow and extend the work of Hansen (1985) and Rogerson (1988) indivisible labour model. The indivisible labour model typically has the separable preference, allowing leisure has its own smoothing parameter and yielding the high contemporaneous impact of the productivity shock on labour and output.

To see the volatility of consumption increases while σ decreases, θ is set near 1. If the θ parameter is set to 1, the model economy is under the fixed labour condition and the economic effects are in table 5.14.⁴⁶

In this case, labour is rigid and the preference presumably contains only consumption. In this way, consumption tends to be more volatile in response to a reduction in σ as in table 5.14.

Table 5.14 The model with different values of intertemporal substitution in fixed labour

Model economy with fixed labour		Volatility		Cross correlation			Persistence
		<i>Std</i>	<i>Std/y</i>	$x(-1)$	x	$x(+1)$	
$\sigma = 10$	Output	2.24	1.000	0.895	1.000	0.895	0.895
	Consumption	0.003	0.152	0.687	0.848	0.834	0.902
	Investment	7.41	3.313	0.899	0.996	0.883	0.895
	Labour	0.00	0.000	0.372	0.409	0.360	0.898
	wages	4.75	2.124	0.422	0.470	0.420	0.898
	Rental rate	4.79	2.141	0.472	0.473	0.395	0.898
$\sigma = 2$	Output	2.24	1.000	0.895	1.000	0.895	0.895
	Consumption	0.008	0.349	0.404	0.526	0.540	0.902
	Investment	7.28	3.251	0.891	0.977	0.857	0.895
	Labour	0.00	0.000	0.330	0.353	0.303	0.898
	wages	4.75	2.120	0.425	0.471	0.418	0.898
	Rental rate	4.78	2.136	0.451	0.471	0.390	0.898
$\sigma = 1$	Output	2.24	1.000	0.896	1.000	0.896	0.896
	Consumption	1.26	0.600	0.241	0.338	0.369	0.901
	Investment	7.59	3.383	0.860	0.934	0.812	0.895
	Labour	0.00	0.000	0.307	0.321	0.268	0.898
	wages	4.75	2.117	0.428	0.471	0.416	0.898
	Rental rate	4.78	2.132	0.452	0.468	0.356	0.897
$\sigma = 0.4$	Output	2.25	1.000	0.896	1.000	0.896	0.896
	Consumption	2.47	1.097	0.049	0.124	0.177	0.900
	Investment	9.27	4.113	0.753	0.798	0.675	0.895
	Labour	0.00	0.000	0.284	0.282	0.220	0.898
	wages	4.75	2.113	0.436	0.473	0.411	0.898
	Rental rate	4.78	2.123	0.455	0.463	0.373	0.897

In other words, the representative agents are more eager to substitute their consumption over time. Although the model performs pro-cyclically for all variables, the degrees of correlation to output are weak, except for investment. The rigidity of labour could be the

⁴⁶ If θ equal to 1, the model economy becomes the fixed labour model economy. According to $D_1u(c, L)AD_2F(k, N) = D_2u(c, L)$, that particular condition could happen by simply setting the value of \bar{N} close to 1.

reasons why labour, wages and rental rates are weakly pro-cyclical. Since capital is able to fluctuate while the labour is almost not, this could decrease correlation between capital and labour, so wages and rental rates are weakly correlated to output. The volatility of consumption and investment in the high substitution economy is higher than that of the low, standard and log utility economy. The volatility of output, wage and rental rate do not change in response to different value of σ .

Comparing to the Thai business cycle, the model economy with variable labour supply can capture important features of the Thai economy during 1993q1 to 2006q3 better than one with fixed labour supply. With $\sigma=2$, the model with variable labour produces a high volatility of investment to 12.64 percent while it is 10.44 in the data. The higher value of σ can bring the investment volatility low, but the volatility of investment relative to output is low. It seems like the problem can solve with the cost of another. The smoothing parameter $\sigma=2$ is still be the reliable parameter for the Thai economy, even though it is not provide a perfect match to Thai economy during the observation period.

5.10 Conclusions

The chapter evaluates the dynamic of a closed economy model calibrated from Thai economy. It investigates the Thai business cycle using both annual and quarter data. The empirical data are divided into two time periods, 1976-2005 annual and 1993q1-2006q3 quarterly. The model and the real economy are filtered by the band pass filter by Baxter and King and the HP filter. For the annual observed data, the model economies filtered by both the band pass and the HP filter exhibit crucial business cycle properties. Namely, consumption and output fluctuate less than investment does. Consumption in the model economy is less volatile than that of output. The models suggest a high degree of persistence and co-movement with output. However, the HP filtered model exhibits lower persistence than that of the band pass filtered model. The striking fact of the Thai business cycle is that consumption fluctuates more than output does. This outstanding fact cannot be explained by the model economy. The Baxter and King band pass filtered model economy results in a strong degree of persistence while the facts suggest the opposite. The model filtered by the band pass filter cannot account for the countercyclical facts in year 2 while the model filtered by the HP filter suggests pro-

cyclical consistency with the facts. This is the difference between the band pass and the HP filter for an annual data.

For quarterly data, the model economies exhibit business cycle regularities. Even though the models cannot explain the high volatility of consumption relative to output, they solve the persistent problem of the model with annual data.⁴⁷ The model economy can mimic the fact better with the band pass filter for the quarterly data.

The model economy is simulated by three different kinds of driving parameters, capital, productivity and government spending. Other shocks cannot generate the real business cycle properties except for the productivity shock. The model is also simulated by multiple shocks and the shocks off the steady state. The idea of the shocks off the steady state is introduced because the economy sometimes is off the steady state such as the periods of rapid expansion and contraction. This chapter replicates the 1997 economic crisis in Thailand and try to minimize the cycle by using the government spending shock. Comparing to the technology shock, the government spending shock has a minimal effect to the aggregation fluctuations.

The calibrated values have an important influence on the model performance. The persistence parameters of the technology shock of the Thai economy during the observation periods are significantly high. The idea that the variation in output can be accounted for by large and high persistence of shocks to productivity is satisfied for Thai economy. In this model economy, the volatility of output is higher than in the Thai economy, concluding that productivity shock can be the reason much, but not all, of the output fluctuation. Similar to the Thai facts, investment in the model economy fluctuates much more than output. However, the model fails to explain the volatility of consumption relative to output for Thai economy. The model predicts that consumption fluctuates less than output does while the Thai facts show the opposite way. The volatility and the contemporaneous correlation with real output of labour, wages and interest rate are low in the data but higher in the models. This discrepancy implies a crucial missing component in this model for capturing the business cycle of developing countries, specifically Thailand. The model economy generates a high persistence and

⁴⁷ For annual data, the model suggests a high degree of the first order auto correlation coefficient while the facts display a weak auto correlation.

strong co-movement, just as it does in the Thai economy for output, consumption and investment.

Chapter 6

The Open Economy Model: Specification and Calibration

6.1 Introduction

Moving on from the closed economy framework presented outline earlier, this chapter will present the results from the simulation of an open economy RBC model. This more realistic assumption is important in the Thai case since Thailand's trade openness is 66.84% of GDP for exports and 59.75% of GDP for imports from 1998q1 to 2007q3. The international real business cycle model improves on the closed economy version in two important ways. One is the international co-movement: the correlation of output fluctuation in one country to that of another country. Another is the relative price determination. The literature focuses on the real exchange rate and terms of trade. Since the model presented in this chapter is the one good two country model, the model has no implication for the terms of trade. The main focus is on international co-movement. The international real business cycle model basically focuses on the channel in which business cycles are transmitted across countries. To explain the international business cycle of Thailand by using the international real business cycle model (IRBC model), there are three main objectives of this chapter. First, the empirical features of international real business cycles are investigated. Secondly, the basic IRBC model driven by the shocks in technology and variations in the fiscal policies, specifically government purchases and taxes, is constructed.

Section 6.2 and section 6.3 deal with the first goal of this chapter. They provide a series of tables and graphs depicting international business cycle properties for several countries and for several variables. These stylized facts are derived using the band pass filter which has become more commonly used in the literature recently. Section 6.2 documents the characteristics of international business cycle for seven developed countries during 1986 to 2006. Because the USA and Japan are the main trading partners for Thailand, section 6.3 presents the business cycle properties for Thailand, US and Japan. The structure of the IRBC Model is presented in section 6.4. It explains the basic structure of this model in terms of

preferences and technology and market structure. The simplest complete market one-good, two-country business cycle model Baxter and Crucini (1995) is used to account for the Thai business cycle fluctuations. Unlike their model, agents in each of country have different preferences, technology parameters and driving processes. The model is used to study the behavior of a pair of countries, Thailand-US. Section 6.5 contains a brief discussion of the methodology used to solve the model. The details are described in chapter 4, methodology. The set of calibrated parameters for the two countries are presented in section 6.6. Finally, the conclusions are given in section 6.7.

6.2 International Business Cycles for 7 OECD Countries

Before evaluating the international real business cycle model, we review some of the salient properties of international business cycles. The data are quarterly from the OECD's Quarterly National Account, except employment from Main Economic Indicators and IMF's International Finance Statistic. Table 6.1 reports a number of properties of the business cycle experience from 1986 to 2006 using quarterly data in seven developed countries: the USA, Canada, Australia, the UK, Japan, Korea and European, an (EU) aggregate constructed by the OECD.¹ This section follows the standard methodology which focuses on volatility, measured by standard deviations; persistence, measured by autocorrelations; and co-movement, measured by correlations. Table 6.1 A presents the volatility of the cyclical component of the main variables in several countries. These figures vary widely across the countries. In general, the output volatilities are less than one except for Canada, Japan and Korea. The volatility of output is 1.22 in Japan but 2.18 in Korea. It is striking that these two Asian countries seemingly are more volatile than other observed countries. The volatility of the ratio of net exports to output is less than one for all countries except for

¹ Europe OECD includes a data for Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom. The civilian employment series for the Europe OECD European Member countries are not available by OECD Main Economics Indicators. They are derived from the summation of each country member series and they are expressed in seasonally adjusted. The employment series are too short in some countries. The employments of Ireland and Greece start from 1998Q1. For Denmark and Belgium, they start from 1995Q1 and 1998Q2 respectively. The employment series of Iceland are not available. The data for the Netherlands is from IMF.

Korea. Consumption fluctuates less than output in all countries except the UK and Korea². With respect to volatility, this section finds that investment has been two to four times more volatile, in percentage term, than output. Nonetheless, there are some differences across countries in the magnitudes.

Table 6.1 A Business cycle properties of 7 OECD countries from 1986q1-2006q4; Volatility

Country	Standard Deviation (%)		Ratio of Standard Deviation to that of y					
	y	nx	c	i	g	x	m	n
United States	0.84	0.22	0.80	3.08	0.73	3.38	3.20	0.72
EU	0.60	0.32	0.84	3.09	0.75	3.00	3.45	3.17
Canada	1.15	0.73	0.68	2.63	0.76	2.13	2.25	0.44
Australia	0.99	0.76	0.82	4.51	0.63	2.15	4.16	1.01
United Kingdom	0.86	0.43	1.17	3.35	0.86	2.17	2.64	1.13
Japan	1.22	0.39	0.67	2.45	0.60	3.23	5.74	0.33
Korea	2.18	2.08	1.51	2.72	0.48	1.64	3.18	0.70

Notes: Business Cycles taken from 1986Q1-2006Q4. Statistics are based on band pass filtered data. All variables are in real terms. Variables are y, output; c, consumption; i, investment; g, government spending; nx, ratio of net exports to output; x, exports; m, imports; n, civilian employment.

The standard deviation of investment relative to output fluctuations ranges from a low of 2.45 in Japan to a high of 4.51 in Australia. The government expenditure fluctuates less than output in all countries. The volatility of employment also has a large spread across countries. There has been even greater variation in the volatility of employment in the EU.³ Relative to output, the volatility of employment in EU is 3.17. Exports are less volatile than import for most cases, except for the USA. Korean experiences less export volatility than others. However, Korean trade is the most volatile series of all observation series. During the Asian crisis 1997, Korean GDP substantially dropped by 6.8% from 1997 to 1998. Imports in Japan and Australia are more volatile than other countries.

² One of the outstanding business cycle features of developing countries is that consumption is more volatile than output. It is important to note that, during the observation period, consumption volatility in the UK is higher than output volatility. Blackburn and Ravn (1992) reports that the UK higher volatility in private consumption than in output because of a high volatility of consumption of durable good. The consumption of non durable good is less volatile than output.

³ Some of this disparity reflects international differences in labour market experience. Also part of the explanation resides in the short of data in some countries in the EU.

Table 6.1 B depicts that all variables in each country perform a high degree of the first order autocorrelation. The cyclical component of all series is highly persistent.

Table 6.1 B Business cycle properties of 7 OECD countries from 1986q1-2006q4; Persistence

Country	The first order autocorrelation							
	y	nx	c	i	g	x	m	n
United States	0.93	0.94	0.93	0.95	0.92	0.91	0.92	0.93
EU	0.93	0.94	0.96	0.95	0.94	0.90	0.92	0.92
Canada	0.92	0.90	0.91	0.90	0.95	0.92	0.90	0.94
Australia	0.92	0.91	0.92	0.90	0.87	0.89	0.89	0.94
United Kingdom	0.93	0.86	0.90	0.89	0.89	0.90	0.87	0.94
Japan	0.94	0.92	0.91	0.94	0.92	0.93	0.94	0.94
Korea	0.91	0.91	0.91	0.92	0.92	0.88	0.90	0.91

Notes: Business Cycles taken from 1986Q1-2006Q4. Statistics are based on band pass filtered data. All variables are in real terms. Variables are y, output; c, consumption; i, investment; g, government spending; nx, ratio of net exports to output; x, exports; m, imports; n, civilian employment.

Table 6.1 C depicts that in most cases the cyclical components of all variables exhibits positive co-movement with output, except for net exports and government spending. Employment has been pro-cyclical in all countries, but the magnitude of the correlation with output varies significantly across countries from a low of 0.15 in EU to a high of 0.92 in Korea. Consumption and investment have also been strongly pro-cyclical and coincident with the output cycle. Consumption in Australia is slightly less correlated to output than other sample countries. Investment correlation to output in Canada is smaller than that in the other countries.

The striking properties are the counter-cyclical nature of net exports in almost all countries, except for Canada.⁴ However, the net exports correlation with output in Canada is quite weak. Korean trade has the highest negative correlation with its own output, -0.74, among the sample countries. This evidence implies that people tend to consume more imported products when they have higher income.

⁴ Corsetti and Muller (2007) documents the business cycle of 10 OECD countries and finds that net export is positively correlated and weak to output for Canada while the other countries show another way during 1973-2005.

Table 6.1 C Business cycle properties of 7 OECD countries from 1986q1-2006q4; Co-movement

Country	Correlation with output							
	y	nx	c	i	g	x	m	n
United States	1.00	-0.47	0.89	0.94	-0.24	0.60	0.87	0.86
EU	1.00	-0.31	0.89	0.91	-0.08	0.85	0.93	0.15
Canada	1.00	0.13	0.86	0.67	-0.15	0.69	0.64	0.64
Australia	1.00	-0.74	0.70	0.83	0.20	-0.45	0.76	0.51
United Kingdom	1.00	-0.26	0.86	0.74	-0.33	0.47	0.68	0.86
Japan	1.00	-0.44	0.89	0.94	0.35	0.77	0.76	0.70
Korea	1.00	-0.89	0.96	0.94	0.24	0.15	0.92	0.92

Notes: Business Cycles taken from 1986Q1-2006Q4. Statistics are based on band pass filtered data. All variables are in real terms. Variables are y, output; c, consumption; i, investment; g, government spending; nx, ratio of net exports to output; x, exports; m, imports; n, civilian employment.

The correlations of net exports to output display considerable variation across countries. Government spending shows the tendency of moving counter-cyclical. One possible explanation is because higher spending leads to higher taxes, which depress private investment. This reduction in private investment is not offset by higher public investment, which appears to have a little impact on growth. The higher spending also undermines economic growth by transferring additional resources from the productive sector to government, which uses them less effectively. Another plausible reason is that when economic growth declines, a government increases its spending to boost growth by injecting purchasing power into the economy. Government expenditures are counter-cyclical in four countries and pro-cyclical in four countries, but the correlations are small for all cases.

Table 6.1 D Business cycle properties of 7 OECD countries from 1986q1-2006q4; International co-movement

Country	Cross correlation with the same US variables							
	y	nx	c	i	g	x	m	n
United States	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
EU	0.38	0.18	0.39	0.46	0.29	0.72	0.61	-0.13
Canada	0.78	-0.42	0.72	0.59	0.56	0.43	0.88	0.55
Australia	0.73	0.32	0.59	0.41	0.15	0.44	0.59	0.48
United Kingdom	0.70	0.45	0.76	0.61	0.36	0.7	0.59	0.60
Japan	-0.08	-0.37	-0.30	0.03	-0.10	0.64	0.02	0.02
Korea	-0.21	-0.38	-0.25	-0.28	0.33	0.59	0.08	-0.23

Notes: Business Cycles taken from 1986Q1-2006Q4. Statistics are based on band pass filtered data. All variables are in real terms. Variables are y, output; c, consumption; i, investment; g, government spending; nx, ratio of net exports to output; x, exports; m, imports; n, civilian employment.

Table 6.1 D displays statistics of an international aspect and documents the cross-correlations of foreign variables with the same US variables.

The international cross-correlation implies to what extent the economies are synchronized. The first column lists the correlation of output fluctuations between each country and the United States. These vary in magnitude for all countries. The cross correlations of output are positive in most cases. The average cross-correlation of output is positive, at 0.47. However, it is noticeable for Japan and Korea that the cross-correlations of output are negative, as well as the correlation of government spending between the USA and Japan. Although the output correlation of Japan and the USA is negative during 1986-2006, it is weak.

The consumption correlation of the USA and the UK, 0.76, is the highest of all sample countries. In general, it appears to be output cross-correlations are higher among neighbours and lower with remote countries. The output correlation of the USA and Canada is 0.78 which is the highest of all sample countries. With respect to consumption, the table finds that the correlations are smaller than those of output for almost all countries, except for the UK and EU. However, the consumption correlation is almost identical to the output correlation for the EU. The average cross-correlation of consumption is also lower than output, at 0.42. Again, it is striking that for Japan and Korea the cross-correlations of consumption are negative. The same facts also arise here, higher among neighbours, lower with remote countries. The average cross-correlation of investment is positive, at 0.40 and the cross-correlation of investment appears to be strongly correlated among neighbor countries. The correlations of net exports, employment and government spending are positive for most cases, except for Korea and Japan. These correlations, however, display significant variations across countries.

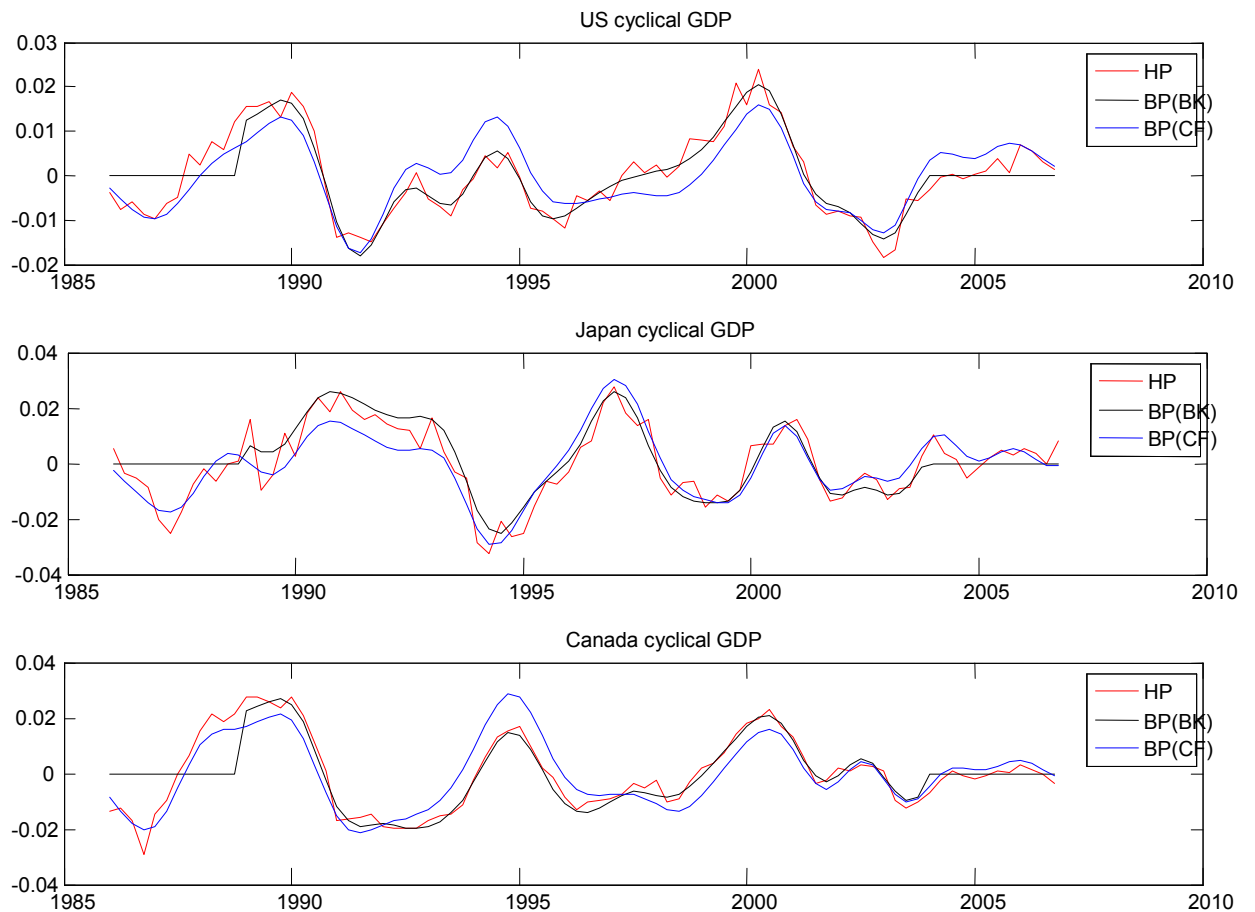
Figure 6.1 shows the cyclical component of real GDP derived by two band pass filters and the hp filter for the sample countries⁵. The figure has shown that the output cycle in the

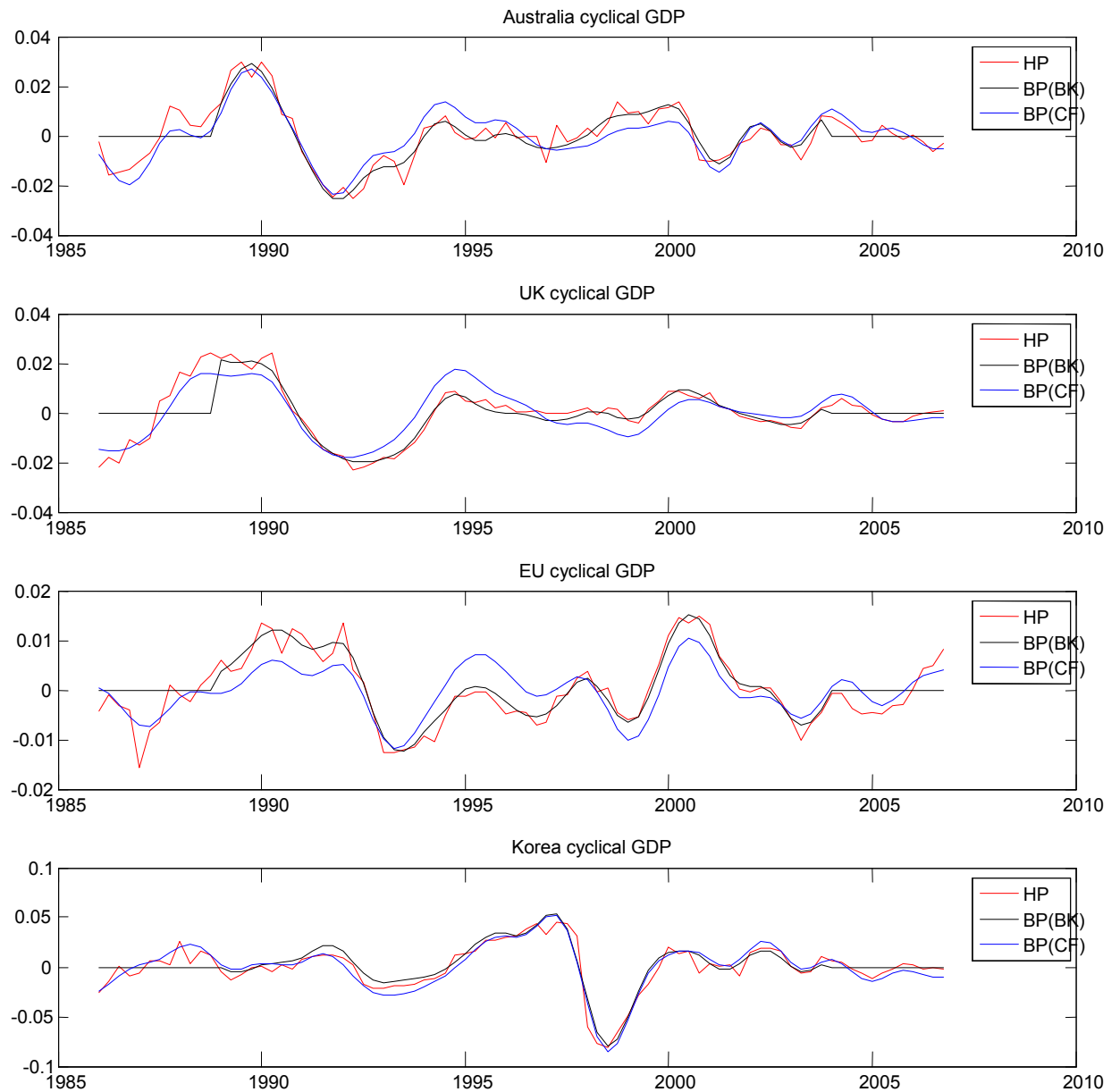
⁵ HP denotes the time series filtered the Hodrick and Prescott filter. BP(BK) stands for the time series filtered by the band pass filter proposed Baxter and King. BP(CF) stands for the time series filtered by the band pass filter suggested by Christiano and Fitzgerald. The y-axis represent the percent deviation from trend.

USA has been similar and synchronous to Canada, Australia, the UK and EU. During the Asian economic crisis of 1997, output in Korea drastically dropped. The same phenomenal can be observed in Japan but to a lower extent. Nevertheless, it is clear that the output of other countries appears not to be affected by the 1997 crisis.

Figure 6.1 The cyclical component of real output

Notes: The cyclical component of real output for the USA, Japan, Canada, Australia, UK, the EU and Korea are depicted. The data is from 1986q1-2006q4. The series are based on a filtered data. Three kinds of filters are used here, the Hodrick-Prescott filter(denoted by HP), the band pass filter by Baxter and King (denoted by BP(BK)) and the band pass filter by Christiano and Fitzgerald (denoted by BP(CF)).





6.3 International Business Cycles for the USA, Japan and Thailand

This section reports the main properties of business cycles in Thailand and compares them with other countries. These properties of the data serve as a basis of comparison with the model economy. The Thai data is from NESDB in the most cases.⁶ The USA and Japanese data are from the OECD's Quarterly National Accounts, except employment from the Main

⁶ NESDB stands for National Economic and Social Development Board of Thailand

Economics Indicators. The statistics are based on quarterly data covering 1993Q1-2006Q4 and have been detrended with a band pass filter. The USA and Japan have been chosen because they are the world's largest and second largest economies and more importantly they are the major trading partners for Thailand.⁷ Table 6.2 contains the results of the cyclical components of these three countries, focusing again on volatility, persistence, co-movement and international co-movement. Table 6.2 A shows that the variation of all variables for the USA and Japan are not considerably different.

Table 6.2 A The business cycle features of the USA, Japan and Thailand 1993Q1-2006Q4; Volatility

Country	Standard Deviation (%)		Ratio of Standard Deviation to that of y					
	y	nx	c	i	g	x	m	n
US	0.74	0.21	0.77	3.32	0.77	4.31	3.85	0.64
Japan	0.94	0.34	0.61	2.32	0.76	3.67	4.12	0.41
Thailand	3.10	3.32	1.12	3.34	0.89	1.05	2.58	0.25

Notes: Business Cycles in the USA, Japan and Thailand from 1993Q1-2006Q4. Statistics are based on band pass filtered data. All variables are expressed in real terms. Variables are y, output; c, consumption; i, investment; g, government spending; nx, ratio of net exports to output; x, export; m, import; n, civilian employment.⁸

Table 6.2 B The business cycle features of the USA, Japan and Thailand 1993Q1-2006Q4; Persistence

Country	The first order autocorrelation							
	y	nx	c	i	g	x	m	n
US	0.94	0.92	0.95	0.95	0.9	0.91	0.90	0.91
Japan	0.91	0.82	0.90	0.91	0.88	0.86	0.89	0.91
Thailand	0.91	0.91	0.92	0.94	0.87	0.82	0.89	0.14

Notes: Business Cycles in the USA, Japan and Thailand from 1993Q1-2006Q4. Statistics are based on band pass filtered data. All variables are expressed in real terms. Variables are y, output; c, consumption; i, investment; g, government spending; nx, ratio of net exports to output; x, export; m, import; n, civilian employment.

On the other hand, the volatilities of all Thai variables are larger than those for the USA and Japan. With respect to output as reported, the volatility of consumption is less than that of output for US and Japan. Nevertheless, consumption in Thailand is more volatile than

⁷ According to Department of Trade and Negotiation, Ministry of Commerce, the USA and Japan are the main export and import destinations for Thailand. Thailand exports goods and services to the USA and Japan roughly 19.5 and 16.4 billions US dollars respectively in 2006.

⁸ Since the employment data is only available for Thailand in quarterly from 2001Q1-2006Q4, the moment calculation for the employment in Thailand in Table 2 uses data from World Bank World Development Indicators annually from 1993-2006 and it is filtered by band pass filter with the truncation point is three. The details can be seen in chapter 3 section 3.2.6. Table 2.1 and 2.2 the Thai employment data is from 1993-2006 annually. In Thailand during 1993-2006 with annual data, the volatility of output is 3.10 and the volatility of employment is 0.79.

output. The output and net exports in Thailand fluctuates about three times more than the output in the USA and Japan during the sample period. The emerging economies are generally more volatile than the developed countries and the variation of consumption is slightly larger than that of output.⁹ The volatility of investment is about three times as high as that of output in all three countries. The government spending in the USA and Japan exhibits the similar variation relative to output at 0.77 and 0.76 respectively. The volatility of government spending relative to output is higher in the case of Thailand at 0.89. The volatility of employment in Thailand is noticeably lower than that of output during the sample period.

Similar to Table 6.1 B, for all three countries, persistence is high for almost all variables as shown in table 6.2 B. However, it is surprising that the persistence of employment in Thailand is so low. Table 6.2 C shows that net exports move counter-cyclicality for all three countries, but strikingly strongly counter-cyclical in the case of Thailand. The interpretation is that people in these three countries consume more imported products when their incomes rise. Consumption, investment and employment are all pro-cyclical in all cases.

Table 6.2 C The business cycle features of the USA, Japan and Thailand 1993Q1-2006Q4; Co-movement¹⁰

Country	Correlation with output							
	y	nx	c	i	g	x	m	n
US	1.00	-0.40	0.82	0.96	-0.41	0.78	0.92	0.85
Japan	1.00	-0.17	0.68	0.94	0.31	0.71	0.88	0.68
Thailand	1.00	-0.94	0.99	0.94	0.46	-0.09	0.85	0.83

Notes: Business Cycles in the USA, Japan and Thailand from 1993Q1-2006Q4. Statistics are based on band pass filtered data. All variables are expressed in real terms. Variables are y, output; c, consumption; i, investment; g, government spending; nx, ratio of net exports to output; x, export; m, import; n, civilian employment.

⁹ Kim, Kose and Plummer (2003) suggest the stylized facts on business cycle in Asian countries during 1960-1996. The average of output fluctuation is 35% roughly higher than the G-7 countries. Mendoza (1995) reports that the volatility of output is two to four times larger than that of developed countries. Neumeryer and Perri (2005) report the similar stylized facts. The numbers of business cycle literatures conducting for developed countries document that consumption less fluctuates than output. See for example Backus, Kehoe and Kydland (1992, 1994), King (1993), Watson (1999), Baxter (1995).

¹⁰ To calculate the correlation of Thai employment with Thai output, the data of output and employment in Thailand are 1993-2006 annually.

Investment has been shown to be more strongly pro-cyclical than consumption and employment in the USA and Japan. Nonetheless, as in the table, consumption in Thailand is more strongly correlated with output than investment. Thai people appear to increase or decrease their consumption by almost the same proportion of an increase or decrease in output. This suggests there is less smoothing of consumption over the cycle than in the other countries. Government spending has been counter-cyclical in the USA, but pro-cyclical in Japan and Thailand. Exports and imports are pro-cyclical for all cases, except for Thailand's exports. However, the degree of negative correlation of Thai exports with output is quite weak, at -0.09.

Table 6.2 D summarizes the cross correlation properties with respect to the US variables. Output, net export, consumption, investment and government spending in Thailand are negatively cross correlated with the same US variables. Output correlation is positive for Japan; however, it is a negative correlation for Thailand.

Table 6.2 D The business cycle features of the USA, Japan and Thailand 1993Q1-2006Q4; International co-movement¹¹

Country	Cross correlation with the same US variables							
	y	nx	c	i	g	x	m	n
United State	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Japan	0.13	-0.38	-0.46	0.27	-0.05	0.81	0.15	0.58
Thailand	-0.25	-0.51	-0.45	-0.33	-0.55	0.38	0.11	-0.50

Notes: Business Cycles in the USA, Japan and Thailand from 1993Q1-2006Q4. Statistics are based on band pass filtered data. All variables are expressed in real terms. Variables are y, output; c, consumption; i, investment; g, government spending; nx, ratio of net exports to output; x, export; m, import; n, civilian employment.

The correlation of net exports, consumption and government spending are negative for Thailand and Japan. The correlation of investment is negative for Thailand, but it is positive for Japan. Government spending is negatively correlated for every country, but these vary in magnitude. The correlation of employment is negative for Thailand. We can therefore definitely say that the Thai business cycle negatively fluctuated with the US business cycle

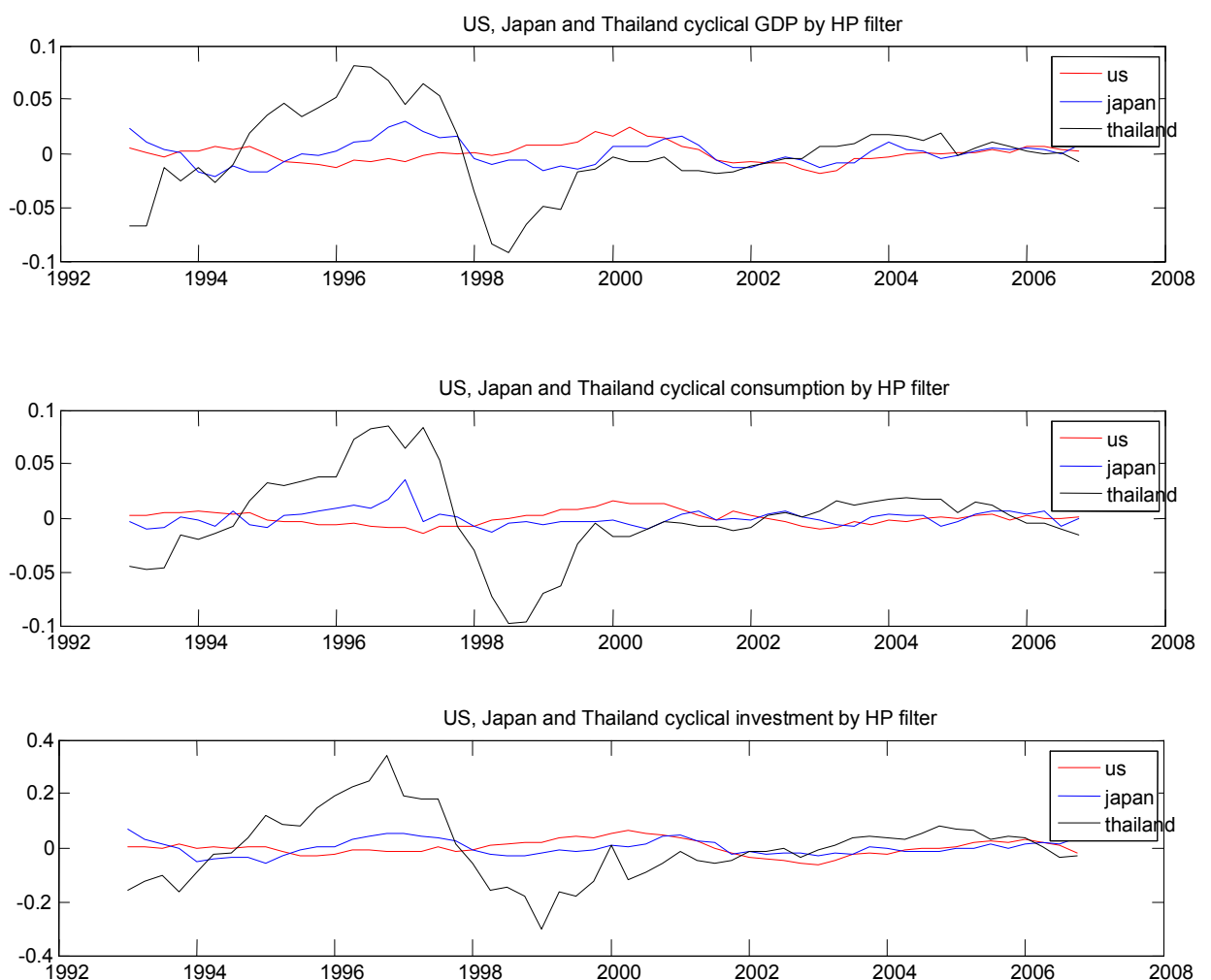
¹¹To calculate the international co-movement of the USA and Thai employment, the data of employment for both countries are 1993-2006 annually.

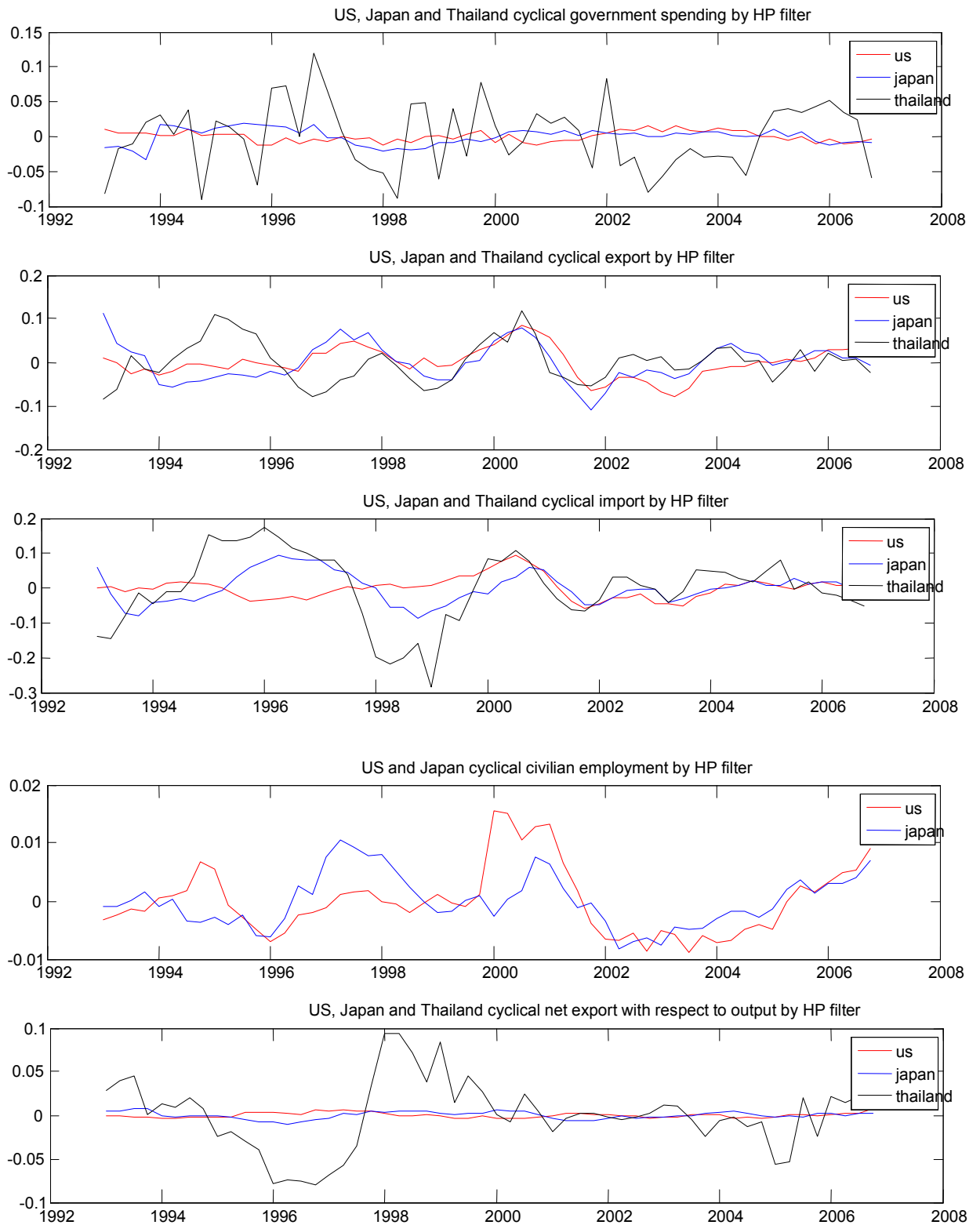
during 1993-2006. However, output and investment in Japan and the USA move in the same direction.

Figure 6.2 exhibits the cyclical component of several variables of the USA, Japan and Thailand by the HP filter for the sample period with $\lambda=1600$ for quarterly data. Comparing volatilities across these three countries reveals that the volatility of Thai variables are larger than those in the USA and Japan. The picture does not plot the employment series of Thailand because the entire series for the sample period is not available.

Figure 6.2 The cyclical component of main macro variables for US, Japan and Thailand 1993q1-2006q4

Notes: The cyclical component of real output, consumption, investment, government spending, exports, imports and employment are depicted for the USA, Japan and Thailand. The data is from 1993q1-2006q4. All series are based on the Hodrick-Prescott filter.





We can see clearly that investment in Thailand is the most volatile series. Thai variables fluctuated even more during the Asian crisis 1997 compared to the entire sample period. In Thailand, consumption, investment, government spending all were positively correlated with output during the crisis, but not, net exports. The reason for the behaviour of net exports is that the Thai Baht was devalued considerably during the crisis.

The important aspect of this chapter is the statistics that measure co-movement across countries. According to table 6.2 D, consumption correlation across countries is lower than output cross correlation. The most crucial point is that the correlations of output across countries are larger than those of consumption. It is essential to see whether and how these salient properties can be captured by the model economy.

6.4 The Basic IRBC Model

This section describes the model economy where the empirical features calculated in the previous section can be explained as the equilibrium of an economy driven by the shocks to total factor productivity and government spending. The two-country version of the standard one-sector stochastic growth model is developed in this study. The model along the lines of Baxter and Crucini (1995) is developed and is used to explain the Thai business cycle. However, each country has its own set of preferences and technology. In this model, we assume the representative agent in each of the two countries produces a single homogenous good; this good is used both for consumption and for investment. The agents provide their capital and labour. They purchase the goods produced by the firms to be consumed and invested. In this model economy, international trade takes place both to smooth consumption and to equalize the cross-country returns to capital. The market structure is complete within countries and capital is allowed to be mobile across countries. However, labour is immobile internationally. All variables are expressed in per capita terms.

6.4.1 Households

In this economy, each country is populated by a large number of identical households and is represented by a single agent. Foreign country variables are denoted by stars or asterisk(*).

Households in each country maximize their preferences which are characterized by an expected utility function given by:

$$E \sum_{t=0}^{\infty} \beta^t U(C_t, L_t); \quad \text{home country} \quad (6.1)$$

$$E \sum_{t=0}^{\infty} \beta^{*t} U(C_t^*, L_t^*); \quad \text{foreign country} \quad (6.2)$$

where C_t and L_t are consumption and leisure at time t and β is discount factor. Consumers, in each country, choose leisure L_t and consumption, C_t , to maximize their utilities. The period utility function, U , can be transformed to a constant-relative-risk-aversion form.

$$U(C_t, L_t) = (1 - \sigma)^{-1} (C_t^\theta L_t^{1-\theta})^{1-\sigma} \quad (6.3)$$

where σ stands for the relative risk aversion or the inverse of the intertemporal elasticity of substitution. The higher the value of σ , the less the intertemporal elasticity substitution of consumption and leisure by representative agents. θ is the consumption share parameter where $\theta \in (0,1)$. $1 - \theta$ is the labour share parameter in the utility function. Households face the constraint that time devoted to work plus leisure cannot exceed the time endowment. For simplicity, the time endowment is normalized to one unit in each country.

$$1 - L_t - N_t \geq 0; \quad \text{home country} \quad (6.4)$$

$$1 - L_t^* - N_t^* \geq 0; \quad \text{foreign country} \quad (6.5)$$

6.4.2 Technology

Output in each country is produced via constant-returns to scale production functions. The production takes place in each country using capital, K_t and domestic labour, N_t and is subject to exogenous shocks to total factor productivity, A_t :

$$Y_t = F_t(K_t, N_t) = A_t K_t^{1-\alpha} (X_t N_t)^\alpha; \quad \text{home country} \quad (6.6)$$

$$Y_t^* = F_t(K_t^*, N_t^*) = A_t^* (K_t^*)^{1-\alpha^*} (X_t^* N_t^*)^{\alpha^*}; \quad \text{foreign country} \quad (6.7)$$

where the variable X_t represents the labour-augmenting technical change, and is assumed to grow at a common, constant (gross) rate γ_x . The law of motion of capital is governed over time by:

$$K_{t+1} = (1 - \delta)K_t + \phi(I_t / K_t)K_t; \quad \text{home country} \quad (6.8)$$

$$K_{t+1}^* = (1 - \delta^*)K_t^* + \phi(I_t^* / K_t^*)K_t^*; \quad \text{foreign country} \quad (6.9)$$

where δ is the depreciation rate of capital. Unlike labour, capital is internationally mobile. Capital formation incorporates capital adjustment costs denoted by the function ϕ where $\phi > 0$, $\phi' > 0$ and $\phi'' < 0$. The parameter ϕ is a function of the ratio of investment and capital. It is set in such a way that the model with and without ϕ yields the same steady state. ϕ is not required to be directly specified. The steady state value of Tobin's q and the share of investment in output are required in order to calculate the value of ϕ .¹² Capital adjustment costs work as a friction of capital mobility between two countries. Their purpose is to slow down the response of investment to location-specific shocks.

6.4.3 Government

The government of each country taxes national output at the rate, τ_t , yielding tax revenue of $\tau_t Y_t$. At each point in time the government consumes domestic goods in the amount, G_t and the government consumption is financed with lump-sum taxation. The government also gives some of the goods back to households in the form of a transfer payment, T_t . The government spending in this model economy does not enter the utility and production function. The government budget constraints for the two countries are given by:

$$G_t + T_t = \tau_t Y_t; \quad \text{home country} \quad (6.10)$$

$$G_t^* + T_t^* = \tau_t^* Y_t^*; \quad \text{foreign country} \quad (6.11)$$

¹² The parameter will be discussed again when calibrating the model.

6.4.4 Market structure

The structure of the markets available to individuals of each country must be specified to be a channel in which two countries are linked.¹³ The structure of markets is important for international business cycles, since an important function of financial markets is to permit individuals to smooth consumption in response to fluctuations in income. The following chapter investigates the simplest market structure, the complete market structure, and applies it to Thailand and its partners. Most IRBC models assume that asset markets are complete, including the original version by BKK (1992). Under complete markets, consumption and investment goods are internationally mobile and there is frictionless trade in output, the world resource constraint for this good is given by:

$$\pi(Y_t - C_t - I_t - G_t) + (1 - \pi)(Y_t^* - C_t^* - I_t^* - G_t^*) \geq 0 \quad (6.12)$$

where π denotes the fraction of the world population that lives in the home country. Baxter and Crucini (1995) suggest that, in the complete contingent claim market, individuals in both countries can trade a full set of contingent claims. Under this particular assumption of market structures, individuals attain the optimal degree of consumption smoothing and pool all idiosyncratic risks.¹⁴ By consumption smoothing, we mean the simple transfer to of a unit of the consumption good from one date to another. By risk-pooling, we mean the transfer of consumption from one state of nature to another. Therefore, individuals equate their marginal instantaneous utilities of consumption across countries and that for all states of the world. A positive technology shock in one country results in an increase in consumption in both countries. Individuals end up smoothing their life time consumption by fully diversifying risk. The cross country consumption correlations are expected to be high

¹³ Two possible market structures, one complete market and another incomplete market, can be studied. The difference between complete and incomplete market lies in the numbers of assets available to the agents in each country. Unlike complete market, the agents in incomplete market restrict their trade only one period risk free bonds, not all type of assets. Agents can engage in consumption smoothing but not risk sharing. Baxter and Crucini (1995) and Kollmann (1996) are among the first literatures to consider the incomplete market.

¹⁴ Canova and Ravn (1996) examines the implications of international consumption risk sharing for a panel of industrialized countries. They conclude that aggregate domestic consumption is almost completely insured against idiosyncratic real, demographic, fiscal, and monetary shocks over short cycles. Crucini (1999) investigates the implications of risk sharing across regions or countries. He finds similar degrees of risk sharing within regions of Canada and the USA that exceeds the risk sharing that occurs across countries. Kose, Prasad and Terrones (2007) evaluate the patterns of risk sharing among different group of countries and how the international financial integration effect these patterns. They find that there is a modest degree of risk sharing between countries. However, it is not near the degree predicted by theory.

theoretically. However, the cross country output correlations are weak, even closed to zero or negative value. The explanation is that individuals are able to shift their investment to the location specific shock where a higher return is offered. Therefore, under complete market structure, the cross country consumption correlation is clearly larger than the cross country output correlation.

6.4.5 The Forcing Process specification

Three driving processes are analyzed in this chapter. The model economy experiences the unanticipated variation to total factor productivity and to fiscal shocks, government expenditure and taxation. The measurement of the exogeneous shocks to total factor productivity with a Cobb-Douglas production function is the so called “Solow residual” or that output growth which cannot be explained by the growth of capital and labour. The exogeneous shocks to government spending are directly measured because the series need not be derived as the residuals. These are defined as the residuals from the production functions and are given by:

$$\log A_t = \log Y_t - (1 - \alpha) \log K_t - \alpha \log N_t; \quad \text{home country} \quad (6.13)$$

$$\log A_t^* = \log Y_t^* - (1 - \alpha^*) \log K_t^* - \alpha \log N_t^*; \quad \text{foreign country} \quad (6.14)$$

It is typically the case that the following is an adequate representation of the bivariate autoregressive stochastic process for productivity.

$$\begin{bmatrix} \log A_t \\ \log A_t^* \end{bmatrix} = \begin{bmatrix} \rho_A \nu_{A^*}^* \\ \nu_A \rho_{A^*}^* \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log A_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{A,t} \\ \varepsilon_{A^*,t}^* \end{bmatrix} \quad (6.15)$$

Under this specification, innovations to productivity which originate in one country (ε_t or ε_t^*) are transmitted to the other country via the “diffusion” parameter, ν . The “persistence” parameter, ρ , is important for the serial correlation of the technology variable within a country. The matrix of technology innovation is the normal distribution with $E(\varepsilon_t) = E(\varepsilon_t^*) = 0$ and given by:

$$\begin{bmatrix} \varepsilon_{A,t} \\ \varepsilon_{A,t}^* \end{bmatrix} : N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \psi \\ \psi & \sigma_\varepsilon^{*2} \end{bmatrix} \right) \quad (6.16)$$

where ψ is the covariance for the innovations to productivity between two countries.

The variance-covariance matrix for the innovations to the productivity process is given by

$$E(\varepsilon_t, \varepsilon_t^*)(\varepsilon_t, \varepsilon_t^*)' = \begin{bmatrix} \sigma_\varepsilon^2 & \psi \\ \psi & \sigma_\varepsilon^{*2} \end{bmatrix} \quad (6.17)$$

The exogenous shock process of government spending are also assumed to follow an AR(1) process. Unlike the shock processes of technology, the shock processes of government expenditure are straight forward to measure. The measure of government spending used in this model is government consumption as recorded in the national income and product account. The series need not to be calculated as a Solow residual. The driving processes of government spending are shown as below

$$\begin{bmatrix} \log G_t \\ \log G_t^* \end{bmatrix} = \begin{bmatrix} \rho_G & \nu_{G^*} \\ \nu_G & \rho_{G^*} \end{bmatrix} \begin{bmatrix} \log G_{t-1} \\ \log G_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{G,t} \\ \varepsilon_{G,t}^* \end{bmatrix} \quad (6.18)$$

Via a similar argument, the shock to government spending which originate in one country (ε_t or ε_t^*) are transmitted to the other country via the “diffusion” parameter, ν . The “persistence” parameter, ρ , represents for the serial correlation of the government spending within a country. The shock process in taxation is similar to that in government spending.

6.5 The Equilibrium and Solution Algorithm

This section summarizes the solution concept behind the real business cycle model. In brief, the households in each country maximize expected life time utility according to (6.1) and (6.2). Outputs in each country are given by the Cobb-Douglas production (6.6) and (6.7). The capital stock accumulations follow the equation (6.8) and (6.9). The shock

processes evolve according to (6.15) and (6.18) where the innovations of technology have the variance-covariance matrix according to (6.17). The market is assumed to be complete; therefore, it satisfies the world resource constraint (6.12). Even though there are several techniques for solving this non-linear system of equations, the real business cycle models are commonly approximated using the solution method of King, Plosser and Rebelo (1988).¹⁵ The model used in this chapter is also solved by the KPR method where the first-order conditions are approximated log-linear around the steady state and this linear system of equations are solved by the standard linear systems procedures. The optimization problems are below:

$$\begin{aligned}
\text{Max } & E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \pi u(c_t, L_t) + E_0 \sum_{t=0}^{\infty} \tilde{\beta}^{*t} (1-\pi) u(c_t^*, L_t^*) \\
\text{Subject to } & y_t = A_t F(k_t, N_t) \\
& \gamma k_{t+1} = (1-\delta)k_t + \phi\left(\frac{i_t}{k_t}\right)k_t \\
& 1 - L_t - N_t \geq 0 \\
& g_t + T = \tau_t y_t \\
& \pi[y_t - c_t - i_t - g_t] + (1-\pi)[y_t^* - c_t^* - i_t^* - g_t^*] \geq 0
\end{aligned}$$

Foreign country equations are the same and indexed by star (*).

Therefore, the Lagrangian of the maximization problem can be expressed in terms of the stationary transformations as

$$\begin{aligned}
\text{Max } L = & E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left\{ \pi u(c_t, L_t) + (1-\pi) u(c_t^*, L_t^*) \right\} \\
& + E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left\{ \pi \lambda_t [(1-\delta)k_t + \phi\left(\frac{i_t}{k_t}\right)k_t - \gamma k_{t+1}] + (1-\pi) \lambda_t^* [(1-\delta)k_{t+1}^* + \phi\left(\frac{i_t^*}{k_t^*}\right)k_t^* - \gamma^* k_{t+1}^*] \right\} \\
& + E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left\{ \pi w_t (1 - L_t - N_t) + (1-\pi) w_t^* (1 - L_t^* - N_t^*) \right\}
\end{aligned}$$

¹⁵ See also Taylor and Uhlig (1990), Judd (1992), Sim (2002) and Christiano (2002). Bacuk, Kehoe and Kydland (1992) approximate the problem with one that has a quadratic objective function and linear constraints. The solution method by KPR incorporates with the discounted sum approach from Hansen & Sargent (1980, 1981).

$$+E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left\{ p_t [\pi(A_t F(k_t, N_t) - c_t - i_t - g_t) + (1-\pi)(A_t^* F(k_t^*, N_t^*) - c_t^* - i_t^* - g_t^*)] \right\}$$

First order necessary conditions:

$$c_t : D_1 u(c_t, L_t) = p_t$$

$$L_t : D_2 u(c_t, L_t) = w_t$$

$$N_t : p_t A_t \Omega_t D_2 F(k_t, N_t) = w_t, \text{ where } \Omega_t = (1 - \tau_t)$$

$$i_t : \lambda_t D\phi\left(\frac{i}{k}\right) = p_t$$

$$k_{t+1} : \tilde{\beta} E_t \lambda_{t+1} \mu\left(\frac{i_{t+1}}{k_{t+1}}\right) + \tilde{\beta} E_t p_{t+1} \Omega_{t+1} A_{t+1} D_1 F(k_{t+1}, N_{t+1}) = \gamma \lambda_t$$

$$w_t : L_t + N_t = 1$$

$$\lambda_t : (1 - \delta)k_t + \phi\left(\frac{i_t}{k_t}\right)k_t = \gamma k_{t+1}$$

$$p_t : \pi(A_t F(k_t, N_t) - c_t - i_t - g_t) + (1 - \pi)(A_t^* F(k_t^*, N_t^*) - c_t^* - i_t^* - g_t^*)$$

$$\text{where } \mu\left(\frac{i_{t+1}}{k_{t+1}}\right) = (1 - \delta) - D\phi\left(\frac{i_{t+1}}{k_{t+1}}\right) \frac{i_{t+1}}{k_{t+1}} + \phi\left(\frac{i_{t+1}}{k_{t+1}}\right)$$

$$\text{TVC: } \lim_{t \rightarrow \infty} \tilde{\beta}^t \lambda_t k_{t+1} = 0$$

Foreign country variables are the same and indexed by star.

The log-linearized equations around the steady state are:

$$c_t : \xi_{cc} \hat{c}_t + \xi_{cL} \hat{L}_t = \hat{p}_t$$

$$L_t : \xi_{Lc} \hat{c}_t + \xi_{LL} \hat{L}_t = \hat{w}_t$$

$$N_t : \xi_{Nk} \hat{k}_t + \xi_{NN} \hat{N}_t + \Omega_t + A_t + \hat{p}_t = \hat{w}_t$$

$$i_t : \zeta \hat{i}_t - \zeta \hat{k}_t + \hat{\lambda}_t = \hat{p}_t$$

$$k_{t+1} : \gamma \hat{\lambda}_t = \tilde{\beta} D\mu\left(\frac{i}{k}\right)\left(\frac{i}{k}\right) \hat{i}_{t+1} - \tilde{\beta} D\mu\left(\frac{i}{k}\right) \hat{k}_{t+1} + \tilde{\beta} \mu\left(\frac{i}{k}\right) \hat{\lambda}_{t+1} - (\tilde{\beta} \mu\left(\frac{i}{k}\right) - \gamma) \hat{q}_{t+1}$$

$$w_t : (1 - N) \hat{L}_t = N \hat{N}_t$$

$$\lambda_t : \gamma \hat{k}_{t+1} - \mu\left(\frac{i}{k}\right) \hat{k}_t = D\phi\left(\frac{i}{k}\right) \frac{i}{k} \hat{i}_t$$

$$p_t : \pi s_k \hat{k}_t + \pi s_N \hat{N}_t + \pi \hat{A}_t - \pi s_c \hat{c}_t - \pi s_i \hat{i}_t - \pi s_g \hat{g}_t \\ + (1 - \pi) s_k^* \hat{k}_t^* + (1 - \pi) s_N^* \hat{N}_t^* + (1 - \pi) \hat{A}_t^* - (1 - \pi) s_c^* \hat{c}_t^* - (1 - \pi) s_i^* \hat{i}_t^* - (1 - \pi) s_g^* \hat{g}_t^*$$

$$\begin{aligned}
\text{where } \xi_{cc} &= c \frac{D_{11}u(c, L)}{D_1u(c, L)}, \xi_{cL} = L \frac{D_{12}u(c, L)}{D_1u(c, L)}, \xi_{Lc} = c \frac{D_{21}u(c, L)}{D_2u(c, L)}, \xi_{LL} = L \frac{D_{22}u(c, L)}{D_2u(c, L)} \\
\xi_{kk} &= k \frac{D_{11}F(k, N)}{D_1F(k, N)}, \xi_{kN} = N \frac{D_{12}F(k, N)}{D_1F(k, N)}, \xi_{Nk} = k \frac{D_{21}F(k, N)}{D_2F(k, N)}, \xi_{NN} = N \frac{D_{22}F(k, N)}{D_2F(k, N)} \\
\zeta &= (D^2\phi(\frac{i}{k})\frac{i}{k}) / D\phi(\frac{i}{k}), \hat{q}_{t+1} = \xi_{kk}\hat{k}_{t+1} + \xi_{kN}\hat{N}_{t+1} + \Omega_{t+1} + \hat{A}_{t+1} + \hat{p}_{t+1}
\end{aligned}$$

For the elasticity of marginal utility, ξ_{xy} is the elasticity of marginal utility of x with respect to y . So for example, ξ_{cc} is the elasticity of marginal utility of consumption with respect to consumption. ξ_{cL} is the marginal utility of consumption with respect to leisure. Similarly, ξ_{LL} is the elasticity of marginal utility of labour with respect to labour and that with respect to consumption is ξ_{Lc} . For elasticity of marginal product, ξ_{kk} denotes the elasticity of marginal product of capital with respect to capital and that with respect to labour is ξ_{kN} . In the same way, denote ξ_{Nk} and ξ_{NN} as the elasticity of marginal product of labour with respect to capital and labour respectively.

For this application with complete contingent claims markets denoted by equation (6.12), the marginal utility of consumption is equated across countries; therefore, country specific risk is fully shared. It is possible to find the equilibrium allocations by solving a planning problem. The equilibrium is computed by maximizing the sum of each country's utility subject to technology and the world resource constraint. The numerical techniques are used to solve for the dynamic equilibrium of the model. Specifically, the model is solved by taking a log-linear approximation to the initial steady state. The detailed descriptions of the solution algorithm can be seen in chapter 4. Under most circumstances, the resulting linear dynamic system moves around the balanced growth path which is described by a linear law of motion. Let the linearized equations of the shock process of (6.15) be given by:

$$\begin{bmatrix} \hat{A}_t \\ \hat{A}_t^* \end{bmatrix} = \begin{bmatrix} \rho & \nu^* \\ \nu & \rho^* \end{bmatrix} \begin{bmatrix} \hat{A}_{t-1} \\ \hat{A}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix} \quad (6.19)$$

where the hatted variables represent percent deviations from the steady state. For the model to display stable local dynamic properties, the eigenvalues of the coefficient matrix of (6.19) must be less than one. In other words, the impact of the technology shock on technology (A) must eventually fade away, with A, A^* returning to their equilibrium values of zero. Ultimately, the equilibrium policy functions of the economy contain a system of linear functions explaining the property of endogenous variables for output, consumption and investment as functions of exogenous variables.

6.6 The Model Calibration

The detailed descriptions of the parameter values used in this model are reported in this section. The calibration procedure typically uses the proper empirical data as part of the characteristics of the theory. Some of the parameter values selected rely on observed properties of real economies, but the determination of others rely heavily on the theory. It is very crucial not to choose the parameters by referring to the previous studies. This section reports the calibrated parameters for Thailand and the USA and Japan so that they are consistent with the long term behaviors of each actual economy. The model parameters are generally determined by the model's first order necessary conditions along with the data set for each country.

By using these calibrated figures, we are able to study the quantitative behaviour of the business cycle in Thailand and the USA and Japan. The steady state growth rate of the economy (γ) is set to match the average quarterly growth rate of real output per capita for each country. These are 0.80%, 0.52% and 0.24% for Thailand, the USA and Japan respectively during 1993Q1-2006Q4. The average quarterly real interest rate is calculated using the quarterly real money market rate at 1.17% for Thailand and 0.99% for Japan. The US 3 month T-Bill rate deflated with CPI deflation at 0.96% represents the US steady state real interest rate. The steady state ratio of government spending to output, chosen to coincide the mean of that ratio over the observation period, is 8.70% in Thailand, and 15.30% in the USA and 16.45% in Japan. Tax revenue as a fraction of output is set to the average of that fraction during the sample period at 16.00% for Thailand, 27.9% for the USA

and 26.45% for Japan. The ratio of tax revenue to output in developing countries is generally lower than that of developed countries.

6.6.1 Preferences

Calibrated parameters 1993Q1-2006Q4

<i>Preferences</i>	Thailand	US	Japan
Steady state real interest rate (r_{ss})	0.01170	0.00960	0.00990
Steady state discount factor ¹⁶ (β)	0.99635	0.99533	0.99260
Relative risk aversion (σ)	2.00000	2.00000	2.00000
Consumption share ¹⁷ (θ)	0.16449	0.12331	0.16106

The steady state real interest rate for each country implies the value of discount factor (β). Another preference parameter (σ) which determines the relative risk aversion of the curvature of utility function and the degree of intertemporal substitution is the most difficult to calibrate. For each country, it is set to 2 following Backus, Kehoe and Kydland (1995) which is commonly used in many literatures for the multiplicative instantaneous utility function.¹⁸ Finally, the steady state consumption share parameter (θ) is the long run representative agent's consumption share in utility function. The details of (θ) can be found section 6.4.

6.6.2 Technology

Calibrated parameters 1993Q1-2006Q4

<i>Technology parameters</i>	Thailand	US	Japan
Growth rate of real output per capita (γ)	0.00799	0.00520	0.00244
Capital depreciation rate (δ)	0.01079	0.01500	0.02000
Capital share ¹⁹ ($1-\alpha$)	0.33686	0.25331	0.34849

¹⁶ The steady state discount factor is calculated from $r_{ss} = (\gamma/\beta) - 1$ where r_{ss} is the steady state real interest rate.

¹⁷ Given the first order necessary conditions, consumption share parameter is computed follow $\theta = (c/y)/((\alpha(1-N)/N) + c/y)$. The steady state consumption share of output equals to the average of consumption ratio to output during the sample period. The c/y is 0.674 for Thailand, 0.690 for the USA and 0.580 for Japan. N represents the time devoted to work at steady state and its value is shown under technology calibration. The c/y ratio equals $1 - (i/y) - (g/y)$ where g/y is exogenous variable and $i/y = [(1-\alpha)(\gamma - 1 + \delta)\beta^*]/[\gamma - \beta^*(1-\delta)]$.

¹⁸ Ortega, Estudios, Espana (1998), Ravn (1997), Baxter and Crucini (1993, 1995) set the curvature parameter to 2.

¹⁹ Capital share parameter is derived from $AD_1 F(k, N) = r_{ss} + \delta$ where r_{ss} is the steady state real interest rate.

Labour share (α)	0.66314	0.74667	0.65151
Steady state working hours ²⁰ (\bar{N})	0.19309	0.16939	0.18132
The elasticity of Tobin's q with respect to the investment capital ratio ($1/\eta$)	0.06670	0.06670	0.06670

The use of estimation technique to estimate the capital stock for the USA and Japan is due to the lack of empirical data for capital in this country. The perpetual inventory method (PIM), which argues that the stock of capital is the accumulation of the stream of past investments, is used in this case²¹.

The capital is accumulated according to:

$$K_t = I_t + (1 - \delta)K_{t-1} \quad (6.20)$$

Using the concept of initial capital stock, K_0 , the construction of the capital stock series follows Nehru and Dhareshwar (1993):

$$K_t = (1 - \delta)^t K_0 + \sum_{i=0}^{t-1} I_{t-i} (1 - \delta)^i \quad (6.21)$$

where, δ is the depreciation rate of capital and K_0 is the capital stock at time zero. Nehru and Dhareshwar (1993) used alternative techniques to estimate the initial capital stock and found the Harberger-approach to be the most effective approach. The value of initial capital stock according to Harberger (1978) used to calculate the series of capital stock is as follow:

$$K_{t-1} = \frac{I_t}{(\gamma + \delta)} \quad (6.22)$$

²⁰ Steady state working hours is the average hours worked in a week (40 hours a week) times fraction of population that works divided by total time available a week (16 hours time 7 days). The fraction of population that works is 0.54 for Thailand, 0.47 for the USA and 0.51 for Japan.

²¹ Abadir and Talmain (2001) provide a data-driven method of estimating the series of depreciation and capital stock. The procedure is first to decide the starting point by minimizing the gross and net investment ratio.

where, γ is the rate of growth of output.²² The other important estimate needed is the depreciation rate (δ). Many studies, such as Nehru and Dhareshwar (1993) and Collins and Bosworth (2003) have chosen 4 percent per year. For the USA in the post war period, Stokey and Rebelo (1995) takes values on the order of 6 percent per year. King and Rebelo (1994), Romer (2001), Backus et al. (1994, 1995) use a conventional value for the depreciation which is 10 percent per year. However, there is evidence that this value is quite high. For the USA in the post war period, Stokey and Rebelo (1995) take values on the order of 6 % per year. The experiment in this chapter adopts the value from Stokey and Rebelo (1995) for the USA, yielding the US capital-output ratio of 2.03.²³ Following the depreciation rate of 8% a year by Laxton, N'Diaye and Pesenti (2006), the capital-output ratio is set to 3.16 in Japan. The capital-output ratio in Thailand can be derived from the provided capital series by NESDB, giving 3.746 during the sample period.²⁴

The capital share parameter ($1 - \alpha$) is set to be 34% for Thailand and 35% for Japan. For the USA during the sample period, this value is 25% which is significantly smaller than the standard value of 36% as in Backus et al. (1992).²⁵ Part of the explanation is that the investment-output ratio was low in the USA during the 1993-2006. Given the capital share parameter, it is straight forward to estimate the labour share parameter as one minus capital share. The steady state of individual time devoted to work for the three countries are quite similar because the fractions of labour force to population are not significantly very different. The adjustment cost parameters $\phi(i/k)$ are set so that the steady state of the model is the same as the one without adjustment cost. This implies its steady state value equals the steady state ratio of investment to capital and the steady state value of (i/k) is

²² Harberger (1978) estimated equation (6.22) based on the assumption of a constant capital-output ratio. He used three-year averages of investment to estimate the initial stock of capital. Nehru and Dhareshwar (1993) modified the Harberger approach. They estimated the initial level of investment using a regression of the log of investment on time. This chapter uses the investment at 1993 to estimate the capital stock at time zero.

²³ The capital-output ratio in the USA is lower than those of in Japan and Thailand because the investment-output ratio is lower in the USA during the sample period. The i/y ratio is 0.282 for Thailand and is only 0.185 for the USA.

²⁴ Capital depreciation rate of Thailand is derived from the capital accumulation equation, $\delta = ((i/y)/(k/y)) + 1 - \gamma$.

²⁵ Backus et al (1992) use the data range from 1970-1990 to calculate the US capital share.

equal to the depreciation rate. The steady state Tobin's q is one. The elasticity of the investment to capital ratio with respect to Tobin's q (η) is set to 15 for all countries.²⁶

6.6.3. Measuring productivity

The total factor productivity, A_t , is normally measured, following the work of Solow (1957), as a residual from a Cobb-Douglas function. The "Solow residuals" would be measured directly using (6.13) and (6.14).²⁷ Measurement of the Solow residual therefore requires measures of output, capital input, labour input and factor shares for each country. The employment data is used to construct the Solow residuals for all three countries.²⁸

The VAR model is estimated to obtain the parameters governing total factor productivity. The forcing processes for productivity follow equation (6.15) and the government spending shock follow equation (6.18) respectively. These driving processes applied in this chapter are not symmetric so as to represent the exogenous processes approximated from the data. The values in the brackets denote the standard error. The ε 's are the innovation to technology:

Calibrated parameters 1993Q1-2006Q4

Technology shocks

US(home)-Thailand(foreign)

$$\begin{bmatrix} \log A_t^h \\ \log A_t^f \end{bmatrix} = \begin{bmatrix} 0.947(0.044) & 0.022(0.034) \\ 0.261(0.281) & 0.864(0.220) \end{bmatrix} \begin{bmatrix} \log A_{t-1}^h \\ \log A_{t-1}^f \end{bmatrix} + \begin{bmatrix} \varepsilon_t^h \\ \varepsilon_t^f \end{bmatrix}$$

²⁶ Let $z = (i/k)$ and it can be shown that Tobin's q under the investment dynamic with adjustment cost is $q = 1/\phi'(i/k)$, it can be shown that the $\eta = d \ln z / d \ln q = (-1/z)/(\phi''(z)/\phi'(z))$. So $\eta = -(\phi'(z)/\phi''(z))(k/i)$ see details in the appendices section A.13. The value η is selected to 15 according to Baxter and Crucini (1995). η is also called the elasticity of the marginal adjustment cost function. The effect of η on Thai business cycles is examined in details on section 6.7.

²⁷ However, there are some problems of taking this method. Firstly, unlike Backus et al. (1992), the capital is included to calculate productivity. The capital stock of the USA and Japan are derived from the estimation of its investment series and the depreciation rate. Secondly, Thai data is available at the annual frequency but not quarterly frequency. The productivities for all three are measured at the annual frequency.

²⁸ Rather than using the total hours worked, the employment data is used here because the labour hours are not readily available for Thailand. Burdett and Wright (1989) document that more of variance in total labour input is explained by hours variation than by employment variation in many European countries.

$$\begin{aligned}
Var(\varepsilon_t^h) &= 0.00669^2 \\
Var(\varepsilon_t^f) &= 0.04264^2 \\
corr(\varepsilon_t^h, \varepsilon_t^f) &= -0.186
\end{aligned} \tag{6.23}$$

Japan(home)-Thailand(foreign)

$$\begin{aligned}
\begin{bmatrix} \log A_t^h \\ \log A_t^f \end{bmatrix} &= \begin{bmatrix} 1.067(0.144) & 0.028(0.101) \\ 0.362(0.377) & 0.765(0.264) \end{bmatrix} \begin{bmatrix} \log A_{t-1}^h \\ \log A_{t-1}^f \end{bmatrix} + \begin{bmatrix} \varepsilon_t^h \\ \varepsilon_t^f \end{bmatrix} \\
Var(\varepsilon_t^h) &= 0.01654^2 \\
Var(\varepsilon_t^f) &= 0.04253^2 \\
corr(\varepsilon_t^h, \varepsilon_t^f) &= 0.529
\end{aligned} \tag{6.24}$$

US(home)-Japan(foreign)

$$\begin{aligned}
\begin{bmatrix} \log A_t^h \\ \log A_t^f \end{bmatrix} &= \begin{bmatrix} 0.886(0.090) & 0.086(0.101) \\ 0.343(0.195) & 0.750(0.218) \end{bmatrix} \begin{bmatrix} \log A_{t-1}^h \\ \log A_{t-1}^f \end{bmatrix} + \begin{bmatrix} \varepsilon_t^h \\ \varepsilon_t^f \end{bmatrix} \\
Var(\varepsilon_t^h) &= 0.00660^2 \\
Var(\varepsilon_t^f) &= 0.01426^2 \\
corr(\varepsilon_t^h, \varepsilon_t^f) &= 0.023
\end{aligned} \tag{6.25}$$

The calibrated parameters imply that the movements in productivity are highly persistent in both countries. The productivity apparently depends to a very large extent on its last period values.²⁹ The eigenvalues of the coefficient matrix of (6.23) are 0.9918 and 0.8192. The model will display the stability local dynamics although it will take a long time before the steady state is obtained following a shock. The positive technology spillovers imply that there is some evidence of transmission of shocks across countries. It is important to note that the technology significantly spillovers from the USA and Japan to Thailand, rather than way around. Total factor productivity in the USA and Japan in $t-1$ accounts for that in Thailand in period t by 26.1% and 36.2% respectively. There are insignificantly different from zero technology spillovers from Thailand to the USA and to Japan. The technology

²⁹ With filtering the total factoring productivity for each country, the persistences are much lower than the values shown above because low frequency variation has been removed from the series. For example, for the pair of Thailand and the USA, the persistency for each country is 0.451 and 0.504. The spillovers from the USA to Thailand is -0.232 and is -0.010 from Thailand to the USA. These results seem impractical. The other pairs of country shows the similar results, low persistence and mix spillovers.

spillovers from Thailand to the USA and Japan are only 2.2% and 2.8% respectively. The calibrated parameters show that technology spillover from the USA to Japan at 34.3%. The previous literature regarding to the international RBC model among the developed countries documents the positive correlation of technology innovation³⁰. However, they are negatively correlated across the USA and Thailand at -0.186, but positive correlated across Japan and Thailand at 0.529. The variances of the innovation in technology are quite small for each country.

Figure 6.3 The Total factor productivity for the USA, Thailand and Japan during 1993q1-2006q4

Notes: The total factor productivity, A_t , is normally measured following the work of Solow (1957), as a residual from a Cobb-Douglas function. The x-axis is time (year) and y-axis is log of total factor productivity.

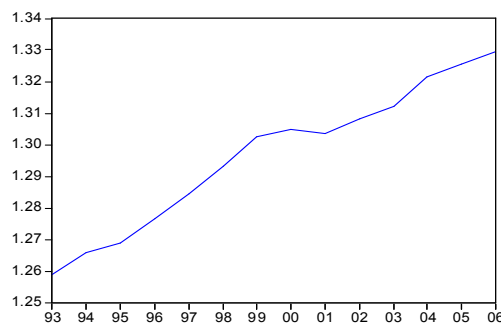


Figure 6.3 A the USA

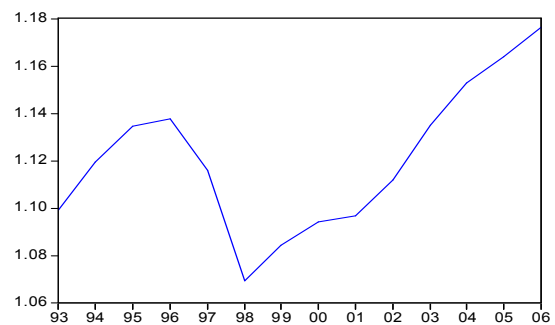


Figure 6.3 B Thailand

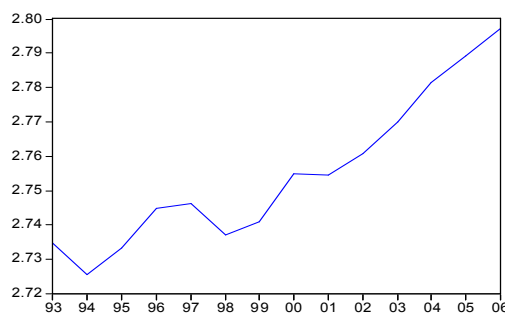


Figure 6.3 C Japan

Figures 6.3 depicts the total factor productivity for the three countries. The three time series have a similar pattern. However, the total factor productivity in Thailand drastically drops

³⁰ See Backus, Kehoe and Kydland (1992), Ravn(1997), Baxter and Crucini(1995), Ortega,Estudios and Espana(1998).

during the 1997 collapse of the Asian economies. In Japan, at the same time, it also declines but much less extent. Total factor productivity in US shows relatively smooth growth and portrays no immediate impact from the Asian crisis 1997.

This section additionally investigates the unit root test for the total factor productivity for each of the two countries.³¹

Table 6.3 Augmented Dickey-Fuller unit root test on total factor productivity of Thailand ,the USA and Japan

Augmented Dickey-Fuller	Thailand		The USA		Japan	
	t-statistic	Prob.	t-statistic	Prob.	t-statistic	Prob.
Test statistic	-0.8285	0.9334	-1.9585	0.5642	0.0336	0.9889
Test critical values: 1% level	-4.8864		-4.9923		-5.1249	
5% level	-3.8290		-3.8753		-3.9333	

The null hypothesis is that the total factor productivity follows a random walk with drift. By using the Augmented Dickey-Fuller (ADF) test, there is evidence that the null hypothesis is not rejected at 1% level of significance. The test critical values at 1% level significance are -4.8864 for Thailand and -4.9922 for the USA. The ADF test statistics for the Solow's residuals are -0.8285 for Thailand, -1.9585 for the USA and 0.0336 for Japan. The co-integrations for these two time series are next explored. The Johansen co-integration is used to test the co-integrating relationship between the total factor productivity time series. The test indicates no co-integration at 5% level of significance.³² However, at a 10% level of significance, the ADF unit root test on the residuals computed from the co-integrating regression is used to test the co-integration. The first step is to regress the total factor productivity of one country on that of another. The second step is to computing the ADF statistics for the residual from the regression.³³

³¹ King, Plosser, Stock and Watson (1991) identify permanent productivity shocks as shocks to the common stochastic trend in output, consumption, and investment. The permanent productivity shock is consistent with the US post war period economy.

³² Wickens (1996) suggests that the Johansen co-integration test is more likely to reject the null hypothesis. He concludes that co-integration analysis needs to take account of structural restrictions.

³³ The details of this method can be found in Stock and Watson (2003).

There is, at 10% level of significance, evidence of co-integration between Thailand and US at the test statistic -1.6758 , and between Japan and the USA the test statistic is -0.8791 . Nevertheless, the evidence for co-integration is weaker for Thailand and Japan at the test statistic -1.1508 . According to these tests, it is impossible to eliminate the hypothesis that the total factor productivity in each country follows a random walk with drift and without spillovers. It is also possible to have the correlation of the technology innovation between the sample countries. Therefore the model performance compares the business cycle properties of data for an actual economy and that of data created by the calibrated model economy. The model examines the sensitivity analysis based on the plausible values of shock processes.

6.6.4. Measuring government spending

The model can also be driven by another source of shock, specifically, the shock to fiscal policy or government purchasing. This shock may yield a different effects and results for the international business cycle. The model economy experiences the alterations in government spending in one country and the main macroeconomic variables dynamically response in the international framework. The government spending in each country is straightforward derived from the observed government expenditure time series, not by calculating the residuals such as the productivity. The calibrated values for each country are as below:³⁴

$$\begin{bmatrix} \log G_t^{us} \\ \log G_t^{th} \\ \log G_t^{jp} \end{bmatrix} = \begin{bmatrix} 1.011(0.012) & 0 & 0 \\ 0 & 0.975(0.011) & 0 \\ 0 & 0 & 0.895(0.051) \end{bmatrix} \begin{bmatrix} \log G_{t-1}^{us} \\ \log G_{t-1}^{th} \\ \log G_{t-1}^{jp} \end{bmatrix} + \begin{bmatrix} \varepsilon_{G,t}^{us} \\ \varepsilon_{G,t}^{th} \\ \varepsilon_{G,t}^{jp} \end{bmatrix}$$

$$\begin{aligned} Var(\varepsilon_{G,t}^{us}) &= 0.007187^2 \\ Var(\varepsilon_{G,t}^{th}) &= 0.059651^2 \\ Var(\varepsilon_{G,t}^{jp}) &= 0.009253^2 \end{aligned} \tag{6.26}$$

³⁴ The persistence of the government spending shock in the USA is 1.011 which is larger than 1. The dynamic system will be explosive. To investigate the effects of the US government spending shock, we consider it as a permanent shock which the persistence is assumed to be 1, rather than 1.011.

This study assumes that there is no evidence of government consumption spillovers across countries. The government spending is highly persistent in each country. However, the variance is relatively high in the case of Thailand. The three series also shows the evidence of random walk at 1% level of significance. The ADF test statistics are 1.476, -1.397 and -2.197 for the USA, Thailand and Japan respectively. Since the government spending is highly persistent, it is possible to study the permanent shock of the government purchasing.

The tax revenues as a percentage of GDP for the USA, Japan and Thailand do not considerably change during the observation periods. The average of the fraction of tax revenue to GDP is 27.9% for the USA, only 16% for Thailand and 26.4% for Japan. We assume that the shocks in taxation for all countries are permanent as in equation (6.27). The permanent shock in taxation will be explored in next section as well as the alteration in persistence of the shock.

$$\begin{bmatrix} \log T_t^{us} \\ \log T_t^{th} \\ \log T_t^{jp} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \log T_{t-1}^{us} \\ \log T_{t-1}^{th} \\ \log T_{t-1}^{jp} \end{bmatrix} + \begin{bmatrix} \varepsilon_{T,t}^{us} \\ \varepsilon_{T,t}^{th} \\ \varepsilon_{T,t}^{jp} \end{bmatrix} \quad (6.27)$$

6.7 Conclusion

This chapter examines the Thai business cycle by using the simple IRBC model, a single good, two country model, with complete market to explain Thai stylized facts. The first part of this chapter has assessed empirically the properties of business cycle for 7 OECD countries (the USA, the UK, EU, Canada, Australia, Japan and Korea) over the period 1986q1-2006q4. It finds that the output volatilities of Japan and Korea are larger than that of other countries. Consumption is less volatile than output for all countries, except for the UK and Korea. With respect to output, investments for all countries have been two to four times more volatile than output. The cyclical components of all series are highly persistent. The trade variables exhibit a negative correlation with output for all countries but Canada. The employment, consumption and investment variables have been pro-cyclical with output. Consumption and investment show a strongly positive correlation with output. The cross-country correlations of the US output are positive in most cases, except for Japan and Korea.

The stylized facts of the Thai economy are explored using quarterly data over 1993Q1-2006Q4 due to the lack of Thai data. The characteristics of the business cycle of Thailand, the USA and Japan are also investigated during this period because the USA and Japan are the main trading partners of Thailand. The model is constructed to particularly explain the empirical study of the USA and Thailand during 1993Q1-2006Q4. This chapter suggests that the volatility of all Thai variables are greater than that of US and Japan variables. The output and trade in Thailand fluctuate about three times more than that of the USA and Japan. All variables for all three countries perform a high persistence. Investment has been shown to be strongly pro-cyclical than consumption and employment in USA and Japan. However, consumption in Thailand is more strongly correlated with output than investment. Output, trade, consumption, investment and government spending in Thailand are negatively cross-correlated with the same US variables. Therefore, the Thai cycle negatively fluctuates with the US cycle during 1993-2006. However, output and investment in Japan are positively cross-correlated with the same US variables.

In this chapter we have examined the properties of these countries, with Japan and the US being, at this time, the two largest economies in the world. The information gained in this analysis will later inform our conclusions. But in the rest of this chapter we focus on building a real business cycle model for the USA and Thailand.

Then this chapter proposed an alternative version of a two-country, single good IRBC model based on Baxter and Crucini (1995), where each country has its own preferences, technology and driving processes, under the complete market structure to capture the Thai facts. Special attention is given to the model implication with the calibrated parameters for Thailand and USA. The capital stock of the USA is derived from the US investment series. The measure of technology shock is based on the Solow residual, consisting both capital and labour input. The calibrated parameters imply that the movements in productivity are highly persistent in both Thailand and the USA. There is an evidence of transmission of shocks across countries. The technology spillovers significantly transfers from the USA to Thailand at 26.1%, but only 2.2% from Thailand to the USA. The contemporaneous correlation of technology innovation for Thailand and the USA is -0.186. The total factor productivity of

each country shows the evidence of random walk. Therefore it is possible to assume and examine the effect of Thai business cycle on the permanent shock.

Chapter 7

The Open Economy Model: Simulation and Performance

7.1 Introduction

Before introducing an international real business cycle model, the preceding chapter examined the properties of business cycle for some developed countries and Thailand. The stylized facts of the 7 OECD countries were investigated. The features of the business cycle of Thailand, the USA and Japan were also explored in the previous chapter because the USA and Japan are the world's largest economies and they are major trading partners of Thailand. The international real business cycle model was introduced to test the relationship between the theory and the data. The model was constructed in the previous chapter in order to explain the features of business cycle in the real economy specifically between the USA and Thailand. Once the model was specified, the next step was to parameterize it. The model was calibrated for the USA and Thailand. Most of the parameters of the model were chosen to match the long run properties of the USA and Thailand.

This chapter focuses on the model's simulation and performance. The model is simulated for three different types of shock, a technology, government spending and tax shocks. The sensitivity to changes in the persistence of shocks is also analyzed. The shocks investigated both are permanent and temporary shocks. A closed economy model, in chapter 5, uses a simple government spending rule which does not allow to examine the effect of a tax shock. A government spending rule in this open economy model allows us to analyze a shock for tax and government spending.¹ By assuming that a large country represents up to 99.99% of the world GDP, this open economy model is also able to investigate a tax and government spending shock for a closed economy.

¹ The government spending rule is simple in chapter 5, a closed economy model. The government spending is financed by lump-sum taxes. Nonetheless, government in the open economy model collects taxes from a representative agent's income by rate τ and has a transfer payment of T . The permanent government spending shock is not explored in chapter 5 since the government spending rule is simple. To study a permanent government spending shock, it needs more complicated government spending rule.

The shocks off the steady state and the multiple shocks are also discussed in the context of the open economy model in this chapter. The model performance is evaluated by comparing the model moments and the moments calculated from the facts. There are four scenarios to test for the model performance, the permanent technology shock, the realistic shock without technology spillovers, the realistic shocks with technology spillovers with changing variance of technology innovation, and the sensitivity analysis.

This chapter, using the one good two country model described in chapter 6, focuses on the model simulation and model performance. The remainders of the chapters are as follows. The model is simulated in section 7.2. The shocks off the steady state for the open economy model are discussed in section 7.3. The multiple shocks for the open economy model are in section 7.4. Section 7.5 documents the model performance and the effect of the parameterization on their economic implications, comparing the stylized facts and the model predictions. Section 7.6 of this chapter concludes.

7.2 Model Simulation

After constructing the model and calibrating the model parameters, the intuition of dynamic responses can be examined by the impulse responses illustrated in this section.² This section analyses the sensitivity of impulse responses to changes in the persistence of technology, government spending and taxation and investigates how each macroeconomic variable responds to these shocks for the case of the USA and Thailand. Since the ratio of Thailand to US GDP is on average 0.76%, the small open economy follows the calibrated parameters of Thailand is assumed to represent 0.76% of the world GDP. To study a large open economy, the calibrated parameters of the USA are used and its GDP is assumed to be half of the world.³ The responses are traced up to 120 quarter periods. The shocks are measured as percentages of their steady state value. Since it is easy to analyze the effect of the shocks in term of good units, the remaining variables are measured as percentages of steady state

² The correlation of the innovation to the each shock is not in part of simulation. The model simulation only gives us the idea of how macroeconomic variables behave after facing the shocks. The correlations are the key parameters in calculating the moments. Section 6.7 will discuss the model performances.

³ To study a large open economy in this model, the size of the USA could not be $(1 - 0.76\%)$. For this size of the USA, the USA is considered as the close economy model. The size of the USA is assumed to be half of the world to study the impact of a large open economy model.

good units. To convert the responds from the shock to good units, the results of the respond from the shocks are multiplied by the ration of each variable to output.⁴

7.2.1 A Permanent Shock

In the preceding chapter, we suggest that there are the possibilities that the shocks can be considered as the permanent driving forces of the economy. This subsection explores the consequences on the general equilibrium of the unanticipated permanent shocks to productivity, government spending and taxation.

7.2.1.1 Simulation US-Thailand: A permanent shock in productivity

The parameterization in the case of permanent technology shock is: technology persistence (ρ_A) in each country is 1 and the technology spillover parameters among two countries are assumed to be zero.

Figure 7.1 A 1% increase in Thailand productivity

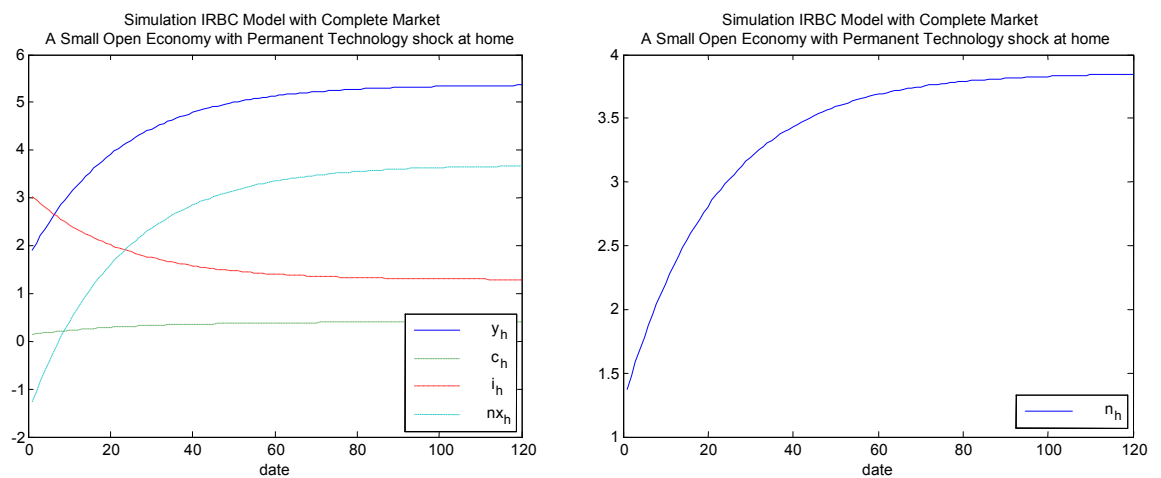


Figure 7.1 A
The responses of variables in Thailand (home)

⁴ For example, the response of investment in good units comes from investment series from the impulse response multiplies by investment output ratio.

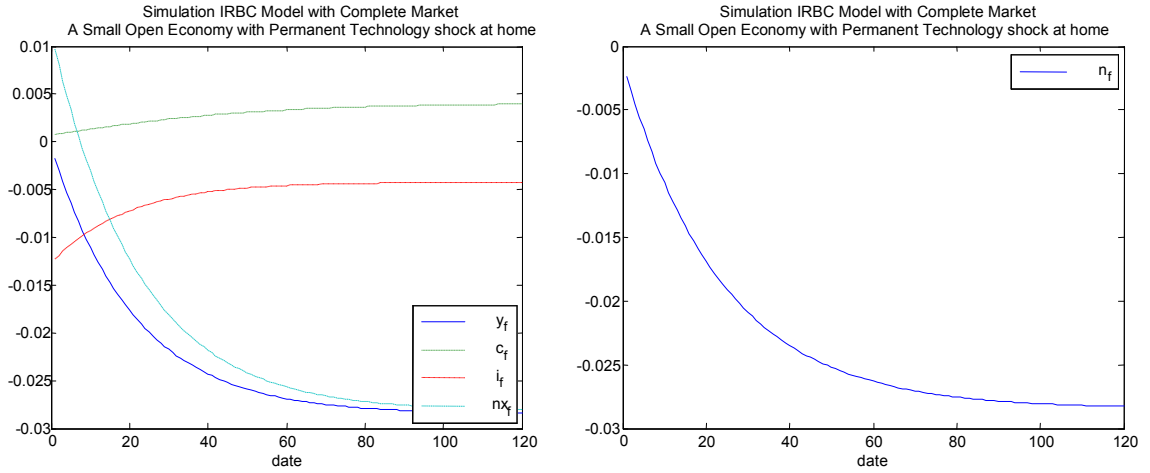


Figure 7.1 B
The responses of variables in the USA (foreign)

Figure 7.1 illustrates the responses to a one standard deviation increase in Thailand's technology innovation starting from the steady state. Figure 7.1 A pictures the responses of Thailand's aggregate variables. Figure 7.1 B pictures the responses of the same aggregate variables for the USA.

The home country (Thailand) responds to a positive productivity shock by increasing investment, consumption and output. The investment and output in the foreign country (USA) decreases following an immediate permanent jump in Thailand productivity because capital moves from the foreign to the home country. Output in the home country rises on the impact by roughly 2 good units and continues to increase toward the new steady state. The marginal product of capital determines the path of investment. After the shock, the marginal product of capital increases in the home country by 2.1 unit goods and gradually declines back to its steady state level (this graph not shown). Apparently, the investment in the home country significantly rises by 3 good units after the shock and declines toward the new steady state. An increase in consumption is negligible not only in the home country, but also in the foreign country because the size of the home country is small. This is because, according to market structure, there is complete risk sharing between the home country and the foreign country. The marginal utilities of consumption of the two countries are the same. Net exports of the home country initially fall by 1.2 good units because consumption and output increases less than investment. Shortly, with the complete market structure, the trade

in the home country moves into surplus because the home country exports goods to the foreign country. Trade in the foreign country initially rises because investment in the foreign country drastically falls and rapidly declines later on. The labour in the home country responds positively to take advantage of the positive shock in productivity and rises towards the new steady state.

The economic implication is that the technology shock under the complete market assumption causes the negative wealth effect in the small country. It increases consumption relatively little, but increases labour supply substantially. Although there is a positive wage effect with a high wage rate which induces households to supply their labour, the negative wealth effect is higher. Therefore, labour in the home country increases significantly. In the foreign country, the labour input exhibits the opposite direction. Compared to the home country which is assumed to be small, the effects of the shock on the foreign country in the aggregate are approximately nothing. This simulation captures the fact that the economic events in Thailand hardly affect the economic activities in the USA, the 1997 crisis in Thailand is a good recent example.

Figure 7.2 A 1% increase in US productivity

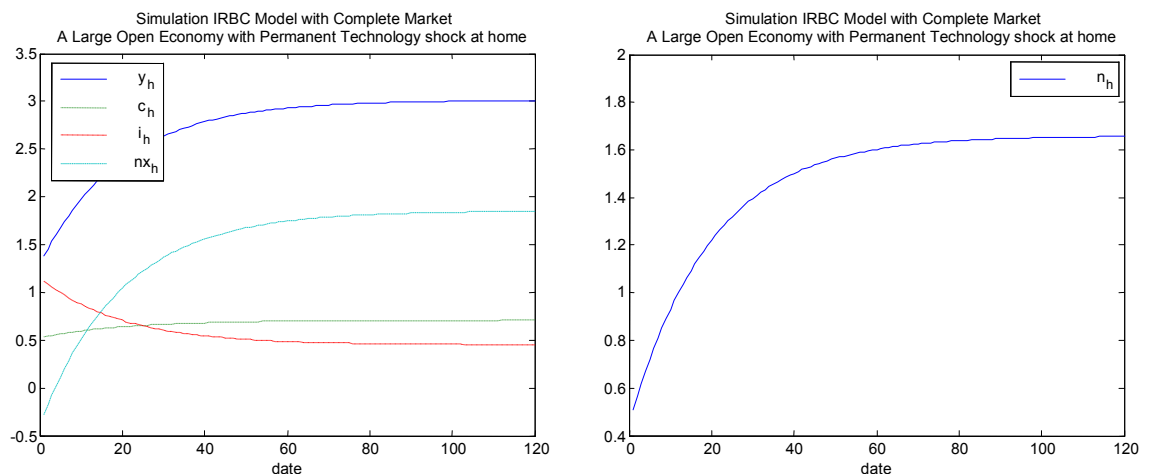


Figure 7.2 A
The responses of variables in the USA (home)

Turning to the permanent technology shock in the USA, figure 7.2 shows what happens in the USA (home) and Thailand (foreign) after permanently increasing productivity 1% in the USA. The impulse responses produced by the model subject to the permanent shock to productivity in large country (USA) are quite similar to the preceding section except for the amplitude of each aggregate variable.

On impact there is an increase in the investment, this is because to take advantage of the permanent productivity shock, the home country increases immediately the investment by 1.1 good units. However, the investment in the foreign country decreases on the impact. Since the home country is large, consumption increases in both the home and foreign country noticeably. Consumption in the home country rises on impact by 0.5 good unit and approximately so does consumption abroad. Output in the large country increases immediately by 1.4 good units since all its components increase except net exports. Following a similar argument to the preceding section, the home country also experiences a negative wealth effect after an unanticipated 1% increase in productivity in the home country.

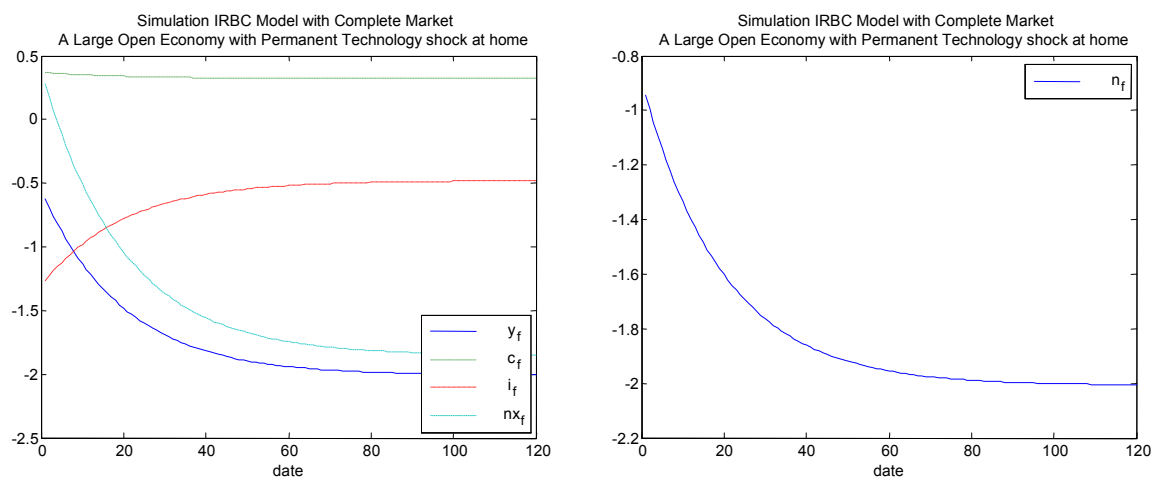


Figure 7.2 B
The responses of variables in Thailand (foreign)

Consequently, labour supply increases in the home country and continues to rise toward the new steady state. Labour input should additionally increase since there is a positive shock

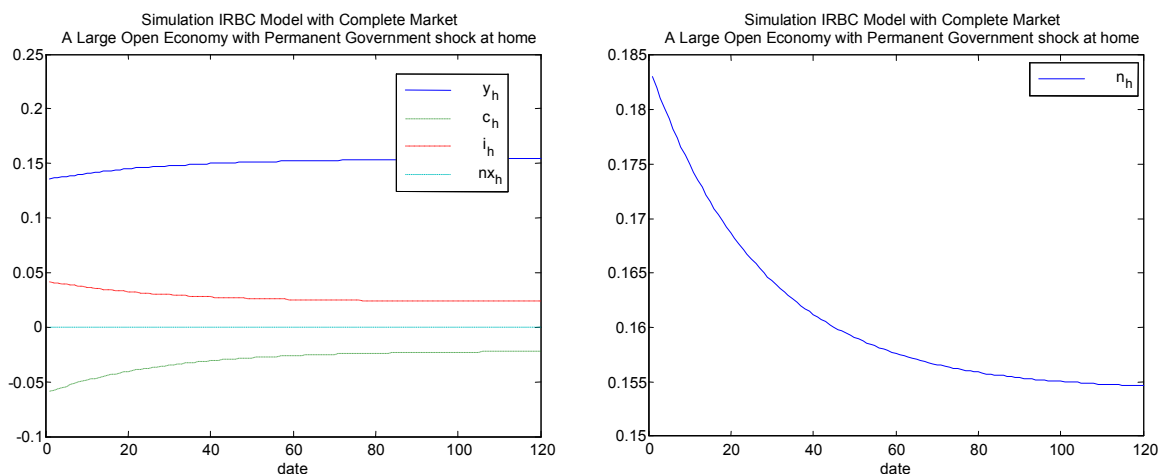
and the wage goes up. This effect generates the incentive to work hard, as it is effectively a supply side response.

Again, the trade in the home country moves into deficit on the impact, by 0.3 good units and quickly improves according to market structure. In the long run, all variables stay at a higher level new steady state. Trade stays also at a higher level to repay the increased consumption and investment from the previous period. The responses in Thailand are significant according to the picture 7.2 B. Output and investment approximately decline on the impact by 0.6 and 1.3 good units. Investment declines due to the fact that capital in Thailand moves into the USA. Output decreases after the impact because of the decline in labour and investment. The trade in Thailand moves into surplus by 0.3 good units after the shock.

7.2.1.2 Simulation US-Thailand: A permanent government spending shock

Since the government purchases in this model are assumed to be financed with lump-sum taxes, an increase in government expenditures typically has a negative wealth effect. That negative wealth effect raises labour input and crowds out a representative agent's consumption.

Figure 7.3 A 1% increase in US government spending (Closed economy); The responses of US variables



Considering the case of the closed economy by assuming that the USA represents up to 99.99% of the world GDP, figure 7.3 illustrates the dynamic outcomes of the US main

variables. The persistence of the government spending (ρ_g) for the USA is assumed to be 1. The economy experiences an increase of 1% in government purchasing.

An increase in government spending is generally a drain on output. A permanent increase in government spending generates the wealth effects by reducing permanent income. The negative wealth effect of an increase in government spending induces individuals to consume less but work harder.⁵ An increase in labour supply by 0.183 unit after the shock in turn leads to an increase in the marginal product of capital and stimulates more investment by approximately 0.05 good unit. The increase in labour input is the main reasons for an increase in output by 0.14 good unit.

Figure 7.4 A 1% increase in US government spending

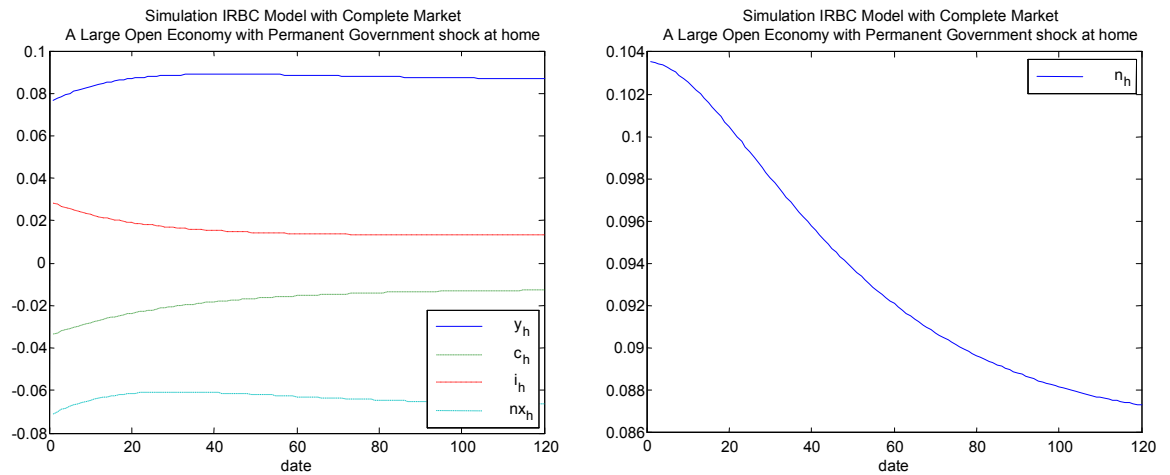


Figure 7.4 A
The response of variables in the USA (home)

Under the cross-country business cycle framework, the transmission of government expenditure variations is analyzed in the context of general equilibrium. Under complete market assumption, the persistence of the government spending (ρ_g) for each country is set

⁵ Theoretically, there is a distinct difference regarding how a change in government spending affects consumption in the real business cycle model (or RBC) and Keynesian model (or IS-LM). An increase in government spending, in RBC model, decrease the after tax income and in turn decline in consumption. However, an increase in government spending according to Keynesian IS-LM model increases after tax income inducing an increase in consumption. The RBC model considers the supply side effect of government spending, but the IS-LM model views the demand side effect. Angeletos and Panousi (2007) examine the supply side effects of government spending under incomplete market. The negative wealth effect leads to a reduction in risk taking and investment.

to be 1. There is no government spending spillover between the two countries. The effect of government purchasing alteration for the large economy (USA) is pictured in figure 7.4.

If the country is large, the negative wealth effect will also increase the interest rate, causing a secondary increase in labour input and also an inter-temporal substitution effect. The large country interest rate dominates and drives up the world interest rate as in figure 7.4 B.⁶

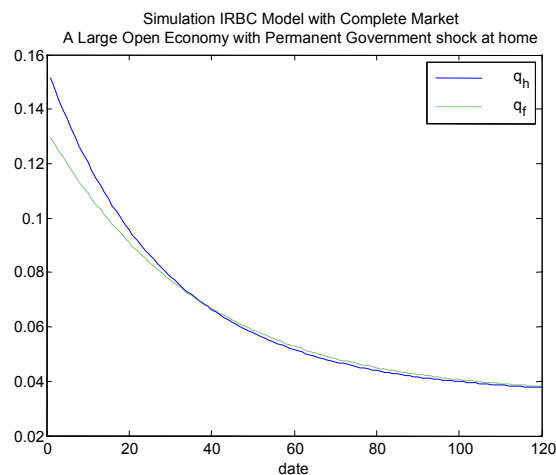


Figure 7.4 B
The responses of the marginal product of capital for both countries

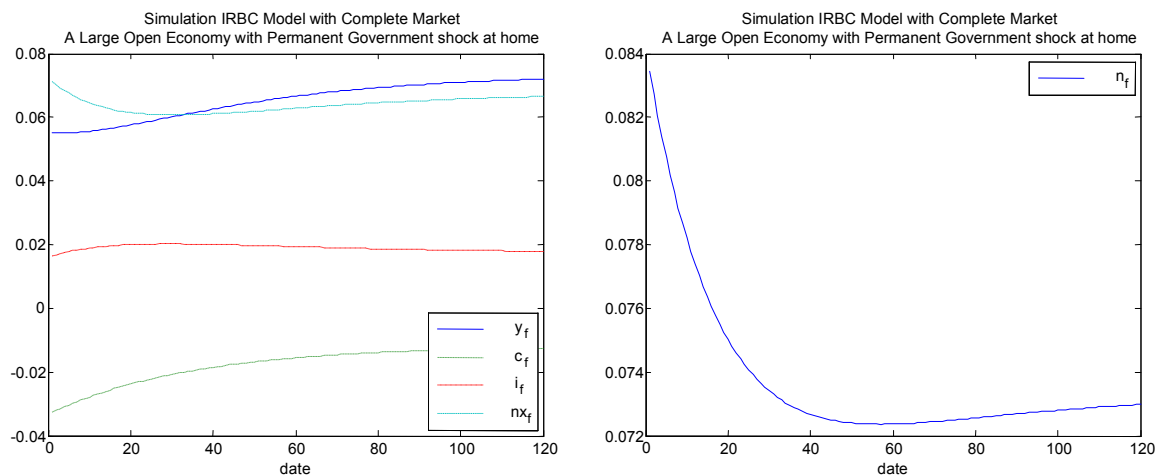


Figure 7.4 C
The response of variables in Thailand (foreign)

⁶ Blankenau, Kose and Kei-Mu (2001) find that the world interest rate shock can play an important role in explaining business cycles in small open economy. They can explain up to one-third of the fluctuation in output and more than half of the variation in net export.

An increase in the interest rate in the large country is transmitted abroad, raising foreign labour input and lower foreign consumption. The model predicts that both countries increase labour input and decrease consumption by approximately the same amount. In the USA, the time path of the interest rate dictates the time path of labour. However, labour market adjustment is more rapid in Thailand than in the USA. The rise in labour input increases the marginal product of capital, which in turn leads to an increase in investment and output in both countries. The home country experiences a worsening trade deficit on the impact and stays in deficit at the new steady state.

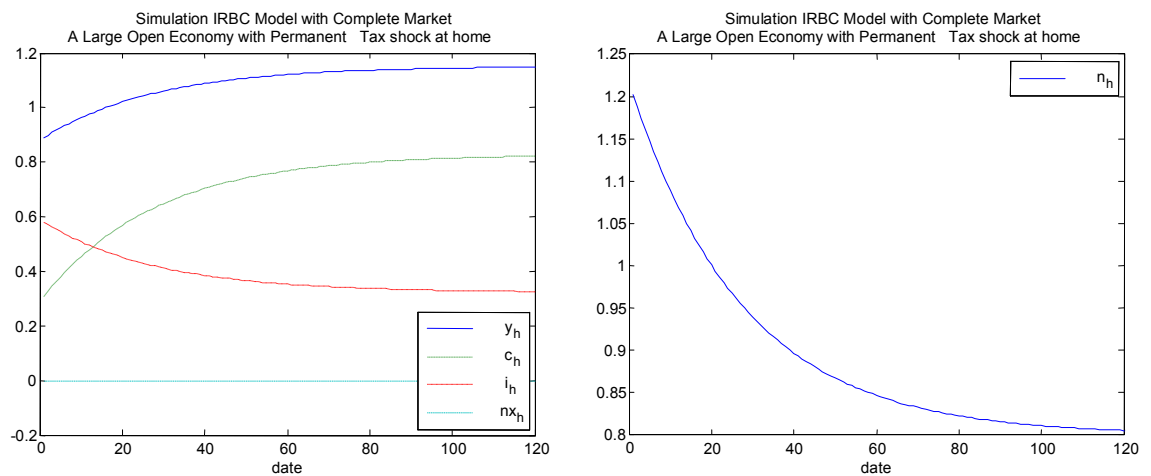
Turning to the government shock in small country (Thailand), there will be approximately zero change in consumption both in home and foreign country. The reason is that the country is very small, there is no shock to the world wealth. The sensitivity of impulse responses to changes in the persistence of government spending shock in Thailand will be investigated more in section 7.5.5.

7.2.1.3 Simulation US-Thailand: a permanent shock in taxes

The structure of the model allows us to examine how important the effect of the variation in taxation on the context of international business cycle is. Since, in this model, government collects tax at a rate (τ) on total output (Y), the decrease in tax rate generates the positive wealth effect. To see the effect of the distorting impact of taxation, consider first the case of the closed economy model by assuming that the USA represents up to 99.99% of the world GDP, figure 7.5 pictures the dynamic outcomes of the US main variables. The persistence of the tax (ρ_T) for the USA is assumed to be 1. The economy here faces a permanent decline of 1% in the tax rate.

The permanent decline in the tax rate creates a positive wealth effect and basically pushes up the real interest rate and the marginal product of labour. While the positive wealth effect induces a lower labour input, the increases in the interest rate and the marginal product of labour raises the incentive to work more. Because the two substitution effects are stronger than the wealth effect, the labour input increases by 1.2.

Figure 7.5 A 1% decline in US tax rate (Closed economy)



The responses of variables in the USA

This effect causes an increase in output by 0.9 and investment by 0.6 good units which then move towards the new steady state. The gradual increase of capital from the previous steady state after the shock is the reason why output increases while labour keeps decreasing after the shock.

Turning to the open economy framework under the complete market assumption, the effect of a 1% permanent decline in taxation in Thailand is pictured in figure 7.6.

Figure 7.6 A 1% permanent tax cut in Thailand

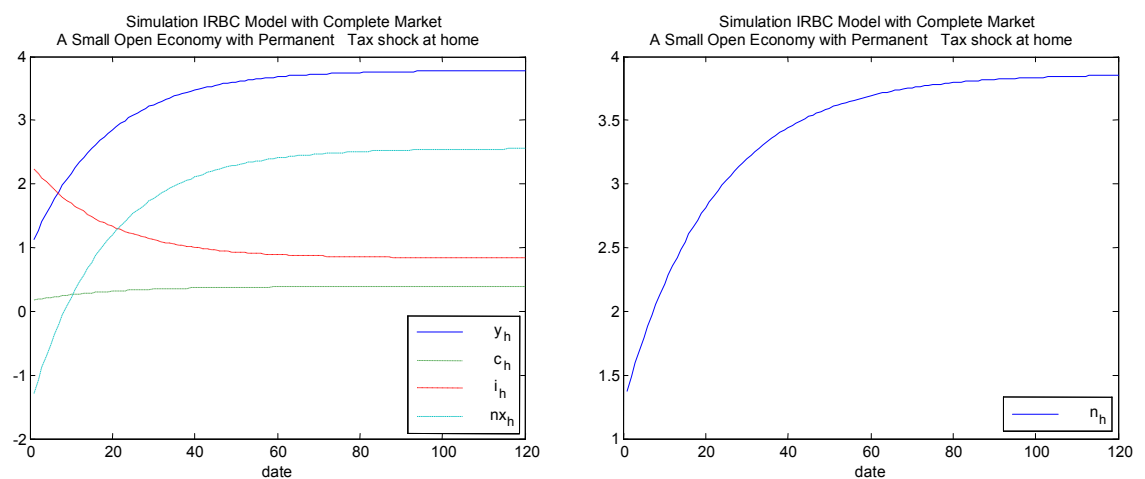


Figure 7.6 A
The response of variables in Thailand (home)

Since Thailand is a small country, consumption in Thailand only increases approximately by 0.2 good units after the impact because of the positive wealth effect.⁷ Because Thailand is labour input increases by 1.4 good units, home investment and output rise accordingly. Thailand faces a trade deficit on the impact which improves overtime.

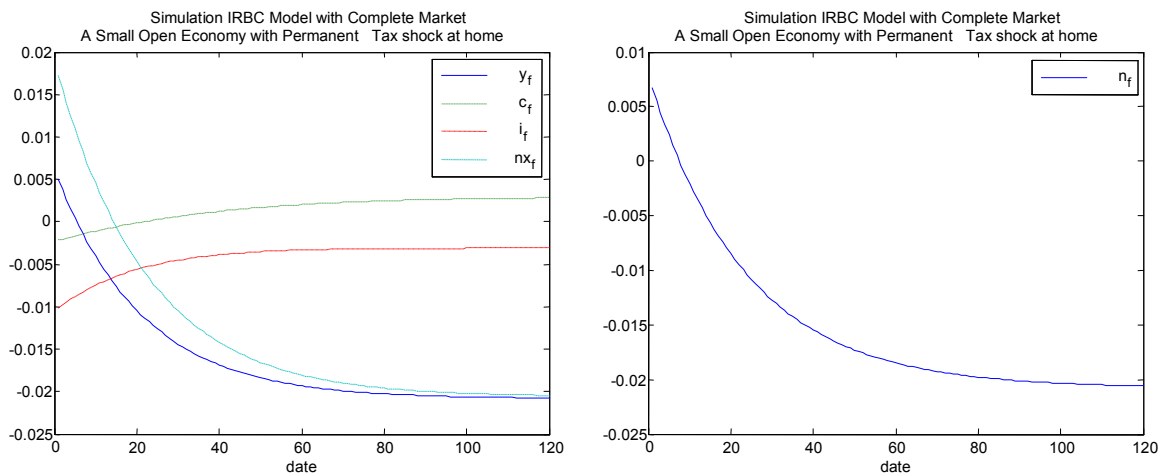


Figure 7.6 B
The response of variables in the USA (foreign)

The new steady state shows a higher trade surplus than before. The switchover point comes after 10 quarters from which trade is higher than before the shock. The response of US variables to the tax shock in Thailand is almost negligible. However, we can see a path of each US variable. Investment in the USA is slightly down following the impact by 0.01 good unit because capital moves to a more productive home country (Thailand). The labour input rises in the USA on impact and then substantially declines overtime. Initially, the US output slightly increases due to an increase in labour input, but then quickly falls to the new steady state as labour input falls. US consumption is almost unaltered.

Considering the effect of a permanent tax cut in the USA, the response of the US variables follow similar paths as in the preceding experiment for the closed economy because the USA is a large country relatively to Thailand. Labour in the USA rises and carries on increasing

⁷ Since the tax revenue as a fraction of output is only 16% in Thailand, a 1% cut in taxes increases disposable income in Thailand by 1.19%. The tax ratio to output in the USA is 27.9 %, a 1% cut in taxes increases disposable income in the USA by 1.37%. The response of consumption in Thailand after cutting taxes is smaller than the response of consumption in the USA.

towards the new steady state. However, investment in Thailand significantly drops on impact by 1.1 good units and moves toward the new steady state which is below the original situation. Consumption in Thailand also increases due to the market structure. The marginal product of labour decreases in the foreign country, causing a decline in labour input and also output. The sensitivity of impulse responses to a change in persistence of taxation in Thailand will be investigated more in section 7.5.5.

Figure 7.7 A 1% permanent tax cut in US

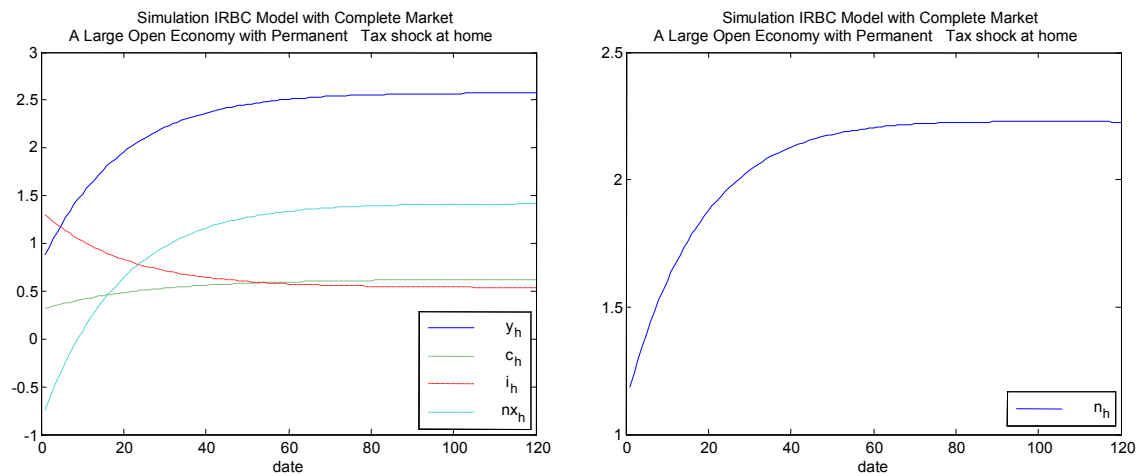


Figure 7.7 A
The response of variables in the USA (home)

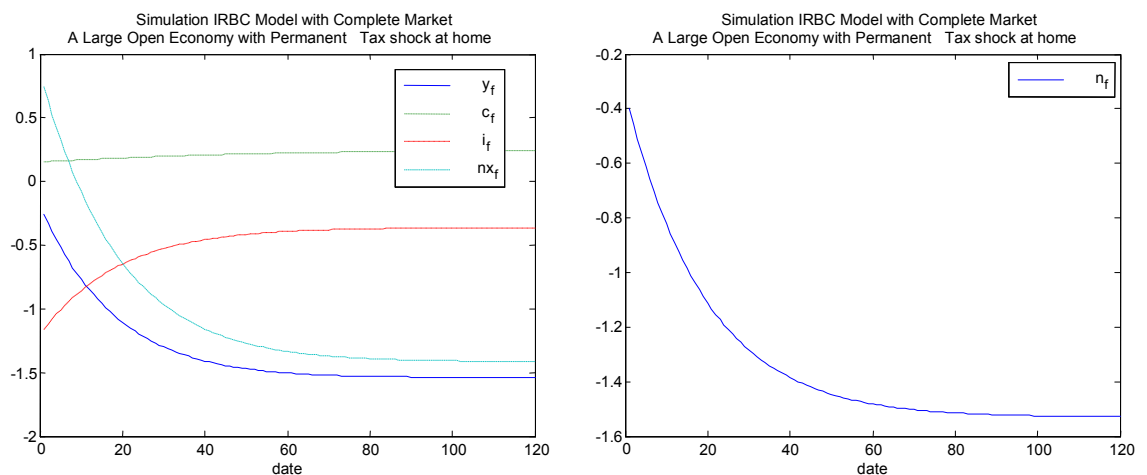


Figure 7.7 B
The response of variables in Thailand (foreign)

7.2.2 A Temporary shock

This section allows us to look at the dynamic responses under different assumptions regarding the forcing processes. Temporary shocks have different effects from permanent shocks. In general, the permanent shocks generate much larger possibilities for wealth effects, while a temporary shock create much greater chances for intertemporal substitution effects. The temporary shocks originating from technology, government spending and taxation result in the similar effects. Specifically, the less the persistence of the shock, the higher the substitution effects due to interest and wage rates and the smaller the wealth effect. Additionally, this section investigates the role of technology spillovers in the international real business cycle framework. According to the actual empirical measurement of the shocks, the driving processes are not permanent even though the evidence of unit roots can be accepted.⁸ It is necessary to simulate and analyze the impulse responses based on the empirical estimation where the shocks are temporary. The simulation takes only the case of a temporary shock in productivity for Thailand-US as an example. The responses to a change in the persistence of taxation and government expenditures in Thailand will be analyzed in section 7.5.5.

Simulation US-Thailand: a temporary shock in productivity

This subsection explores how the economy responds under an unexpected temporary change in total factor productivity. Since the shocks are not permanent, the wealth effect is small. Typically, the temporary shocks induce a small positive wealth effect while there are more possibilities for inter-temporal substitution via the interest and wage rates. The forcing parameters across the countries as in equation 6.23 are used in this study. The calibrated parameters are as below;

Technology shocks

US(home)-Thailand(foreign)

$$\begin{bmatrix} \log A_t^h \\ \log A_t^f \end{bmatrix} = \begin{bmatrix} 0.947 & 0.022 \\ 0.261 & 0.864 \end{bmatrix} \begin{bmatrix} \log A_{t-1}^h \\ \log A_{t-1}^f \end{bmatrix} + \begin{bmatrix} \varepsilon_t^h \\ \varepsilon_t^f \end{bmatrix}$$

where variance of technology innovation (ε_A) in each country is 1. (7.1)

⁸ Chapter 6 suggests that the shocks process can also be treated as a permanent shock since because of the evidence of a unit root.

Figure 7.8 and 7.9 illustrates the predicted responses that are obtained when the model economy is subject to a temporary productivity shock with technology spillovers in Thailand and the USA. An increase in a temporary technology shock in Thailand (home) raises investment by 0.7 good units and labour input by 1.3 good units, resulting in an increase in output by 1.9 and consumption by 0.2 good units. The labour input in Thailand increases because the wage effect crowds out the positive wealth effect. Because the shock is not permanent, all Thai variables increase to take advantage of the positive shock and rapidly decline until quarter 16. They decline only slightly after that. After 16 quarters, they gradually fall toward the steady state level. But it is interesting to note that even after 120 periods the system is still approaching the previous steady state. In this sense a temporary shock with relatively high persistence produces an impact which if not permanent is at least very long-term. The temporary productivity shock with spillovers results in a surplus in trade for the home country. The US (foreign) trade deficit must be offset by the surplus in the home country. The technology shock must have a very high persistence to create a trade deficit in the country where the shock originates.⁹

Figure 7.8 A temporary shock in productivity with technology spillovers in Thailand

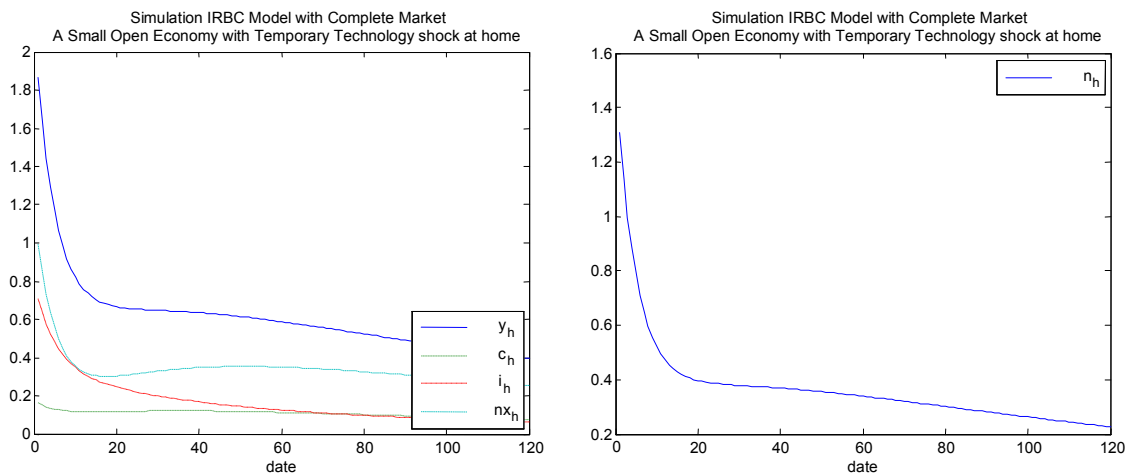


Figure 7.8 A
The response of variables in Thailand (home)

Both home and foreign country consumption increases due to the risk sharing condition. However, the output, investment and labour input of the USA decreases and have a negative

⁹ For the case of permanent technology shock, the persistence is assumed to be 1. In the case of a temporary shock persistence is as in (7.1).

value immediately after the shock. Then they rapidly increase because the shock originating in Thailand does not last forever. Nonetheless, the effects to the US variables are small. The effects of the technology spillovers from Thailand to the USA are also clearly negligible.

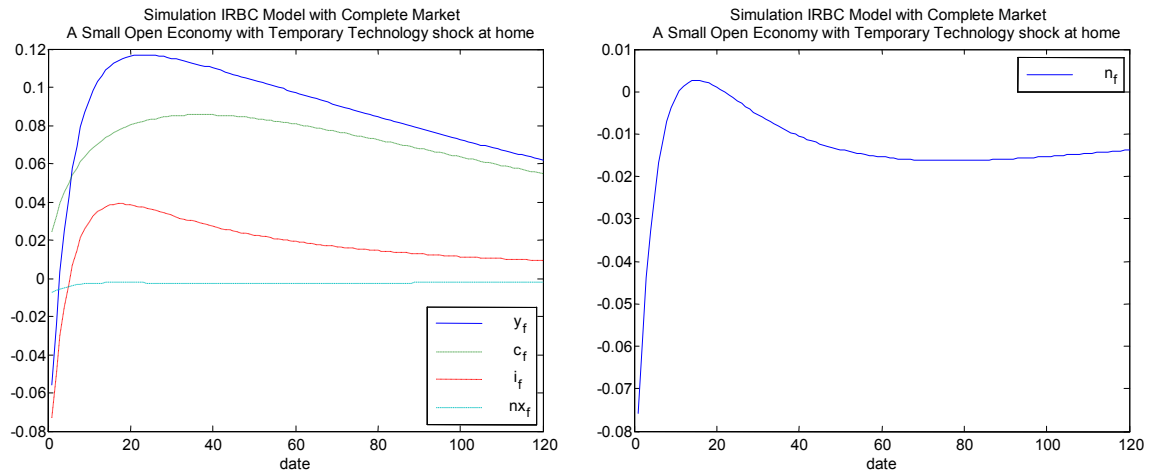


Figure 7.8 B
The response of variables in the USA (foreign)

Figure 7.9 A temporary shock in productivity with technology spillovers in the USA

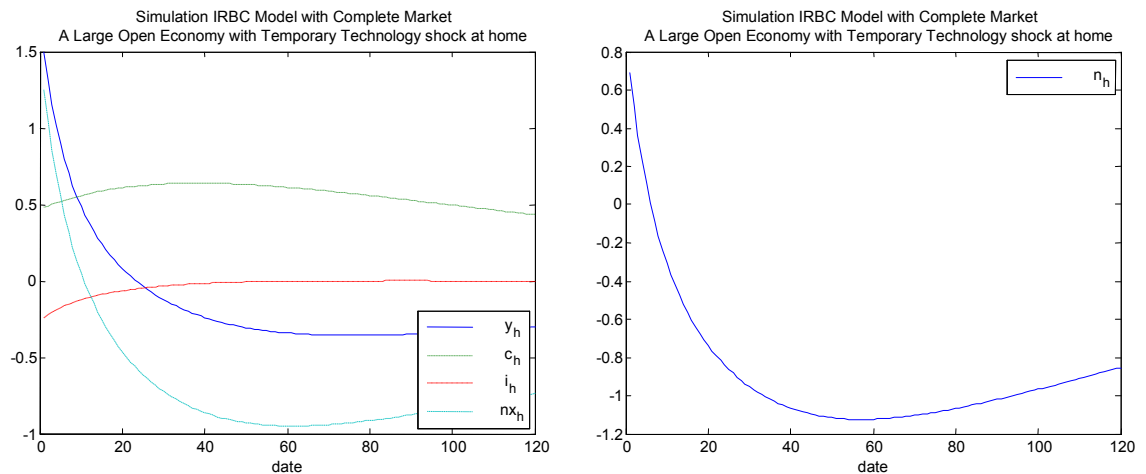


Figure 7.9 A
The response of variables in the USA (home)

However, figure 7.9 illustrates how important the technology spillovers are when the shock originates in the large open economy. The shock in the USA this case is not permanent and

it spillovers to Thailand 26.1% as in (7.1). The surprising results are that investment in the USA (home) falls on the impact of temporary shock in the USA while the labour input rises.

In closed economy, the shock in technology induces an increase in investment and labour input in the country specific shock. For the complete market, the result is similar under a large open economy with a permanent shock.

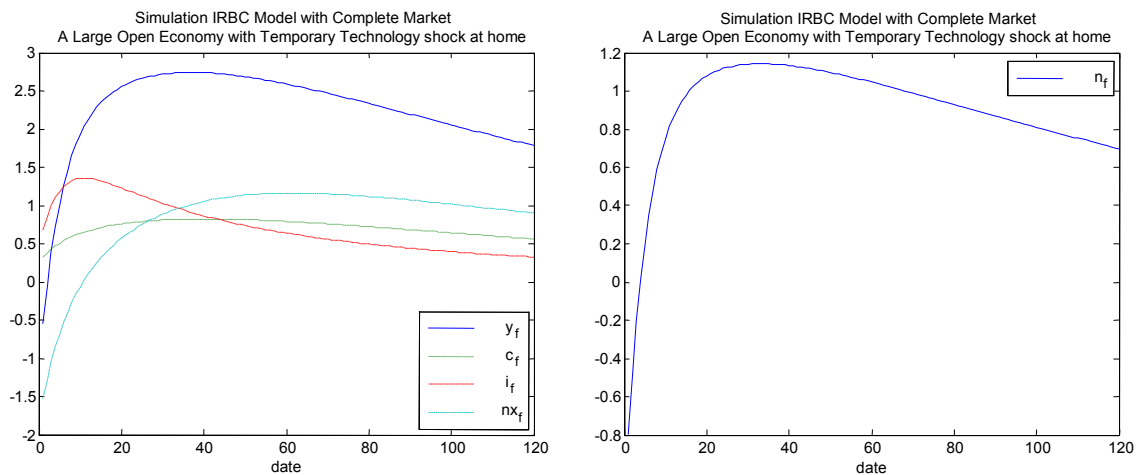


Figure 7.9 B
The response of variables in Thailand (foreign)

A possible explanations of the behaviour of the US variables when the shock is temporary lies with the market structure, the effect of the technology spillovers and the size of the home country which is large. There are obviously large technology spillovers from the USA to Thailand, specifically 26.1% quarterly. Under the complete risk sharing assumption, the temporary technology shock originating in the USA is transmitted to Thailand rapidly enough, and with sufficient size, that US investment falls and Thailand investment increases. Labour input in the USA increases because the substitution effect is larger than the wealth effect. It is easy to see that the technology spillovers have more effect to an investment than the substitution effect in this case.

To verify our intuition, figure 7.10 plots the responses of the variables in the USA following the temporary shock in the USA with several values of technology spillovers. In these figures, we clearly see that the transitional effects of the technology spillovers from the USA to Thailand.

Figure 7.10 A A temporary shock in productivity in the USA with technology spillovers 0.20

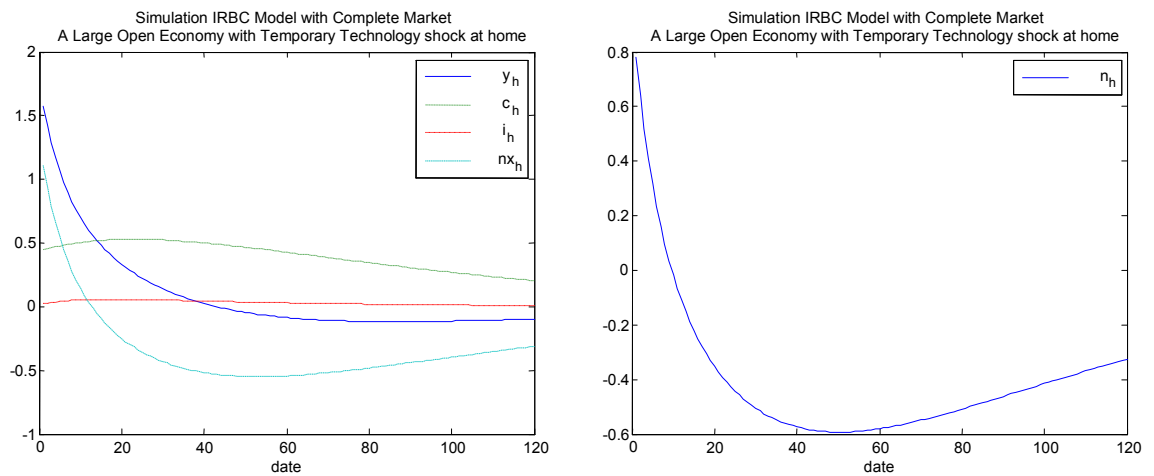


Figure 7.10 B A temporary shock in productivity in the USA with technology spillovers 0.15

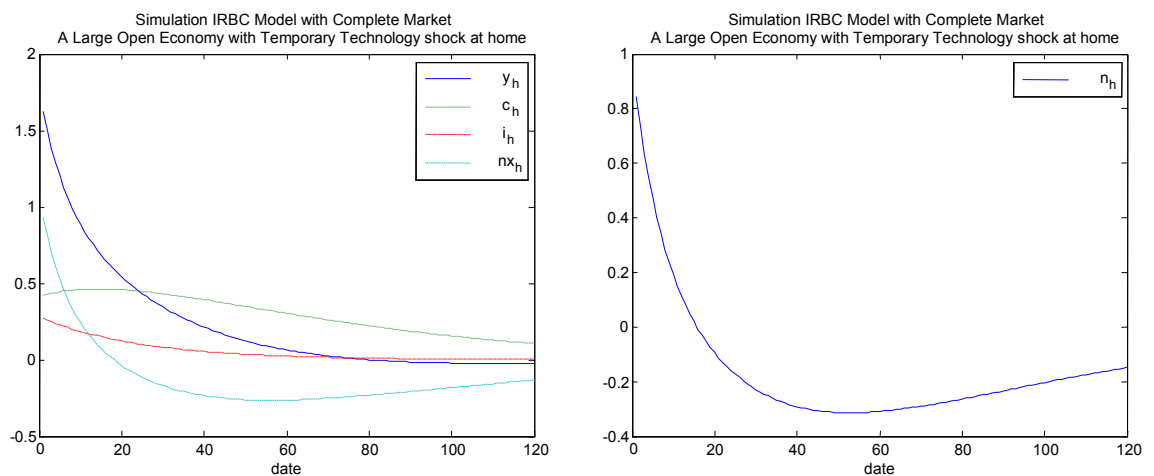


Figure 7.10 C A temporary shock in productivity in the USA with technology spillovers 0.10

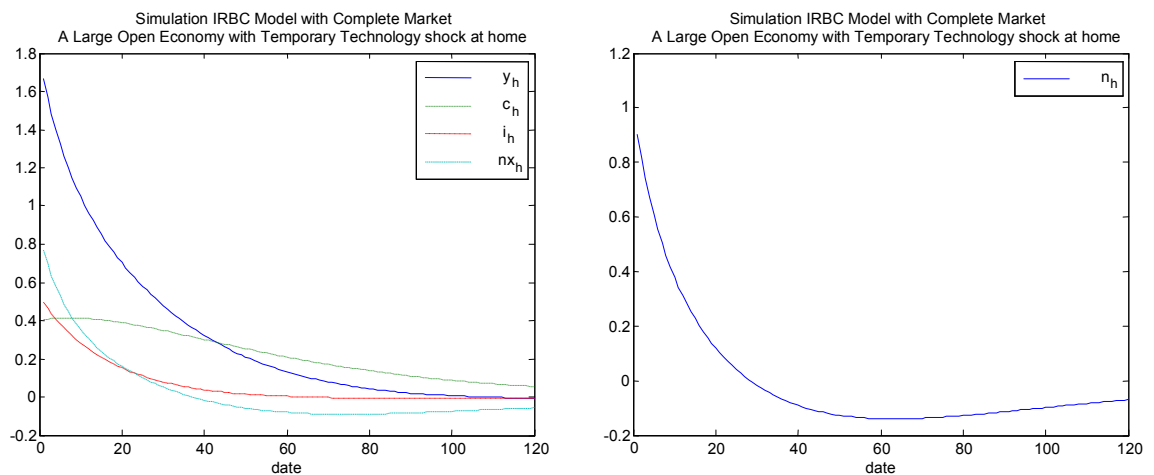


Figure 7.10 D A temporary shock in productivity in the USA with technology spillovers 0.05

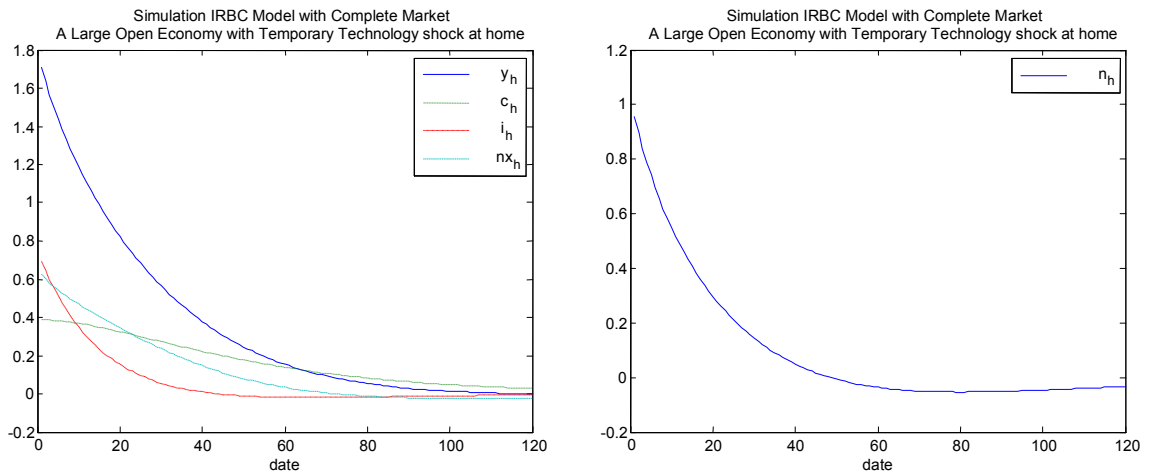
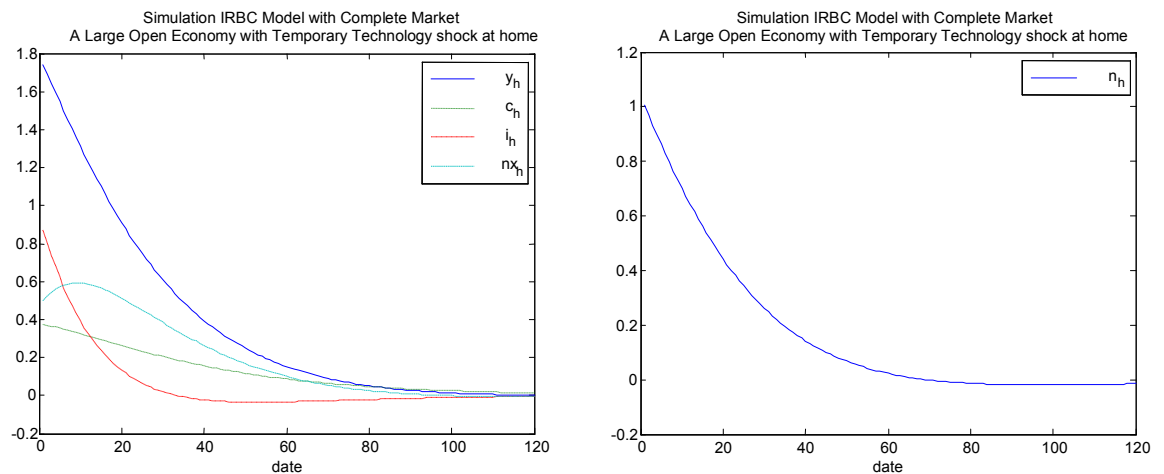


Figure 7.10 E A temporary shock in productivity in the USA without technology spillovers



The response of variables in the USA (home)

The smaller the technology spillovers are, the higher the positive impact investments and the output in the USA. All variables in the foreign country fall except for consumption because of the complete market assumption (not plotted in the picture).

Without technology spillovers as in figure 7.10 E, the large open economy takes advantage of an increase in temporary technology shock by increasing consumption, labour input, investment, output and net exports. Then they decline back to the steady state level because the shock does not last forever.

Looking back to the shock with spillovers in figure 7.9, consumption in both countries rises by approximately the same amount due to the market structure equalising the marginal utilities of both countries. The US trade is in surplus on the impact and it moves into deficit rapidly because the shock is not permanent. The large trade deficit is the main reason for the decrease in output in Thailand after the shock. There is evidence that this is return to zero but only slowly. Labour input in Thailand declines on the impact because the wealth effect is larger than the wage effect.

7.3 The shocks off the steady state in the open economy model

This section examines the reaction of the open economy model to the shocks off the steady state or a shock after another shock has already impacted. The general idea of how to shock off the steady state can be seen in chapter 5 section 5.7.5. The key difference between the shocks off the steady state in the closed and in the open economy model is that we can investigate the responses of the variables of both home country and foreign country. Using the shocks off steady state, we can also study the effect on the variables of fiscal policies such as an increase in government spending or a cut in taxes in the context of the open economy model.

7.3.1 Simulation US-Thailand: The permanent shocks off the steady state

The parameterization in the case of permanent shocks off the steady state is: technology persistence (ρ_A) is 0.99 the technology spillover parameters among two countries are assumed to be zero.

Figure 7.11 depicts a 1% positive permanent technology shock at period 1 followed in period 3 by 1 % positive permanent technology shock in the USA. The figure illustrates the responses to the shocks of output, consumption and investment for both the USA and Thailand. Figure 7.12 exhibits a 1% positive permanent technology shock at period 1 followed in period 3 by 1 % positive permanent technology shock in Thailand. The figure shows the responses to the shocks of output, consumption and investment for both countries.

Figure 7.11 A 1% increase in a permanent technology shock after another 1 % increase in a permanent technology shock in a large open economy (the USA)

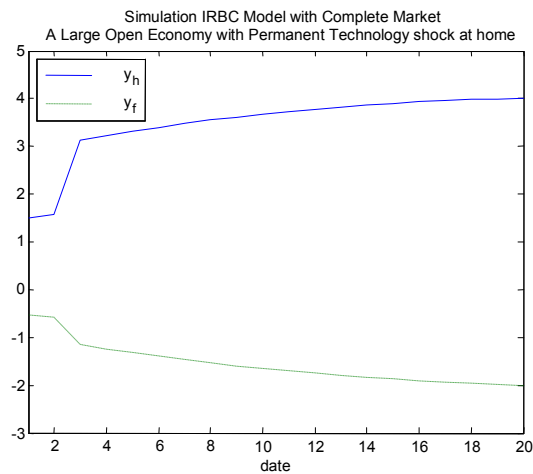


Figure 7.11 A output

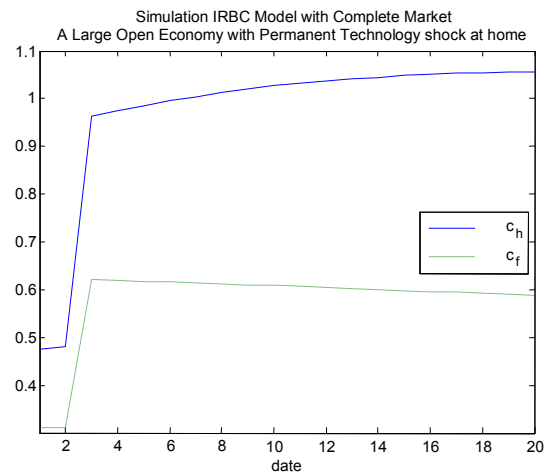


Figure 7.11 B consumption

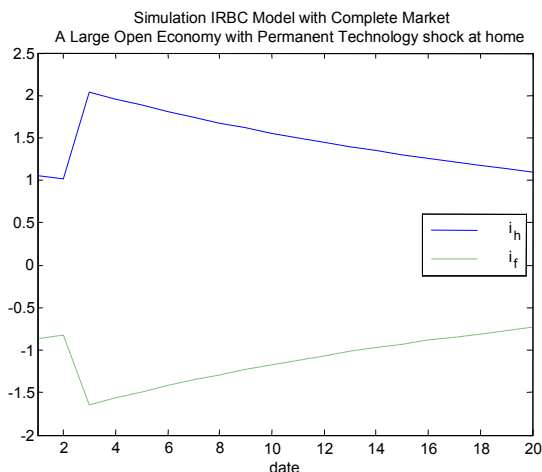


Figure 7.11 C investment

The home country (the USA) responds to a 1% permanent positive productivity shock by increasing output 1.4 good units on impact. At period 3, the output increases to 3 good units because of another 1% permanent positive productivity shock. However, the output in the foreign country decreases on the impact approximately 0.5 good units and further in the period 3. Consumption in both countries increases because of the complete market assumption in which the marginal utilities of consumption are equated across countries. Investment increases in the USA by approximately 1 good unit on the first impact and jumps to approximately 2 good units on the second impact. Investment in the foreign country

decreases on the impact because the capital flows into the country experiencing the shock (the USA). After the second shock, investment in the USA continues to fall toward the steady state while the investment in Thailand rises toward the steady state. The USA responds to a positive productivity shocks by increasing investment, consumption and output. Investment and output in Thailand decreases following the shocks in the USA productivity because the capital is shifted from the Thailand to the USA.

Turning to a permanent technology shock off the steady state originating in Thailand, figure 7.12 shows what happens in Thailand and in the USA. Output in the home country (Thailand) rises on the impact by roughly 2 good units and jump to approximately 4 good units after the second shock at period 3. Then the output continues to increase toward the new steady state. The increase in consumption in Thailand is very little, roughly 0.15 good units after the first impact and 0.3 good units after the second shock.

Figure 7.12 A 1% increase in permanent technology shock after another 1 % increase in permanent technology shock in a small open economy (Thailand)

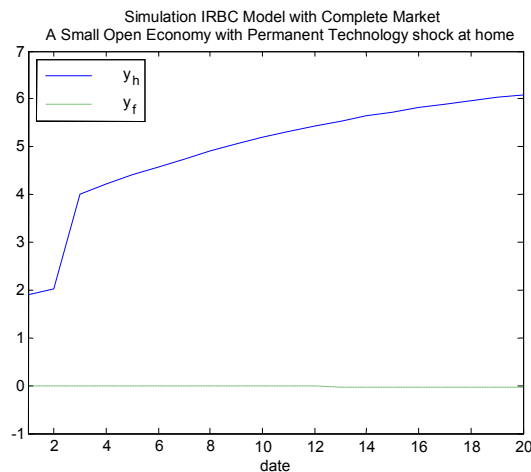


Figure 7.12 A output

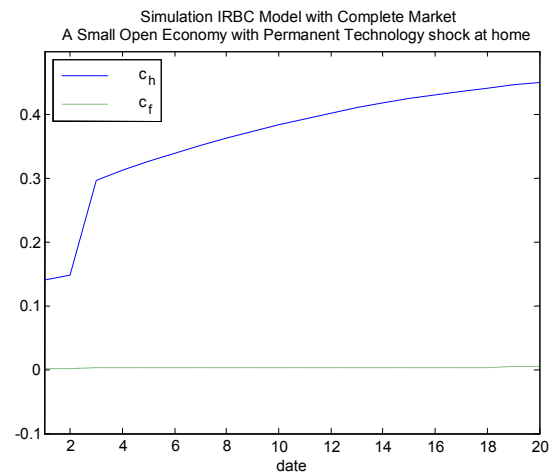


Figure 7.12 B consumption

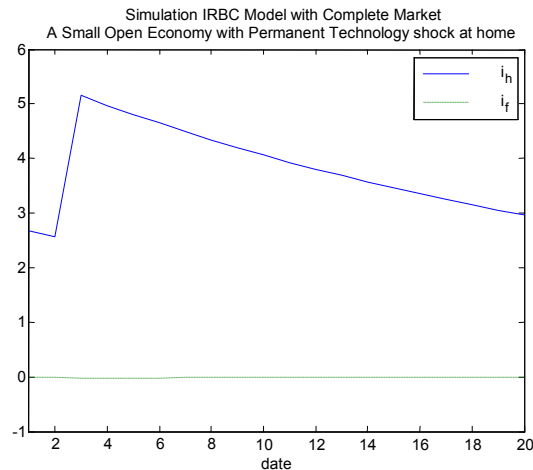


Figure 7.12 C investment

Investment in the home country (Thailand) increases approximately 2.8 good units after the first permanent technology shock and responds to 5 good units after the second shock then it declines back to its steady state level. The response of output, consumption and investment in the USA are negligible. Since Thailand is small compared to the USA, the effects of the Thai shock on the USA variables are approximately negligible. Apparently, the USA does not respond to Thailand shocks whereas Thailand particularly is dominated by the USA shocks.

7.3.2 Simulation US-Thailand: The permanent shocks off the steady state and fiscal policies

An alternative way to examine the effectiveness of the fiscal policies such as government spending and taxes can be achieved by simulating one shock after another. The model is first simulated by 1% negative permanent technology shock then using fiscal policies to minimize the cycle and return the output to the steady state.

Figure 7.13 A 1% decrease in permanent technology shock after another 22% increase in permanent government spending shock in a large open economy (the USA)

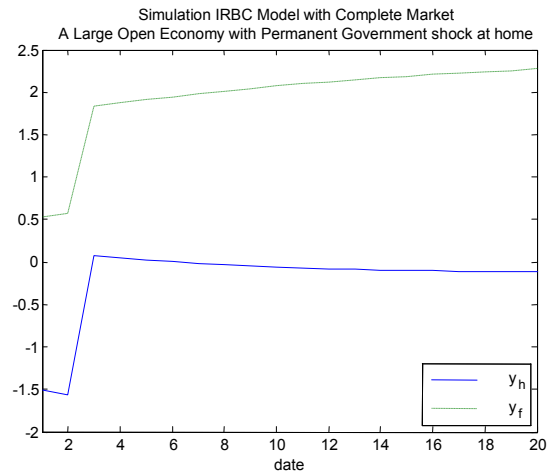


Figure 7.13 depicts a 1% decrease in the form of permanent technology shock followed in the following period by a 22 % increase in permanent government spending shock in the USA. Compared to the technology shock, the government spending shock has a small effect. It takes up to a 22% change in government spending to have the same effect to output as a 1% change in technology. In the USA, a 1% negative permanent technology shock decreases output to 1.5 unit goods where as a 22% increase in government spending again returns output to its original level. Consider the effect of shocks and the policy in the USA to the output in Thailand, the output increases to 0.5 good units after the permanent technology shock and jumps to approximately 1.8 good units right after the permanent government spending shock. The negative technology shock in the USA causes the movement of the capital to Thailand. Therefore, the investment and output in Thailand increase. The government spending shock generates the negative wealth effect, inducing people to work more in the shock specific location. The output in the USA increases mainly because the labour input rises. The output in Thailand increases since the government spending shock in the USA drives up the interest rate in Thailand. An increase in interest rate in Thailand induces Thai people to work more, raising the output in Thailand.

Figure 7.14 A 1% decrease in permanent technology shock after another 1.9% permanent cut in tax in a large open economy (the USA)

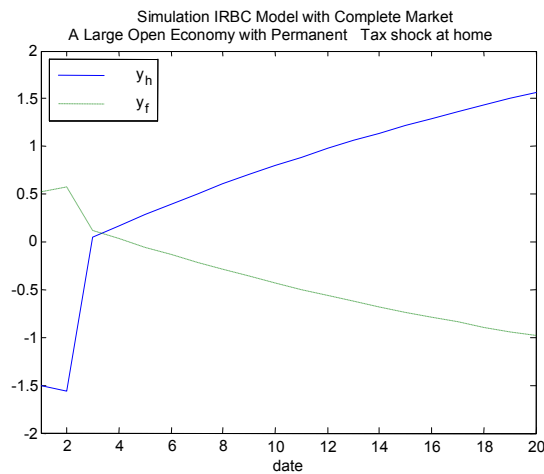
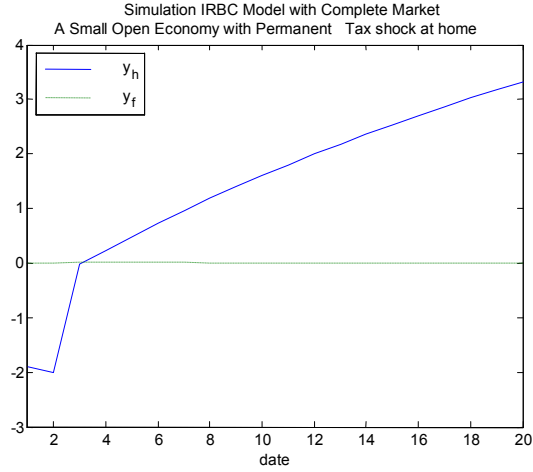


Figure 7.4 shows the impact of a 1 % decrease in permanent technology shock followed in the following period by 1.9% permanent cut in tax in the USA. Compared to the government spending shock as in figure 7.13, the tax shock has a larger effect on the variables. It requires only a 1.9% permanent cut in tax to return the output in the USA back to the steady state level. On impact of the first shock, output of Thailand increases to 0.5 good units since capital is shifted to Thailand. Nonetheless, the permanent cut in tax in the USA leads to a decline in output in Thailand.

Considering the effect of policy in Thailand on the USA, figure 7.15 illustrates a 1 % decrease in permanent technology shock followed in the following period by 1.9% permanent cut in tax in Thailand. The figure shows that in Thailand a 2.3% permanent cut in tax can return the output back to the steady state level. The responses of US variables to the shocks in Thailand are negligible.

It has been noticeable that whilst a fiscal correction can neutralize the impact of a productivity shock in the USA, it exacerbates the impact of the original shock in Thailand. Hence stabilizing fiscal policy may destabilize Thailand. The feedback effects of this destabilization on the US are negligible because the Thai economy is small. But the bigger trading partners such as Japan and the EU it is possible that the destabilizing effect could feedback to the USA making corrective fiscal policy more problematic.

Figure 7.15 A 1% decrease in permanent technology shock after another 2.3% permanent cut in tax in a small open economy (the Thailand)



7.3.3 Simulation US-Thailand: The temporary shocks off the steady state

Since the realistic shock is not permanent, this section investigates the shock after a temporary shock in the USA and Thailand.¹⁰ The parameterization in the case of temporary technology follows the equation (7.1). The calibrated parameters are again as below;

Technology shocks

US(home)-Thailand(foreign)

$$\begin{bmatrix} \log A_t^h \\ \log A_t^f \end{bmatrix} = \begin{bmatrix} 0.947 & 0.022 \\ 0.261 & 0.864 \end{bmatrix} \begin{bmatrix} \log A_{t-1}^h \\ \log A_{t-1}^f \end{bmatrix} + \begin{bmatrix} \varepsilon_t^h \\ \varepsilon_t^f \end{bmatrix}$$

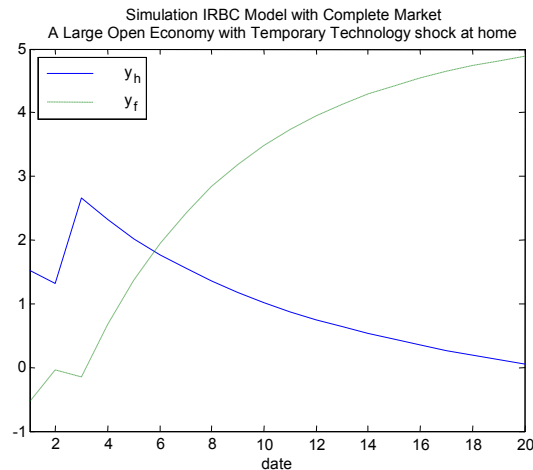
This section also examines the effect of fiscal policies on a temporary technology shock both in large and small open economy. The fiscal policies are assumed to be permanent for both countries. Government spending and tax persistence in each country are 1.

Figure 7.16 shows a 1% increase in temporary technology shock followed in the following period by a 1% temporary technology shock in the USA. After the shocks, the US output tends to decline to its steady state level. A 1% increases in temporary technology shock causes the US output rises on the impact to approximately 1.5 unit goods. After the second

¹⁰ The realistic shock is derived from the calibration using the data from the USA and Thailand in chapter 6. The realistic shock has a technology spillover both from the USA to Thailand and vice versa. However, the spillovers from the USA to Thailand are quite sizable, 26.1%.

temporary technology shock, the US output increases to approximately 2.8 unit goods. Then the path of output declines toward its steady state level.

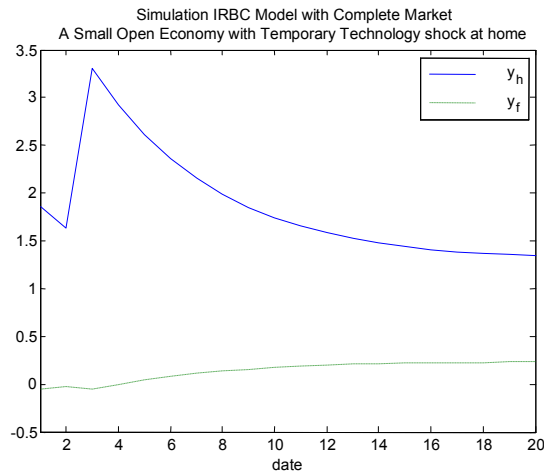
Figure 7.16 A 1% increase in temporary technology shock followed in the following period by 1% increase in temporary technology shock in a large open economy (the USA)



Output in Thailand declines 0.5 unit good on the impact of the first shock then increases to approximately the steady state level at period 2. Because of the technology spillovers from the USA to Thailand, output in Thailand increases during the transition period. At period 3 on the second impact, output in Thailand slightly declines below the steady state. Then it increases and will decline to the steady state later on.

Figure 7.17 shows a 1% increase in temporary technology shock followed in the following period by 1% temporary technology shock in Thailand. An increase in the temporary shock in Thailand raises output by 1.8 good units. Because the shock is not permanent, output in Thailand increases to take advantage of the positive shock and rapidly declines as shown in period 2. On the impact of the second shock at period 3, the output increases to 3.3 good units. The output of the USA slightly decreases and has a negative value immediately after the first impact. Then it increase and has a positive value because the shock originated in Thailand does not last forever.

Figure 7.17 A 1% increase in temporary technology shock followed in the following period by 1% increase in temporary technology shock in a small open economy (Thailand)



7.3.4 Simulation US-Thailand: The temporary shocks off the steady state and fiscal policies

Since the realistic shock is a temporary shock, this section examines the fiscal policy shocks after the temporary shock in the USA and Thailand.¹¹ The model is first simulated by a 1% negative temporary technology shock and then again using fiscal policies to minimize the cycle and return the output to the steady state.

Figure 7.18 shows that a 1% decrease in the temporary technology shock followed in the following period by an 18% increase in permanent government spending shock in the USA. A 1% decrease in temporary and permanent technology shock results in approximately the same level of decrease in output, specifically 1.5 good units. Comparing the size of government spending shocks to return the US output back to the steady state level, it requires 18% increase in permanent government spending after a 1% negative temporary technology shock where as it requires 22% increase in permanent government spending after a 1% negative permanent technology shock. After the second shock, the US output continues to increase. Thailand output increases immediately to 0.5 good units after the first shock. Then it declines to zero in period 2. The permanent government spending shock in

¹¹ The realistic shocks are derived from the data from the USA and Thailand as in equation (7.1).

the USA causes the output in both countries rise. However, the output in Thailand declines after the second shock as in figure 7.18.

Figure 7.18 A 1% decrease in temporary technology shock followed in the following period by 18% increase in permanent government spending shock in a large open economy (the USA)

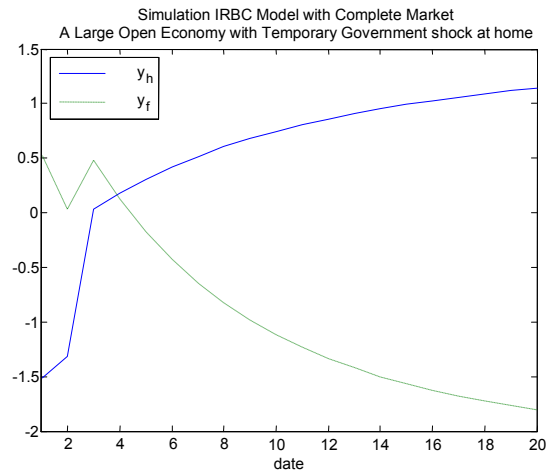


Figure 7.19 A 1% decrease in temporary technology shock followed in the following period by 1.2% permanent cut in tax in a large open economy (the USA)

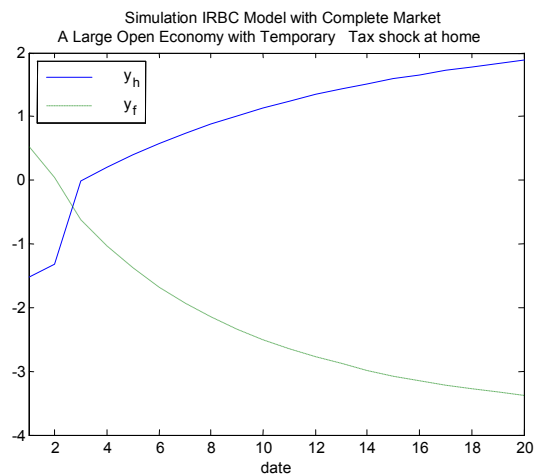


Figure 7.19 shows that a 1% decrease in a temporary technology shock followed in the following period by a 1.2% cut in tax in the USA. Unlike fiscal policy after the permanent technology shock, a 1.2% permanent cut in tax can minimize the cycle and return the US output back to the steady state level. Output in Thailand increases to 0.5 good units on the

impact of the first shock. Output in Thailand declines after the second shock and it moves toward a new steady state.

Figure 7.20 A 1% decrease in temporary technology shock followed in the following period by 1.6% permanent cut in tax in a small open economy (Thailand)

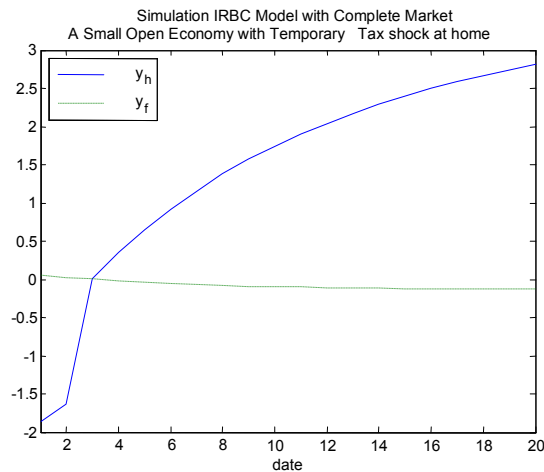


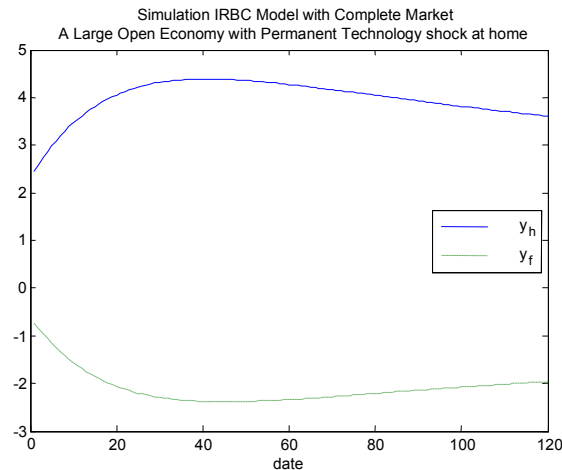
Figure 7.20 shows that a 1% decrease in temporary technology shock followed in the following period by 1.6% permanent cut in tax in the Thailand. A 1% decrease in temporary technology shock in Thailand decreases output by 1.9 good units. A 1.6% permanent cut in tax can return the output back to the steady state level. Then the Thai output increases and moves toward the new steady state level. On the impact of the first shock, the output of the USA barely increases. Immediately after the second shock, the output of the USA slightly declines and is below the steady state. However, the overall impacts on the US variables are negligible.

7.4 The multiple shocks in the open economy model

This section will present the results from the simulation of an open economy model with multiple shocks. The analysis of multiple shocks is crucial since the Thai and the US economy, in the reality, may not fact only one shock at a time. It could be the case that they face more than one shock simultaneously. The effects of multiple shocks are investigated by using technology, government spending and tax shocks at the same time both in Thailand and in the USA.

Simulation US-Thailand: The multiple shocks in the open economy model

Figure 7.21 A 1% increase in permanent technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a large open economy (the USA)



First the model is simulated by 1% increase in permanent technology shock, a 1% increase in the government spending shock and also a 1% permanent cut in tax as in figure 7.21. These shocks stimulate the economy, causing the output in the USA to rise by approximately 2.5 good units immediately after the impact. A 1% increase in permanent technology shock itself increases output by 1.5 good units. Output increases by approximately 0.08 good units immediately after a 1% increase in government spending. A 1% permanent cut in tax leads to an increase in output in the USA by approximately 0.9 units. The multiple shocks are essentially the combination of each shock, 2.5 good units coming from 1.5, 0.08 and 0.9 good units from the technology shock, government spending and tax shocks. The response of output in Thailand is also the summation of the responses from each shock. Output in Thailand declines by 0.8 good units and it exhibits a mirror reverse of the output in the USA.

Figure 7.22 shows the result of the simulation of 1% decrease in permanent technology shock and 1% increase in government spending shock and also 1% permanent cut in tax. This differs from the previous analysis in that the technology shock is now negative. A 1% decrease in permanent technology shock dominates the other two shocks. Therefore, the US output drops approximately 0.5 good units then continue to increase to the new steady state which is higher than the previous steady state. Output in Thailand increases by approximately 0.3 good units and it exhibits a mirror image of the output in the USA.

Figure 7.22 A 1% decrease in permanent technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a large open economy (the USA)

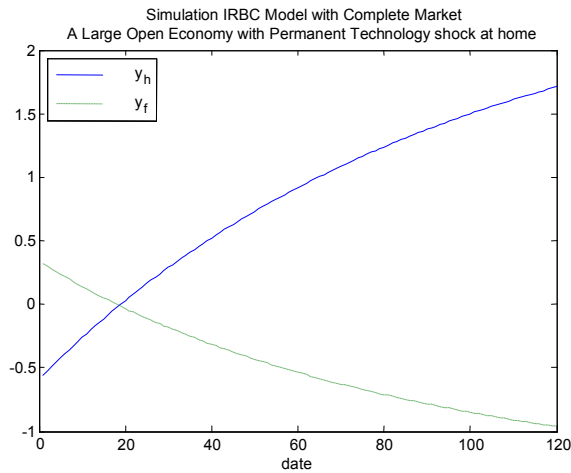
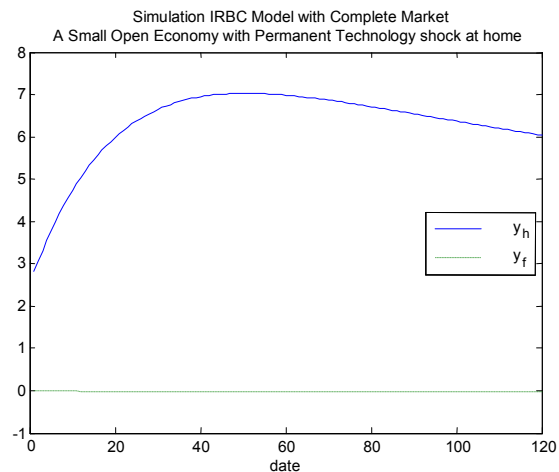


Figure 7.23 shows the shocks that stimulate the economy in Thailand. These shocks stimulate the economy, causing the output in Thailand to rise by approximately 2.8 good units immediately after the impact. A 1% increase in permanent technology shock itself increases output by 1.8 good units.

Figure 7.23 A 1% increase in permanent technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a small open economy (Thailand).



Output increases by approximately 0.08 good units immediately after a 1% increase in government spending. A 1% permanent cut in tax leads to an increase in output in the USA by approximately 0.9 units. Once again the multiple shocks are approximately the

summation of each shock. The effect of the multiple shocks from Thailand on the US output is clearly negligible.

Figure 7.24 shows that a 1% decrease in the permanent technology shock initially outweighs the government spending and tax shocks. Therefore, Thailand output decreases by 1 unit goods on the impact. The US output clearly does not respond to the multiple shock in Thailand. However, the Thai economy gradually approaches a new, higher equilibrium. The switchover point at which output exceeds the old equilibrium is in period 24. This means in the long run that the two fiscal shocks dominate the technology shock.

Figure 7.24 A 1% decrease in permanent technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a small open economy (Thailand)

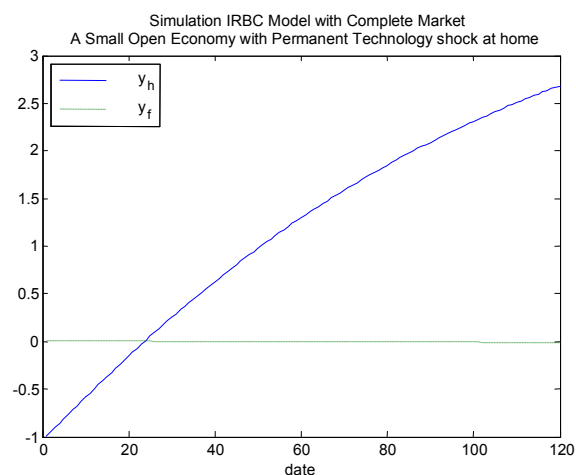


Figure 7.25 shows the multiples shocks, a 1% increase in temporary technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in the USA. A 1% positive temporary shock alone increases output by approximately 1.5 good units. The three shocks combined create the economic expansion. Overall the US output increases by approximately 2.5 unit goods which arises from the combination of these three shocks. The response of the US and Thailand output is similar to the response of the output to a 1% increase in temporary shock.

Figure 7.25 A 1% increase in temporary technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a large open economy (the USA)

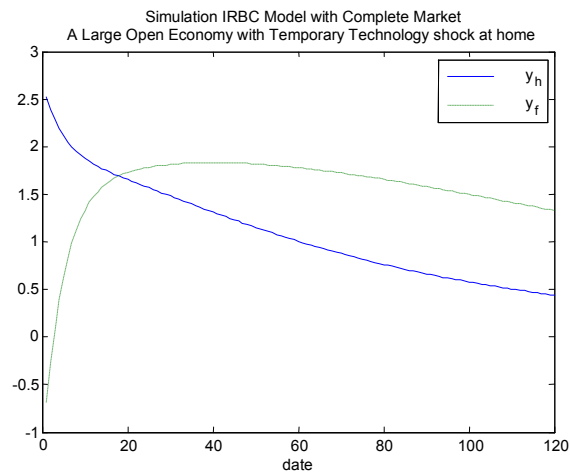
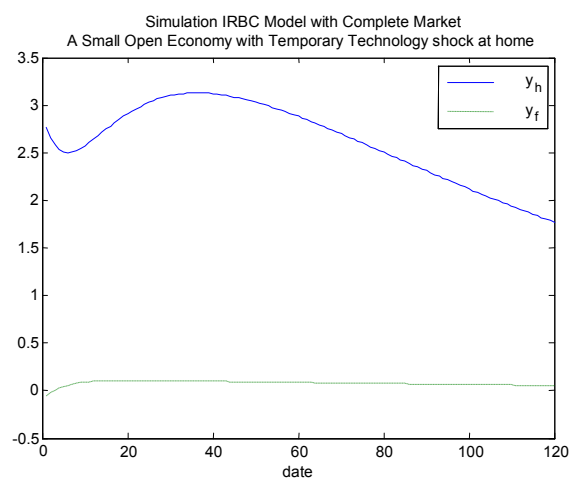


Figure 7.26 illustrates multiple temporary shocks in a small open economy. A 1% increase in the temporary technology shock itself increases output by 1.9 good units. Output increases by approximately 0.08 good units immediately after a 1% increase in government spending. A 1% permanent cut in tax leads to an increase in output in the USA by approximately 0.9 units. The overall responses on the impact are that the output in Thailand increases by approximately 2.7 unit goods which is slightly less than the summation of the individual shocks.

Figure 7.26 A 1% increase in temporary technology shock, a 1% positive government spending shock and a 1% permanent cut in tax in a small open economy (Thailand)



After the impact Thailand output declines because of the effect of the temporary technology shock. Later the output increases according to a 1% increase in government spending shock and a 1% cut in tax. The effect of the multiple shocks from Thailand on the US output is clearly negligible. However, on impact, the US output slightly declines mainly because of the effect of the temporary technology shock in Thailand.

7.5 Model Performance

As with the closed economy model, the model performance is evaluated by comparing characteristics of model moments with facts. To test the model performance, three business cycle properties, volatility, persistence and co-movement are calculated. However, the open economy model also allows us to study the international co-movement of variables between two countries. The sensitivity analysis is typically used to determine how sensitive the model is to changes in certain key parameters. By doing sensitivity analysis, it is possible for us to investigate the effect of some uncertainty associated to variables on the model simulation. This section examines the sensitivity by altering certain key parameters. By showing how the model behaviour responds to change in parameter values, sensitivity analysis is a useful tool to evaluate the model performance. The model is evaluated for only the USA and Thailand. In doing this we once more stress that we are not testing the validity of the model to replicate the economic cycle. This is the product of multiple shocks over different time periods. Rather we are evaluating the ability of a single shock to generate characteristics similar to those observed in the real world. In part the purpose is to determine the ability of a single shock, permanent or temporary, to replicate actual data characteristics.

The values of the model's parameters can largely influence how the quantitative properties of the model economy behave. In this study, by changing values of certain key parameters, four alternative model specifications are derived.

Scenario 1: the permanent technology shock, the parameterization in the case is: technology persistence (ρ_A) in each country is 0.995, and variance of technology innovation (ε_A) in each country is 1. The technology spillovers among the two countries are 0. The

contemporaneous correlation of technology innovation takes the realistic value, which is -0.186 for the USA and Thailand.¹² The purpose of the first scenario is to examine how the importance the persistence of the technology shock is.

Scenario 2: the realistic shock without technology spillovers which follows the equation 6.3. The realistic shock processes applied in this scenario represents the temporary shock processes approximated from the data. The parameters are the same as the case of realistic shock with 1% variance of technology innovation, but there are no technology spillovers between the two countries. Using the realistic shock, the effect of the spillovers is analyzed in the second scenario

Scenario 3: the realistic shocks, following the equation 6.3, with low and high variance of technology innovation. The third scenario is the case of realistic shock with low and high variance of technology innovation. The variance is set to 0.5% for the low and 1% for the high case. The effect of variance of technology innovation is examined in the third case.

Scenario 4: the sensitivity analysis. This section additionally examines how the change in persistence of fiscal shocks and the change in capital adjustment cost, variance of technology innovation, correlation of technology innovation and country size influence the model performance. The parameters are the same as the case of realistic shock with 1% variance of technology innovation with technology spillovers between the two countries.

7.5.1 A permanent technology shock; under scenario 1

Considering the model results under a permanent technology disturbance for Thailand and the USA reveals, in table 7.1 that the technology shock in Thailand generates higher volatility in Thailand. The relatively small amount of capital from Thailand flows into the US, leading to a high volatility in investment at 1.14 and output at 1.26 in the US. However, the shock in Thailand does not strongly affect the volatility of the US variables.

¹² The realistic shocks are derived from the data from the USA and Thailand as in equation (7.1).

The volatility of output is lower in the model than in the Thai data, but it is higher in the model than in the US data. There is a significant large labour response to the shock, leading to a high volatility of labour. It in turn causes a high volatility of investment and output. The model cannot capture the high volatility of consumption in Thailand. Consumption is the smoothest series in country specific shock due to the risk sharing condition

Table 7.1

The model economy with permanent technology shock for Thailand and the USA under scenario 1

Table 7.1 A The volatility, persistence and co-movement of Thailand and US variables; Permanent technology shock

		Model					
		Data		Shock in Thailand		Shock in US	
	Moments	Thailand	US	Thailand	US	Thailand	US
Volatility	y	3.10	0.74	2.75	1.26	2.31	1.79
	nx	3.32	0.21	1.93	0.01	0.80	0.66
	c/y	1.12	0.77	0.32	0.41	0.39	0.49
	i/y	3.34	3.32	6.69	1.14	6.36	5.81
	n/y	0.25	0.64	0.91	0.04	1.03	0.50
Persistence	y	0.91	0.94	0.91	0.90	0.91	0.91
	nx	0.91	0.92	0.91	0.91	0.93	0.93
	c	0.92	0.95	0.90	0.90	0.90	0.90
	i	0.94	0.95	0.90	0.90	0.90	0.90
	n	0.14	0.91	0.90	0.90	0.90	0.91
co-movement with respect to output	y	1.00	1.00	1.00	1.00	1.00	1.00
	nx	-0.94	-0.40	-0.57	-0.50	-0.42	-0.23
	c	0.99	0.82	-0.26	0.99	0.18	0.90
	i	0.94	0.96	0.91	0.99	0.94	0.89
	n	0.83	0.85	0.96	0.93	0.95	0.93

Notes: Volatility is measured by the percentage of standard deviation of the series. Volatility with respect to output is measured by the percentage of standard deviation of the series over the percentage of standard deviation of output. Persistence is measured by autocorrelation. Co-movement with respect to output is measured by the correlation coefficient of the series of output and another series.

The volatility of output is lower in the model than in the Thai data, but it is higher in the model than in the US data. There is a significant and large labour response to the shock, leading to a high volatility of labour. It in turn causes a high volatility of investment and output. The model cannot capture the high volatility of consumption in Thailand. Consumption is the smoothest series following country specific shock due to the risk sharing condition.

In contrast, the technology shock in the USA induces higher volatility in Thailand because a large amount of capital shifts from the USA to Thailand. The shock in the USA obviously affects the movement of the Thai variables. But it is interesting that a shock in the USA can have a greater impact in Thailand than on the USA itself. The model can capture the pattern of business cycle properties. It predicts the volatility of investment with respect to output is higher than the volatility of consumption with respect to output. Nevertheless, there is the difference in magnitude between the data and model prediction. The model can quite well replicate the empirically strong persistence both of the shocks originated in Thailand and in the USA. The model can mimic the negative sign of the correlation of net exports with respect to output in both Thailand and the USA. The model predicts the counter cyclical nature of consumption in Thailand for the shock originated in Thailand. The possible explanation is that Thailand is a small country and consumption slightly increases after the shock. The model produces a strongly pro-cyclical for the rest of the variables. Labour is more highly correlated with output because of fully risk sharing.

Table 7.1 B The international co-movement of Thailand and US variables; Permanent technology shock

		Model		
		Data	Shock in Thailand	Shock in US
International co-movement	y	-0.25	-0.52	-0.65
	nx	-0.45	-1.00	-1.00
	c	-0.33	0.96	0.58
	i	-0.51	-0.70	-0.94
	n	-0.50	-0.66	-0.96

Notes: International co-movement is measured by the correlation coefficient of the series of two countries.

Considering another important aspect of international business cycle, the international co-movement, the model shows that the international correlation of output is much lower than that of consumption while the empirical data evidences the opposite direction, as can be seen from table 7.1 B. This is the consumption-output anomaly in the international business cycle model and it happens under complete market structure. Table 7.1 B reports the negative international co-movement for all variables in empirical data. The model cannot replicate the negative international co-movement of consumption, but it produces the negative international co-movement for the rest of all variable. The model generates a strongly

positive international co-movement of consumption and negative in output because of the assumption of complete market structure.

7.5.2 The realistic technology shock without technology spillovers; under scenario 2

Table 7.2 illustrates the behaviour of the main macro-economic variables for Thailand and US in the case of a realistic technology disturbance where the variance of the technology innovation is 1 without international technology spillovers. The persistence of technology shock in Thailand and the USA is 0.864 and 0.947 respectively. This suggests that, the technology shocks are temporary.

Table 7.2

The model economy with realistic technology shock without technology spillovers for Thailand and the USA under scenario 2

Table 7.2 A The volatility, persistence and co-movement of Thailand and US variables

		Model					
		Data		Shock in Thailand		Shock in US	
	Moments	Thailand	US	Thailand	US	Thailand	US
Volatility	y	3.10	0.74	2.48	1.58	2.13	2.11
	nx	3.32	0.21	1.90	0.01	1.02	0.83
	c/y	1.12	0.77	0.28	0.39	0.25	0.32
	i/y	3.34	3.32	1.47	2.45	2.20	3.20
	n/y	0.25	0.64	0.87	0.21	0.74	0.60
Persistence	y	0.91	0.94	0.89	0.90	0.89	0.90
	nx	0.91	0.92	0.90	0.90	0.90	0.90
	c	0.92	0.95	0.90	0.90	0.90	0.90
	i	0.94	0.95	0.90	0.90	0.89	0.90
	n	0.14	0.91	0.89	0.90	0.89	0.90
co-movement with respect to output	y	1.00	1.00	1.00	1.00	1.00	1.00
	nx	-0.94	-0.40	0.92	0.76	0.86	0.86
	c	0.99	0.82	-0.16	0.99	0.47	0.91
	i	0.94	0.96	0.93	1.00	0.98	0.96
	n	0.83	0.85	0.97	0.99	0.97	0.98

Notes: Volatility is measured by the percentage of standard deviation of the series. Volatility with respect to output is measured by the percentage of standard deviation of the series over the percentage of standard deviation of output. Persistence is measured by autocorrelation. Co-movement with respect to output is measured by the correlation coefficient of the series of output and another series.

For the shock originating in Thailand, the volatilities of Thai variables are lower than that of Thai variables in the case of permanent technology shock. The shock in US has a strong

influence on the volatility of variables in Thailand, particularly output and investment. Similar to the case of a permanent shock, a temporary shock performs quite well in being able to replicate high persistence in the data. Since the shocks are temporary, they are not persistent enough to create the counter-cyclicality of trade within country. As with the permanent technology shock, the temporary shock predicts negative international correlations of output, net exports, investment and labour, but positive cross-country correlations of consumption as in table 7.2 B.

Table 7.2 B The international co-movement of Thailand and US variables

		Model		
		Data	Shock in Thailand	Shock in US
International co- movement	y	-0.25	-0.49	-0.51
	nx	-0.45	-1.00	-1.00
	c	-0.33	0.94	0.81
	i	-0.51	-0.26	-0.17
	n	-0.50	-0.69	-0.81

Notes: International co-movement is measured by the correlation coefficient of the series of two countries.

7.5.3 The realistic technology shock with altering variance of technology innovation; under scenario 3

The technology shock process in this case is taken from the section 6.6. The variances of technology innovation are set to 0.5% and 1% to analyse how a change in value of the variance can alter the characteristics of the business cycle.¹³ This section also allows us to investigate how important the international technology spillovers are to the business cycle regularities. Table 7.3 depicts a number of characteristics of behaviour of the main macro-economic variables for Thailand and the USA.

¹³The model with realistic variance of technology innovations following section 6.6 fails to replicate the empirical data because the variances of technology innovation are too low (does not show) for both countries. We also generate the variance of technology innovation by randomly select from the normal distribution with mean zero and variance one. The mean of the variance is too low and closed to the mean. It is fails to replicate the facts using a very low variance of technology innovation. To investigate the effect of a change of the real variance, the variance is assumed to be 0.5% in this section and 1.0% for section 6.8.3. BKK (1992) used the variance of technology for the USA and the aggregate EU 0.7 and so does Baxter and Crucini (1995).

The model can replicate the facts in the sense that variables in Thailand in general have more volatility than those in the USA. Clearly, the higher the variance of technology shock, the higher volatility of each variable. The volatility of variables depends on their filtered spectral density. The filtered spectral density is a function of the variance of technology shock. For those reasons the higher variance of technology shock causes a higher volatility.

The model can again replicate the pattern of business cycle properties. Namely, it generates a volatility of investment with respect to output higher than the volatility of consumption with respect to output. As with the permanent technology shock case, the model poorly mimics the high volatility of consumption in Thailand.

Table 7.3

The model economy with realistic technology shock with variance 0.5% and 1% for Thailand and the USA under scenario 3

Table 7.3 A The volatility, persistence and co-movement of Thailand and US variables

		Model									
		Shock in Thailand						Shock in US			
		Data		v=0.5		v=1		v=0.5		v=1	
	Moments	Th	US	Th	US	Th	US	Th	US	Th	US
Volatility	y	3.10	0.74	2.20	0.97	2.99	1.37	1.87	1.32	2.55	1.81
	nx	3.32	0.21	3.10	0.02	4.19	0.03	1.49	1.22	2.01	1.65
	c/y	1.12	0.77	0.26	0.32	0.29	0.34	0.38	0.41	0.42	0.45
	i/y	3.34	3.32	3.30	1.42	3.63	1.45	3.08	0.99	3.41	1.03
	n/y	0.25	0.64	0.81	0.11	0.79	0.11	1.02	0.59	0.97	0.57
Persistence	y	0.91	0.94	0.90	0.89	0.90	0.89	0.90	0.89	0.90	0.89
	nx	0.91	0.92	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89
	c	0.92	0.95	0.91	0.90	0.91	0.90	0.92	0.90	0.92	0.90
	i	0.94	0.95	0.91	0.89	0.91	0.89	0.92	0.89	0.92	0.89
	n	0.14	0.91	0.89	0.87	0.89	0.89	0.89	0.89	0.89	0.89
co-movement with respect to output	y	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	nx	-0.94	-0.40	0.73	0.94	0.66	0.94	0.72	0.99	0.64	0.99
	c	0.99	0.82	0.10	0.99	0.19	0.99	0.52	0.81	0.56	0.81
	i	0.94	0.96	-0.06	0.98	0.05	0.99	0.64	-0.97	0.65	-0.97
	n	0.83	0.85	0.97	0.92	0.96	0.92	0.97	0.94	0.96	0.93

Notes: Volatility is measured by the percentage of standard deviation of the series. Volatility with respect to output is measured by the percentage of standard deviation of the series over the percentage of standard deviation of output. Persistence is measured by autocorrelation. Co-movement with respect to output is measured by the correlation coefficient of the series of output and another series. Th stands for Thailand; V stands for variance of technology innovation.

The shock originating in Thailand has only a small impact on the volatility of the US variables. The possible explanations are that the country specific shock is small and the technology spillover from Thailand to the USA is almost negligible. On the other hand, a shock in the USA produces a sufficiently high volatility in both countries. An increase in the variance of technology innovation has a very small effect on the persistence and co-movement with respect to output within the country. In contrast to a permanent shock, a realistic or a temporary shock leads to pro-cyclicality of trade in all cases. The model needs a very high persistence of technology shock to produce counter-cyclicality of trade. Technology spillovers between the two countries play an important role in depressing the persistence of the shock. The shock originated in the USA leads to a counter-cyclical movement of investment in the USA because of a strong technology spillover from the USA to Thailand. To create a pro-cyclical nature of investment in Thailand, the model needs a high volatility of technology innovation. The change in variance does not substantially alter the international co-movements as in table 7.3 B. It is striking that the realistic or temporary shock with technology spillovers leads to the positive cross-correlation of investment when the shock takes place at the small open economy. The possible explanation could be the technology spillovers from Thailand to the USA are very small.

Table 7.3 B The international co-movement of Thailand and US variables

			Model			
			Shock in	Shock in US	Shock in	Shock in
			Thailand		Thailand	US
Data		v=0.5		v=1		
International co-movement	y	-0.25	-0.47	-0.61	-0.36	-0.52
	nx	-0.45	-1.00	-1.00	-1.00	-1.00
	c	-0.33	0.91	0.83	0.92	0.87
	i	-0.51	0.83	-0.43	0.84	-0.53
	n	-0.50	-0.90	-0.95	-0.86	-0.93

Notes: International co-movement is measured by the correlation coefficient of the series of two countries. V stands for variance of technology innovation

7.5.4 The effect of the technology innovation correlation; under scenario 4

The correlation of technology innovation for the USA and Thailand in the realistic case is - 0.186 as shown in equation 6.23. To investigate how important the correlation of technology innovation is to the business cycle, the realistic shock with spillovers and a 1%

variance of technology innovation is used in this subsection.¹⁴ With these specifications, the value of technology innovation correlation is increased from 0 to 0.9. Table 6.6 reports only the international correlations of the key variables because the correlation of technology innovation does not substantially alter the variances, persistence and co-movement within a country.

The table 7.4 shows that the higher the correlation of technology innovation, the larger the value of the co-movement. The model still exhibits a negative correlation of trade between two countries. The output correlation is low in the model for a 0 correlation, as is in the data. However, the consumption correlation is too high in the model compared to the data. The shock in the USA induces a negative cross-correlation of investment while the shock in Thailand creates the opposite. The model exhibits a negative cross-correlation for the trade variable and labour input.

Table 7.4
The model economy with varying correlation of technology innovation for Thailand and US

	Data	Model							
		Shock in Thailand				Shock in US			
		correlation of technology innovation				correlation of technology innovation			
		0	0.3	0.6	0.9	0	0.3	0.6	0.9
y	-0.25	-0.23	-0.01	0.26	0.60	-0.40	-0.18	0.11	0.49
c	-0.45	0.92	0.95	0.97	0.99	0.90	0.94	0.96	0.99
i	-0.33	0.86	0.89	0.92	0.97	-0.61	-0.73	-0.84	-0.93
nx	-0.51	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
n	-0.50	-0.81	-0.71	-0.55	-0.22	-0.92	-0.87	-0.78	-0.57

Notes: The figures are the international co-movement for Thailand and the USA. International co-movement is measured by the correlation coefficient of the series of two countries.

7.5.5 The effect of the fiscal shocks; under scenario 4

This section analyses the importance of the persistence of government spending and tax shocks to the Thai business cycle, under the international real business cycle model. Expansionary policies are considered to facilitate the comparison. The international implications of those shocks are shown in table 7.5 and 7.6 which reports the impact effect in good units.

¹⁴ The realistic shocks are derived from the data from the USA and Thailand as in equation (7.1).

Government spending shock

Table 7.5
The response of Thai's variables for a 1% increase in government spending

	1% increase in government spending				
	in Thailand				in US
	rho=0	rho=0.8	rho=0.9	rho=1	rho=1
y	1.500*	2.000*	2.300*	4.150*	0.055
c	-0.850*	-1.150*	-1.400*	-2.500*	-0.043
i	-6.100*	-5.200*	-4.200*	0.730*	0.018
nx	-0.086	-0.086	-0.086	-0.086	0.030
n	5.500*	2.590*	3.500*	6.100*	0.008

* denotes a basis point, rho denotes the persistence of the shock

As in table 7.5, the responses are more sensitive when the persistence is high. A 1% increase in government spending in Thailand is not substantial in its impact on Thai variables.¹⁵ In fact, they are approximately negligible. However, if we look at them in detail, we can see the pattern of a wealth effect. After a purely temporary shock, consumption falls by 0.850 basis points, but by 2.5 basis points after a permanent shock. Output increases when the economy faces shocks. An increase in labour input is the main factor behind the higher output. The increase in government spending creates a negative wealth shock, falling consumption and rising labour input. The lower the persistence is, the less the negative wealth effect. Labour input rises by 6.1 basis points on the permanent impact and by 5.5 basis points on the purely impact temporary shock. The high labour input is the main reason for the positive investment after a permanent shock. Net exports are negative on the impact and not sensitive to persistence. The shock in the USA creates a high world interest rate, affecting consumption, labour input, investment and output in Thailand. Consumption in Thailand decreases by 0.043 and labour increases by 0.008 because of the full diversifying of risk between two countries.

Turning to another example of fiscal policy, a decrease in taxation, table 7.6 suggests that a cut in taxes leads to a positive wealth effect which induces more consumption. The agents in

¹⁵ Corsetti and Muller (2006) examine the fiscal expansions in the IRBC model for Australia, CA, the UK and the USA. They find that the fiscal expansions have no effect to the trade balance if the economy is not very open to trade and if the shocks are not too persistent.

Thailand consume more to take advantage of a cut in tax. However, consumption is not sensitive to the persistence until the persistence is really closed to 1. Consumption rises to 0.2 under the permanent shock and 0.14 under the temporary shock ($\rho=0.9, 0.8$, and 0.0).

Tax shock

Table 7.6
The response of Thai's variables for a 1% cut in taxes

	1% cut in taxes				
	in Thailand				in US
	$\rho=0$	$\rho=0.8$	$\rho=0.9$	$\rho=1$	$\rho=1$
y	0.900	0.900	0.900	0.900	-0.250
c	0.140	0.140	0.140	0.200	0.200
i	0.005*	0.500	0.900	3.050	-1.200
nx	0.700	0.270	-0.150	-2.250	0.750
n	13.000	1.000	1.370	1.350	-0.400

* denotes a basis point, ρ denotes the persistence of the shock

The tax cut in addition increases real wage and interest rates, inducing agents to work more. The increase in labour supply in Thailand is due to the large substitution effects. The substitution effects are higher when the shock is less persistent. With the pure temporary shock, labour rises by 13 due to a large substitution effect, but increases only 1.35 for the permanent shock due to the wealth effect. Output and investment increase mainly because labour input rises. Investment is very sensitive to the high persistence which is really close to 1. Output is not sensitive to the persistence of the shock. A trade deficit for Thailand occurs when the persistence approaches unity. Table 7.6 also reports the effect of a permanent cut in tax in the USA on the Thai business cycle. Under complete risk sharing, the wealth effect dominates the substitution effects in Thailand, leading to an increase in consumption by 0.2 basis points and a decrease in labour supply by 0.4. Given the decline in labour supply in Thailand, investment declines and so does output. Trade is in surplus in Thailand after the shock because in the US trade is deficit.

Compared to the government spending shock, the tax shock induces more responses in the Thai business cycle. However, the responses of the business cycle to the tax shock are less than that of a technology shock. The explanation is that the shock given by government

spending generates only a wealth effect. A government spending shock alone cannot explain the business cycle due to the creation of the negative wealth effect and the responses to the shock are small.

7.5.6 The effect of the capital adjustment cost; under scenario 4

The final issue to be discussed is how important the capital adjustment cost function is to the international business cycle behaviors. To consider how the Thai variables respond to changes in the marginal adjustment cost function, the model with temporary shocks and technology spillovers between the USA and Thailand is used in this section.¹⁶

Table 7.7 The behaviour of Thai key variables with the alternate values of the marginal adjustment cost function.

Table 7.7 A The volatility, persistence and co-movement of Thailand variables.

	Moment	Thai data	Model							
			Shock in Thailand					Shock in US		
			eta=5	eta=15	eta=25	eta=50	eta=160	eta=5	eta=15	eta=25
Volatility	y	3.10	2.96	2.99	3.02	3.10	3.37	2.42	2.55	2.61
	nx	3.32	3.78	4.19	4.36	4.60	6.07	2.00	2.01	2.00
	c/y	1.12	0.31	0.29	0.28	0.26	0.24	0.46	0.42	0.41
	i/y	3.34	2.30	3.63	4.29	5.30	8.12	2.19	3.41	4.08
	n/y	0.25	0.83	0.79	0.78	0.77	0.77	0.90	0.97	1.02
Persistence	y	0.91	0.90	0.90	0.91	0.91	0.92	0.90	0.90	0.90
	nx	0.91	0.89	0.89	0.90	0.90	0.92	0.89	0.89	0.90
	c	0.92	0.91	0.91	0.91	0.92	0.92	0.92	0.92	0.92
	i	0.94	0.90	0.91	0.91	0.92	0.91	0.92	0.92	0.92
	n	0.14	0.89	0.89	0.90	0.90	0.91	0.99	0.89	0.89
co-movement with respect to output	y	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	nx	-0.94	0.81	0.66	0.56	0.38	-0.29	0.78	0.64	0.55
	c	0.99	0.07	0.19	0.24	0.28	0.31	0.60	0.56	0.54
	i	0.94	-0.08	0.05	0.14	0.30	0.51	0.54	0.65	0.70
	n	0.83	0.95	0.96	0.96	0.96	0.97	0.94	0.96	0.96

Notes: Volatility is measured by the percentage of standard deviation of the series. Volatility with respect to output is measured by the percentage of standard deviation of the series over the percentage of standard deviation of output. Persistence is measured by autocorrelation. Co-movement with respect to output is measured by the correlation coefficient of the series of output and another series. eta stands for the marginal adjustment cost function

¹⁶ The capital adjustments cost, as in equation (6.8) and (6.9) work as a friction of capital mobility between two countries. Their purpose is to slow down the response of investment to location-specific shocks.

The model faces an unanticipated increase of 1% in a technology shock in Thailand and the USA with 1% of the variances of technology innovation for both countries. Table 7.7 shows that an increase in the marginal adjustment cost leads to an increase in the volatility of all Thai variables, except for consumption and labour input. The model captures quite well the persistence of Thai data. The shock taking place in the USA induces a lower volatility of output, trade and investment in Thailand than a shock originating in Thailand. To exactly match the volatility of output (3.10) in Thailand requires the marginal adjustment cost to increase up to 50. The persistence of all variables is strong corresponding to any value of marginal adjustment cost. The marginal adjustment costs do not have any impact on the persistence of all variables. The characteristics of the trade balance are also sensitive with regard to the marginal adjustment cost function. The model fails to explain the negative correlation of trade and output with a low value of the marginal adjustment cost.

A value of 160 of the marginal adjustment cost can produce a counter cyclical movement of trade in Thailand as depicted on table 7.7. The shock originating in the USA produces a stronger pro-cyclical behaviour for consumption and investment in Thailand than the shock originating in Thailand itself. The higher the marginal adjustment cost, the higher the degree of co-movement of investment.

Table 7.7 B The international co-movement of Thailand variables

			Model					
			Shock in Thailand			Shock in US		
	Moment	Data	eta=5	eta=15	eta=25	eta=5	eta=15	eta=25
International co- movement	y	-0.25	-0.36	-0.36	-0.36	-0.57	-0.52	-0.49
	nx	-0.45	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
	c	-0.33	0.94	0.92	0.91	0.89	0.87	0.86
	i	-0.51	0.92	0.84	0.77	-0.16	-0.53	-0.77
	n	-0.50	-0.95	-0.86	-0.85	-0.99	-0.93	-0.90

Notes: International co-movement is measured by the correlation coefficient of the series of two countries.

The model cannot explain the negative cross correlation of consumption. The high value of the marginal adjustment cost causes the weak cross correlation of investment and

consumption. The cross correlation of output and trade does not rely heavily on the value of the marginal adjustment cost.

7.6 Conclusions

The starting point of investigation is the impulse responses. The impulse response is done for permanent and realistic shock. Three forcing processes, technology, government spending and taxes, are examined. The technology shock in Thailand increases investment, consumption and output in Thailand. The investment and output in the USA declines on the impact because capital is shifted to Thailand. An increase in consumption is negligible in both countries because of the small country specific shock location. The responses in Thailand are significant if the shock is originated in the USA. Consumption noticeably increases in both countries in this case. The realistic shock based on the calibration of the USA and Thailand is investigated as a temporary shock with technology spillovers. The realistic shock results in a trade surplus for home country. The technology shock must have a very high persistence to induce a trade deficit in home country. Because of the effect of technology spillovers, investment in the USA falls while the labour input rises on the impact of the realistic shock originated in the USA. The effects of persistence of government spending and taxation on Thai business cycle are also investigated. Government spending creates a pure wealth effects while tax shock generates both wealth and substitution effect. The higher the persistence is, the higher the wealth effect is.

The shocks off the steady state are discussed in the context of the open economy model. A 1% increase in a permanent technology shock in the USA increases output in the USA approximately 1.5 good units while decreases output in Thailand 0.5 good units. A 1% increase in permanent technology shock in Thailand increases output in Thailand approximately by 2 good units whereas the output in the USA does not respond to the shock at all. The USA requires a 22% increase in permanent government spending shock to offset a 1% negative technology shock. A permanent government spending shock in Thailand has a very small effect to output in Thailand. It would require a huge amount of government spending shock in Thailand to return the output to the steady state after a 1% negative permanent technology shock in Thailand. A 1.9% permanent cut in tax in the USA equalizes

a 1% negative permanent technology shock. After a 1% negative permanent technology shock, a 2.3% permanent cut in tax can return output of Thailand to the steady state level. A 1% increases in temporary technology shock in the USA causes the US output rises on approximately 1.5 good units while the output in Thailand decreases by 0.5 good units. Comparing to the size of fiscal policies to equalize a 1% permanent negative technology shock, the size of the fiscal policies to offset a 1% negative temporary technology shock are smaller. The multiple shocks are in fact the combination of each shock. The responses of the output due to the multiple shocks are essentially the combination of the output from each response of the output from each shock.

There are two important points concluding from the shocks off the steady state analysis. Firstly, it is obvious from this analysis that large countries do not respond to small country shocks. In the reality, for instant, the Asian crisis in 1997 which originated from Thailand did not have any effect to the US economy. During the crisis, output in Thailand drastically dropped. However, output in the USA appeared not to be affected. Secondly, small countries, particular openness, are dominated by large country shocks.

Comparing the predictions of the model with the data, the model is evaluated for the permanent technology shock, realistic technology shock with and without technology spillovers, realistic technology shock with technology spillovers with altering in variance of technology innovation. The realistic technology shock with technology spillover with high variance (1%) captures the Thai stylized facts better than others. The model requires a high variance of technology innovation (1%) to explain the Thai facts. Even though, the model cannot exactly match the data, the model can capture the business cycle pattern of volatility. It performs quite well to replicate the persistence. The shock in the USA can explain the co-movement in Thailand better than the shock originates in Thailand itself. However, the model fails to explain the counter-cyclical behavior of trade in Thailand and in the USA. In term of international co-movement, the model performs poorly to exactly match the data. It only can match the sign of some variables. It fails to mimic the negative cross correlation of consumption of the USA and Thailand. The shock in the USA can capture the negative sign of investment cross correlation, but not the shock originated in Thailand. The model

predicts that the cross correlation of consumption is higher than that of output. However, the fact shows another way around.

Chapter 8

Filter comparison

8.1 Introduction

The macroeconomics time series are often filtered prior to analysis to eliminate elements such as trends and seasonal. These features are a nuisance from the point of view of the theoretical models for example the real business cycle models. The models try to explain the cyclical component of data, not the trend or slow moving component of the data. The objective of the filter is to extract the trends from the time series. Chapter 3 describes how data filters work and the theoretical rationale that lies behind them both in time and frequency domain. Using Thai data, it focuses mainly on the Hodrick-Prescott and the band pass filter by Baxter and King. In this chapter, several methods of filtering a time series are presented with the application to Thai data. It explores the filter comparison of the first difference, the centered moving average¹, the Hodrick-Prescott, the band pass by Baxter and King, and the band pass by Christiano and Fitzgerald. Further, since model evaluation involves comparison of model moments with moments computed from the data, it is crucial to apply these filters both to the Thai data and to economic model. In part the purpose of this section is to establish that the filter we have used is the appropriate one. This chapter is organized as follows. Section 8.2 portrays the effects of alternative filters on Thai macroeconomic time series. The moment implications, both the moment computed from the data and the model moments, are discussed in section 8.3. This section focuses on three important characteristics of time series, volatility, persistence and co-movement of Thai key macroeconomics time series. The moment comparison is also discussed in section 8.3.3. Section 8.4 of this chapter concludes.

8.2 The effects of alternative filters on Thai data

A standard method of removing a linear trend from the cycles is to apply a log function to macroeconomic time series. However, this procedure cannot remove a unit root from the time series. The linear detrending is undesirable for most macroeconomics time series for this reason. The first difference can separate the cyclical component from a

¹ The center moving average in this chapter refers to the 3 points equally weighted moving average.

time series and also can remove a unit root component. Nonetheless, the major problem involved with the first difference is a phase shift in the filtered series. The first difference filter cannot also separate the cyclical component within a specific band frequency. The equally weighted moving average is symmetric and can remove a stochastic trend. Like the first difference filter, it cannot isolate the cyclical component within a particular band frequency.² The Hodrick-Prescott filter removes the periodicities higher than 32 quarters or a slow moving component while retaining those lower than 32 quarters. This filter is basically the high pass filter because the high frequency component can pass through. It can remove unit roots and is symmetric.³ The band pass filter retains a periodic component of a time series within a particular frequency band. The ideal filter follows Burn and Mitchell's business cycle definition where the business cycles are the components with more than 6 quarters and less than 32 quarters.⁴ The main objective of the band pass filter both by Baxter-King and Christiano-Fitzgerald is to capture the ideal band pass. However, the band pass filter introduced by Baxter-King sacrifices the beginning and the end of the data to retain the major business cycle. For quarterly data, the truncation points are 12. Therefore, we lose three years of data at each end of the plots.⁵

This chapter investigates the Thai business cycle during 1993q1 to 2006q3 using the same set of data from chapter 5.⁶ The aim is to study the business cycle of the filtered time series using the first difference filter, the equally weighted moving average filter, the Hodrick-Prescott filter, the band pass filter by Baxter and King and the band pass filter by Christiano and Fitzgerald. This chapter essentially examines how the estimation of the Thai business cycle depends upon the filters being used. It is crucial to apply the filters both to the data and to the business cycle model because the model is evaluated by comparing the Thai business cycle properties and the model predictions. The

² The band frequency is the frequency between 6 and 32 quarters according to Burn and Mitchell's definition.

³ The filter is symmetric when the number of its lags and the leads are the same. Therefore, there is no phase shift in the filtered series.

⁴ The transfer function of the ideal filter has two cutoff frequencies, $2\pi/32$ and $2\pi/6$. The frequency response function of the ideal filter is 1 in the period between 6 and 32 quarters and 0 elsewhere.

⁵ See also Baxter and King (1990) for the details.

⁶ The data set is from National Economic and Social Development Board of Thailand except for real interest rate and labour. The real interest rate is obtained from IMF (International Finance Statistic). The labour input for quarterly frequency provided by National Statistical Office ranges from 1998q1 to 2006q3 due to the short and limited data of Thailand.

comparison of the model moments and the moments computed from the data is discussed in section 8.3.

8.2.1 Inspecting the results for Thai GDP

This section compares the results of applying five filters to the logarithm of Thai real GDP per capita. Throughout the six graphs, the cyclical components of several filters, the Hodrick-Prescott filter is used as the benchmark because it is popular and widely used for measuring business cycles.

Figure 8.1 A, the upper panel, shows the quarterly real GDP of Thailand with its trend during the observation period. Thai Real GDP per person grows over time but this normal growth is sometimes interrupted by periods of declining income as in the 1997 crisis. The GDP before 1997 increases significantly above its trend. Nonetheless, it decreases clearly below its trend during the financial crisis of 1997. The GDP is sometimes below its trend but sometimes it is not. The trend line is generated by the Hodrick-Prescott filter since the filter removes a slow moving component. In this sense, the Hodrick-Prescott is a trend estimator.

Figure 8.1 The effects of alternative filters on Thai GDP

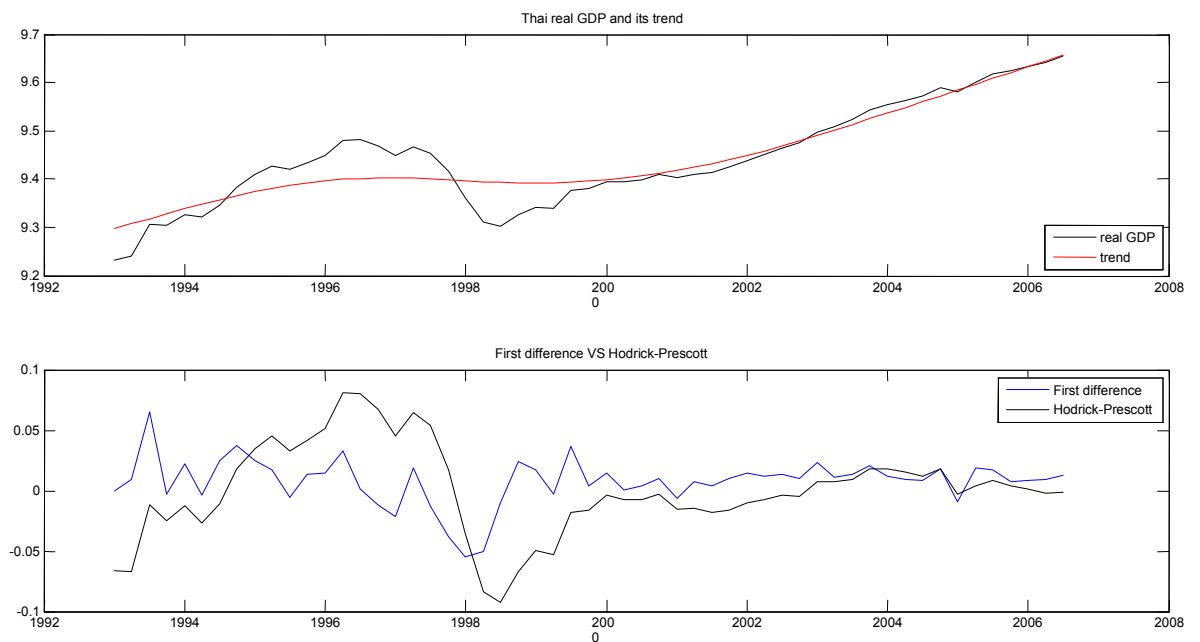


Figure 8.1 A

Figure 8.1, A the lower panel, portrays the results of the first difference filter versus the Hodrick-Prescott filter with the parameter $\lambda = 1600$ for quarterly data.⁷ It essentially shows the quarterly growth rate of Thai GDP compared to the Hodrick-Prescott filter. The first difference filter puts a high weight on the higher frequency and a low weight on the lower frequency. The first difference eliminates the trend component, but it intensifies the effect of high frequency components of the data, leading to the jagged appearance of the filtered time series. The Hodrick-Prescott filter does not amplify the high frequency components. However, it still passes as much as of the high-frequency noise outside the business cycle band. The correspondence between the time series produced by the first difference and the Hodrick-Prescott is small. Three years before and after the crisis, the first difference and the Hodrick-Prescott produce substantially different filtered time series.

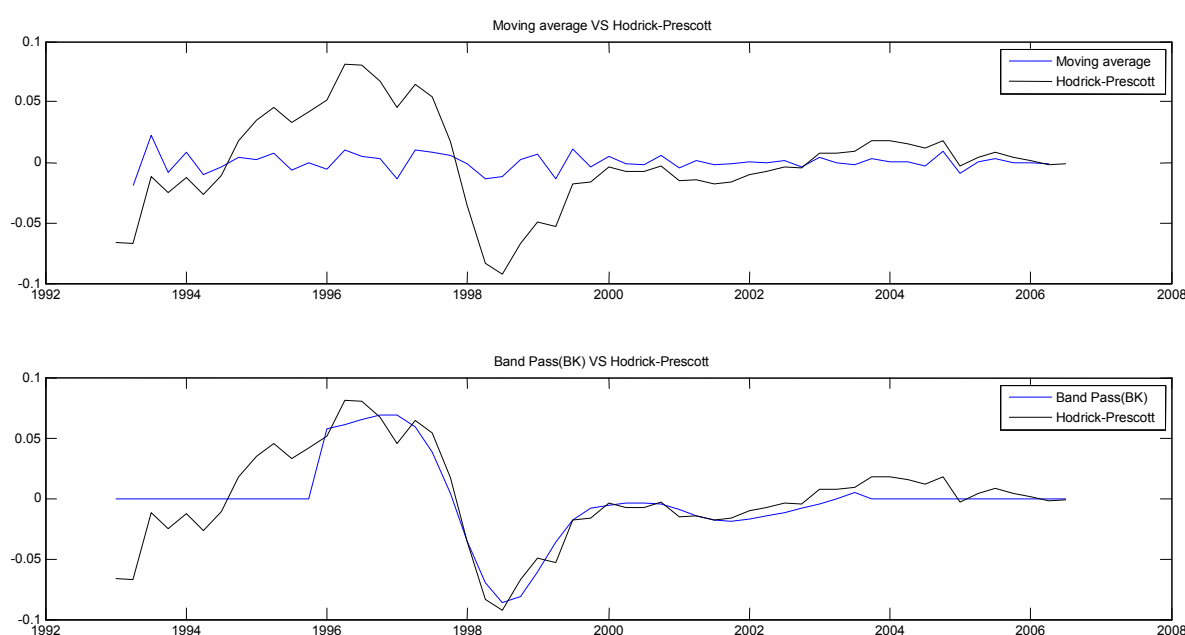


Figure 8.1 B

In figure 8.1 B, the upper panel, displays the equally weighted moving average filtered real GDP of Thailand⁸. This filter serves to separate trends and it is symmetric. The moving average filter can pick up many of the peaks and troughs. The deviations of the moving average filtered time series from the Hodrick-Prescott filtered time series are

⁷ The details of the first difference filter can be found in chapter 3 section 3.2.3.

⁸ The details of the 3 point equally weighted moving average are shown in chapter 3 section 3.2.4.

sizable during the crisis in 1997. The moving average filter produces a less volatile series that bears little resemblance to the Hodrick-Prescott filter.

Figure 8.1 B, the lower graph, displays the band pass filter by Baxter and King, compared to the Hodrick-Prescott filter. There is a very close correspondence between the cycles isolated by the Hodrick-Prescott filter and those generated by the band pass filter by Baxter and King. Although, the band pass filter by Baxter-King loses 12 data points in the beginning and the end of the data, it is somewhat less volatile. The reason of the resemblance is that the low frequency variation is much more important as a source of overall variation in output. Output does not have important variations at a high frequency. The Hodrick-Prescott works as the high pass filter where the high frequency components can pass through. Therefore, both the Hodrick-Prescott filter and the band pass filter remove the low frequency variation. The band pass filters are also much smoother than the Hodrick-Prescott filter.

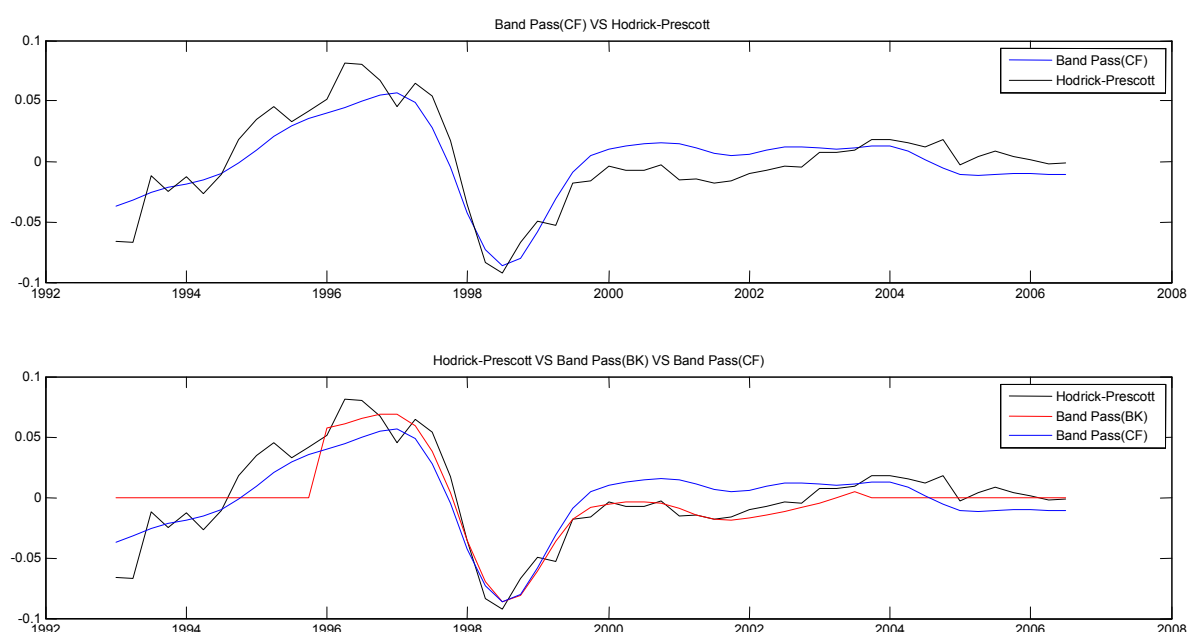


Figure 8.1 C

In figure 8.1 C, the upper graph, portrays the band pass filter by Christiano and Fitzgerald with the Hodrick-Prescott filter. There is a very close correspondence between the two filtered time series. The reason, once again, is that low frequency variation is a source of overall variation in GDP. Therefore, both the band pass filter and

the Hodrick-Prescott filter eliminate the low frequency component of GDP. However, the Hodrick-Prescott is somewhat less smooth. The lower panel of figure 8.1 C illustrates measures of the cyclical component of real GDP per capita in Thailand that emerge from the Hodrick-Prescott filter, the band pass filter by Baxter and King and the band pass filter by Christiano and Fitzgerald. Clearly from the graph, we can see how these three filtered series are similar.

8.2.2 Inspecting the results for Thai consumption

The results of applying the same five filters to the logarithm of Thai real consumption per capita are displayed in figure 8.2. Throughout the six graphs, the Hodrick-Prescott filter is again used as the reference point.

Figure 8.2 The effects of alternative filters on Thai consumption

In figure 8.2 A the upper panel displays the quarterly real consumption of Thailand with its trend during the observation period. The trend line is generated by the Hodrick-Prescott filter.

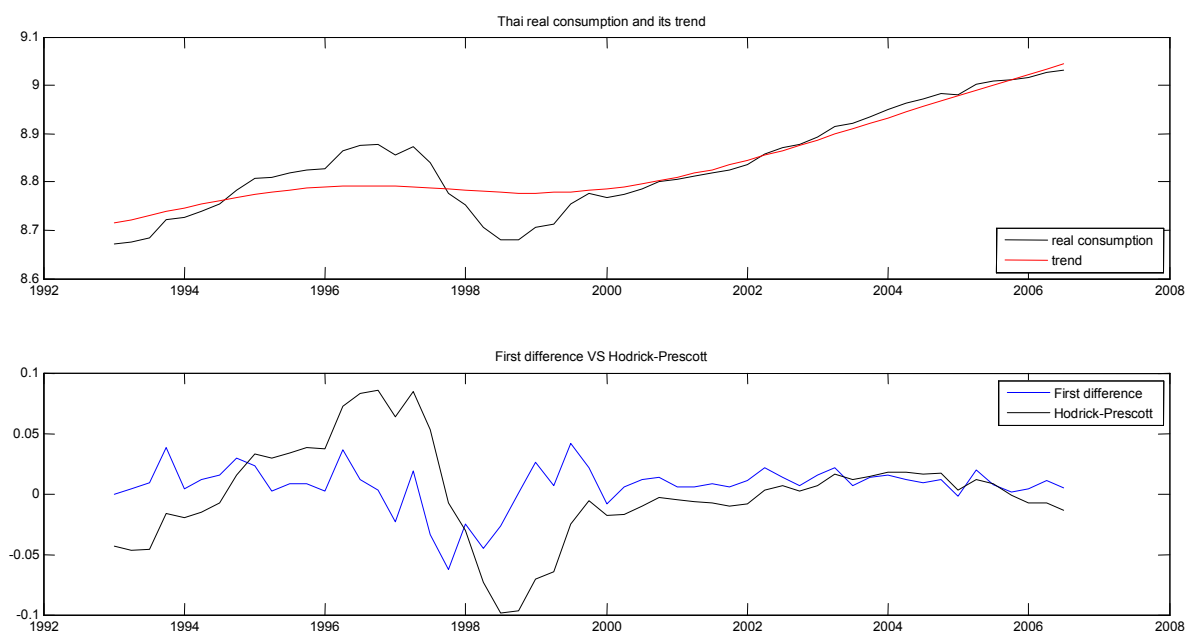


Figure 8.2 A

The consumption in Thailand exhibits a high correlation with output as seen from the figure 8.1 A and 8.2 A the upper panel. The two graphs are very much similar. When

the Thai economy is in the period of expansion, consumption rises. However, when it is in the period of contraction, consumption obviously declines. Like output, consumption before the financial crisis 1997 increases above its trend. However, it decreases dramatically below its trend during the crisis. Once the economy recovers from the crisis it follows the trend more closely than in the pre-crisis period. A lower panel of figure 8.2 A shows the quarterly cyclical component of consumption derived by the first difference and the Hodrick-Prescott filter. There is little correspondence between the two filtered time series.

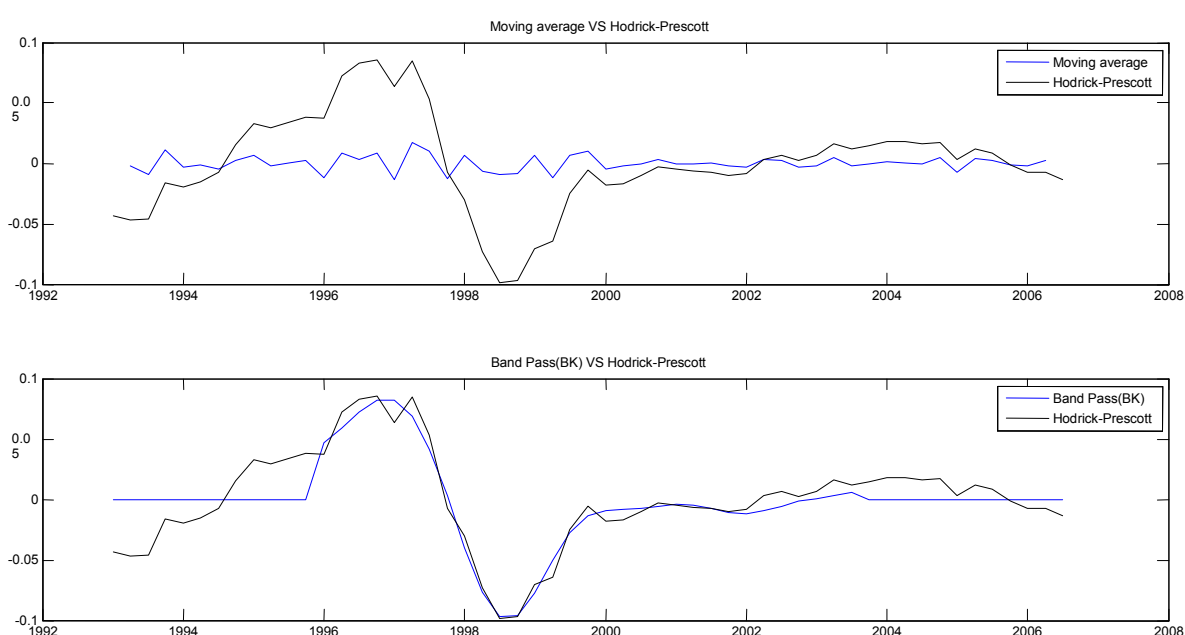


Figure 8.2 B

In figure 8.2 B, the upper figure shows the filtered time series by the equally weighted moving average and the Hodrick-Prescott filter. The Hodrick-Prescott filtered series is somewhat more volatile. As expected, figure 8.2 B bottom panel displays a close correspondence between the band pass by Baxter and King filtered time series and the Hodrick-Prescott filtered time series. Since the low frequency component is a crucial source of the variation of consumption, the two filtered time series are similar. The band pass filtered series is somewhat less volatile. However, we lose three years of data at each end of the plots for the band pass filter.

The upper graph of figure 8.2 C top figure illustrates a very close correspondence between the cycles isolated by Christiano and Fitzgerald band pass filter and those generated by the Hodrick-Prescott filter. Once again, the reason is that both filters eliminate the low frequency components which are the main source of the variation of consumption.

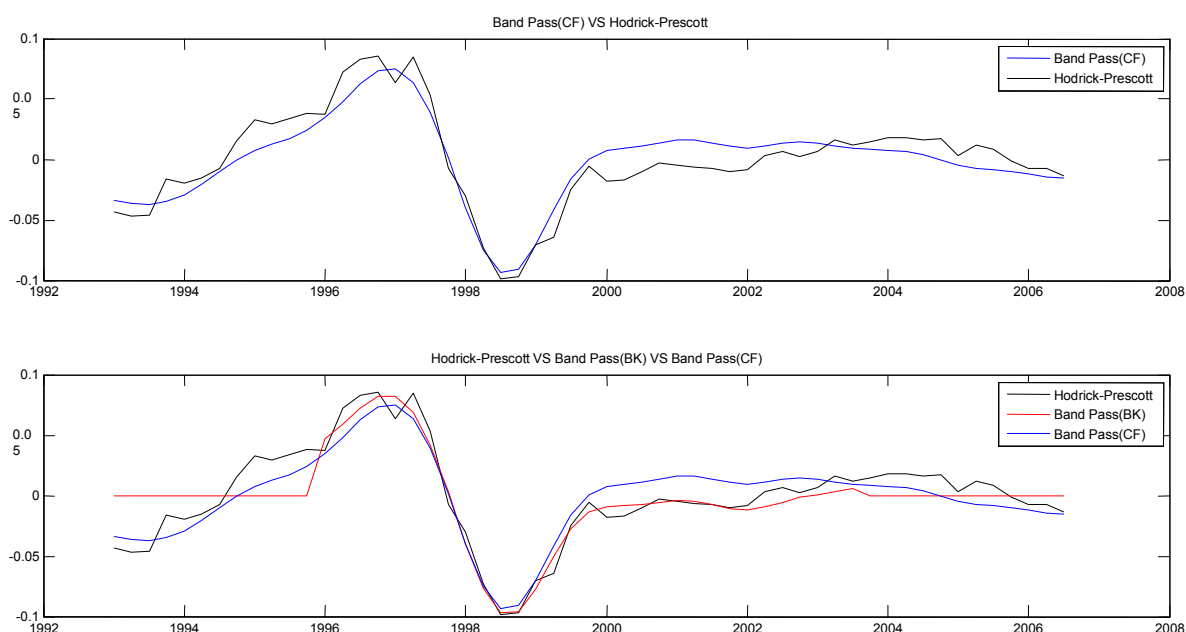


Figure 8.2 C

Nonetheless, the Hodrick-Prescott filtered series is more volatile. In figure 8.2 C the lower figure illustrates measures of the cyclical component of real consumption per capita in Thailand that emerge from the Hodrick-Prescott filter, the band pass filter by Baxter and King and the band pass filter by Christiano and Fitzgerald. The three filtered time series have a very close correspondence to each other, although the Christiano and Fitzgerald band pass filter shows a quick recovery from the crisis than the other two filters.

8.2.3 Inspecting the results for Thai investment

In figure 8.3, the same five filters are applied to the natural logarithm of investment in Thailand. As before, throughout all the graphs, the Hodrick-Prescott is used as the benchmark.

In figure 8.3 A the upper panel displays quarterly real investment of Thailand with its trend during 1993q1-2006q3. The trend line is again generated by the Hodrick-Prescott

filter. Like consumption, investment in Thailand exhibits a high correlation with output as clearly seen from the figure 8.1, 8.2 and 8.3. However, investment is the most volatile of the three series we have analysed. In addition it appears to have been the slowest to have recovered from the crisis. Investment obviously fluctuates above its trend before the crisis 1997 and suddenly drops below its trend after that. In figure 8.3 A the lower graph portrays the growth rate of investment versus the Hodrick-Prescott filter. Compared to output and consumption, investment has a more significant deviation around its trend. As before, the first difference filter generates a highly volatile time series that shows little likeness to the Hodrick-Prescott filter.

Figure 8.3 The effects of alternative filters on Thai investment

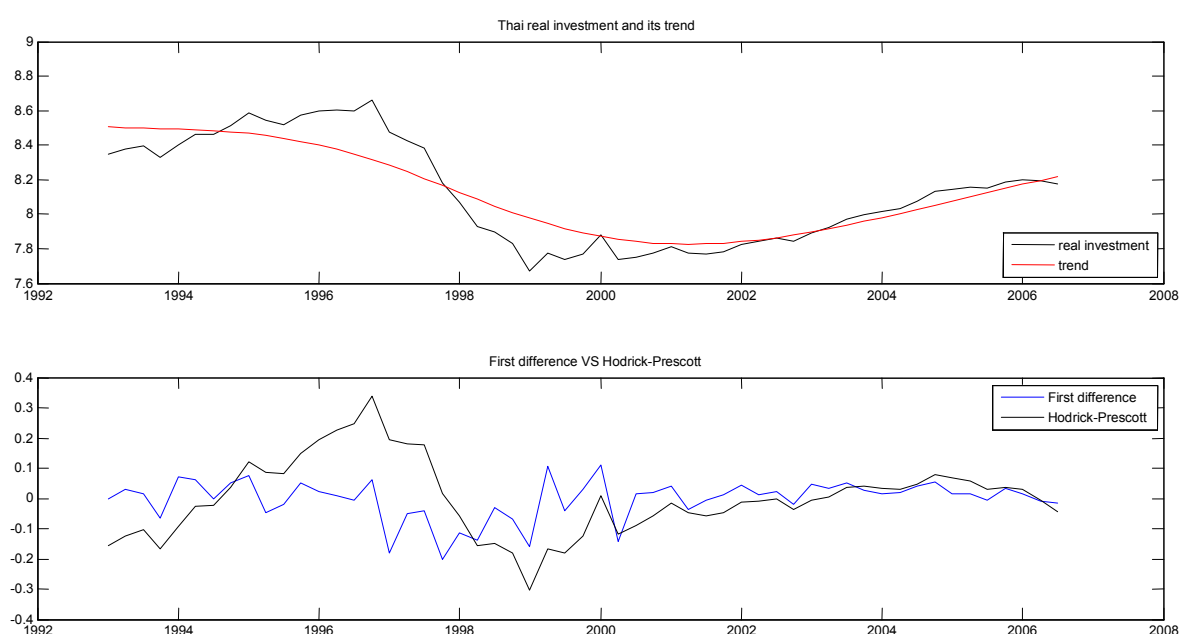


Figure 8.3 A

In figure 8.3 B, the top figure shows the filtered time series by the equally weighted moving average and the Hodrick-Prescott filter. Comparing to the weighted average filter, the Hodrick-Prescott filtered series is more volatile. The lower panel of figure 8.3 B exhibits again a close correspondence of the band pass by Baxter and King filtered time series and the Hodrick-Prescott filtered time series. The reason for the resemblance again is that the low frequency component is the main cause of the variation of investment. However, the Hodrick-Prescott filtered series is less smooth.

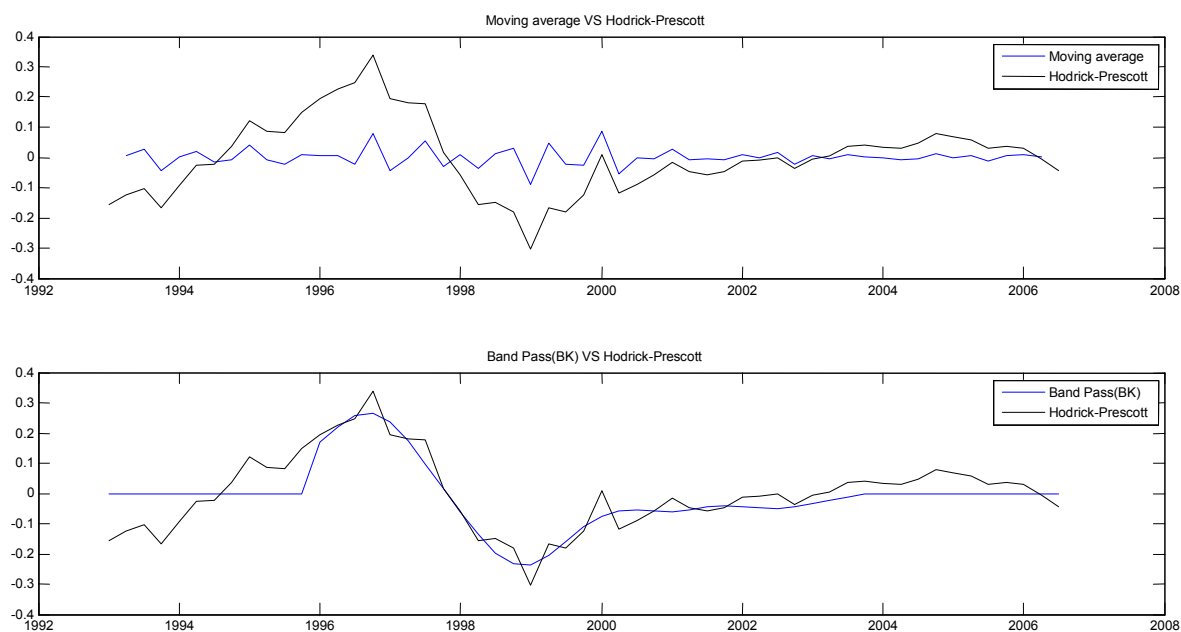


Figure 8.3 B

In figure 8.3 C, the upper panel shows the results for the band pass filter by Christiano and Fitzgerald and the Hodrick-Prescott filter. Like the Hodrick-Prescott filter, the band pass filter removes the slow moving component of investment.

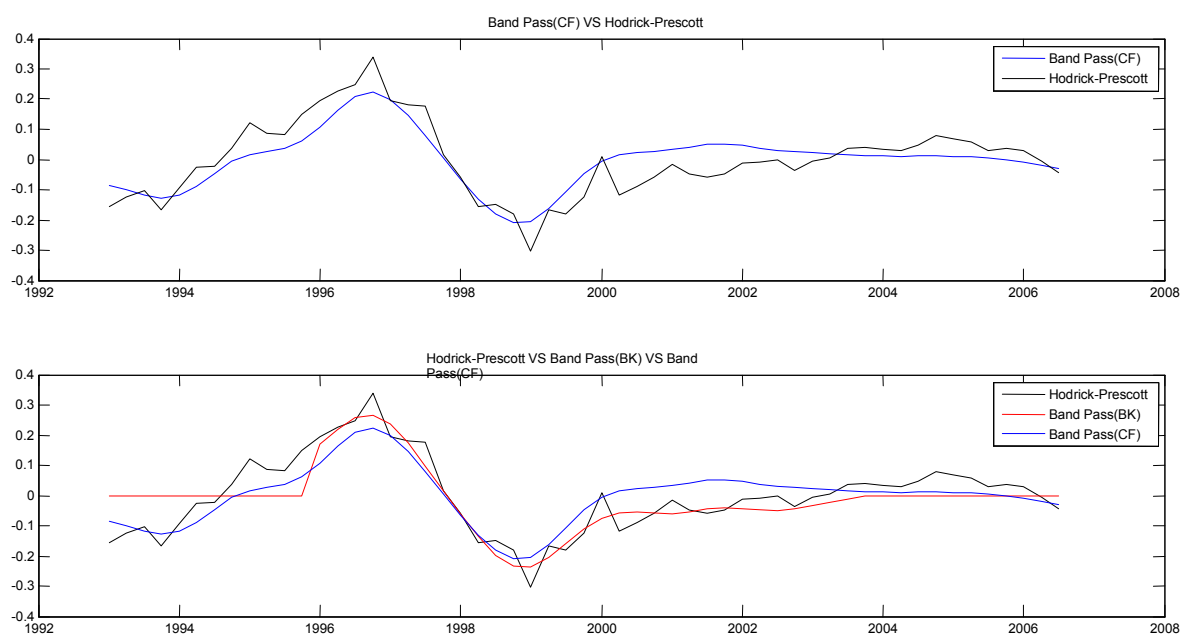


Figure 8.3 C

The measures of the cyclical components by the Hodrick-Prescott filter, the band pass filter by Baxter and King and the band pass filter by Christiano and Fitzgerald are illustrated in figure 8.3 C in the lower figure.

8.3 Moments Analysis

This section compares the main properties of the Thai business cycle using the same five alternative business cycle filters as discussed in section 8.2. As is common, the three main characteristics of the business cycle, volatility, persistence and co-movement with output are calculated. Volatility is measured by the percentage of standard deviation of the series. Persistence is widely measured by the first order auto regressive coefficient. Co-movement is measured by the correlation coefficient of the series with respect to output. The volatility gives information on the magnitude of fluctuations. The persistence determines the degree of the previous period value influencing the current one. It is critical in determining the longevity of the impact of a shock. The co-movement provides the degree of movement between two variables. Using the same set of data from the pervious section, Table 8.1 suggests how the application of these alternative filters affects the moments computed from the main macroeconomics time series of Thailand during 1993q1 to 2006q3.⁹

8.3.1 Moments computed from the Thai data

8.3.1.1 Volatility

Table 8.1 shows that the one widely used measure of volatility, the percentage of standard deviation, is sensitive to the choice of filter. Overall, the results suggest that with respect to the Thai key macroeconomic time series a clear pattern emerges. Consumption fluctuates less than output for the first difference and the moving average filter, but more than output for the remaining filters. Investment is the most volatile series for all filters. It is also about three times more volatile than output for all filters. The first difference and the equally weighted moving average filters give a significantly lower volatile series for output, consumption and investment. The moving average is

⁹ The results of the Hodrick-Prescott filter in table 8.1 are in fact the same as the table 5.11 A for volatility and persistence and table 5.11 B for the co-movement. The band pass filtered cyclical statistics of the Thai key variables in table 8.1 are the same as the table 5.10 A for volatility and persistence and table 5.10 B for the co-movement.

somewhat lower than the others. The first difference filter produces smaller measures of volatility for output, consumption and investment because it removes more of the low frequency components of these series than the Hodrick-Prescott and the two band pass filters. The source of variation of output, consumption and investment are at this lower frequency. However, the source of variation of labour, wages and rental rate are at a high frequency. The pattern mentioned above is reversed for labour, wages and interest rate. The first difference filter and the moving average produce a high fluctuation series for labour, wages and interest rate. These three series contain considerably high frequency components in which are intensified by the first difference filter.

Table 8.1 The effect of filtering on moments from the Thai data; Volatility

	FD		MV		HP		BP(BK)		BP(CF)	
	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>
Output	1.96	1.000	0.74	1.000	3.77	1.000	3.13	1.000	3.06	1.000
Consumption	1.85	0.946	0.64	0.870	3.98	1.056	3.52	1.125	3.49	1.139
Investment	6.67	3.405	2.94	3.997	12.32	3.268	10.45	3.339	9.37	3.060
Labour	4.05	2.069	2.13	2.895	2.77	0.735	0.03	0.010	0.06	0.020
Wages	4.48	2.286	2.70	3.674	2.99	0.793	1.22	0.390	1.54	0.504
Interest rate	3.02	1.544	1.65	2.237	2.69	0.714	2.19	0.700	2.22	0.726

Note: Volatility is measured by the percentage of standard deviation of series. FD stands for the first difference filter. MV is the moving average. HP denotes the Hodrick-Prescott. BP(BK) is the band pass filter by Baxter and King. BP(CF) is the band pass filter by Christiano and Fitzgerald.

The Hodrick-Prescott filter and the two band pass filters show similar volatility for all Thai variables. The Hodrick-Prescott filter generates volatility statistics that exceed those of the two band pass filters, even though in many cases not by a large amount. This is due to the fact that the Hodrick-Prescott filter retains some high-frequency volatility that is removed by the band-pass filter. Output, consumption and investment do not have a great deal of power at high frequencies, so including these components leads to only small increase in the volatilities of the filtered series. However, labour, wages and interest rate have sizably greater power at high frequencies. Therefore, the Hodrick-Prescott filter produces a volatility statistic that exceeds considerably those of the two band pass filters. The band pass filters by Baxter and King and by Christiano and Fitzgerald are similar in term of volatility. This is because they both are designed to capture the ideal band pass filter. They try to eliminate the same low and high frequency component of the series and retain the range of components in between. Nonetheless,

due to the construction of the filter, the band pass filter by Baxter and King is somewhat slightly less smooth.

8.3.1.2 Persistence

Table 8.2 The effect of filtering on moments from the Thai data; Persistence

	FD		MV		HP		BP(BK)		BP(CF)	
	($x+1$)	($x+2$)	($x+1$)	($x+2$)	($x+1$)	($x+2$)	($x+1$)	($x+2$)	($x+1$)	($x+2$)
Output	0.384	0.240	-0.403	0.103	0.859	0.656	0.908	0.709	0.917	0.719
Consumption	0.481	0.335	-0.361	-0.105	0.897	0.725	0.921	0.726	0.923	0.726
Investment	0.153	0.239	-0.551	-0.064	0.866	0.752	0.937	0.792	0.938	0.777
Labour	-0.179	-0.654	-0.269	-0.485	-0.068	-0.754	0.661	0.266	0.838	0.498
Wages	-0.586	0.142	-0.729	0.267	-0.115	0.093	0.879	0.589	0.790	0.299
Interest rate	-0.287	-0.175	-0.542	-0.070	0.468	0.271	0.917	0.702	0.899	0.635

Note: Persistence is measured by the first and the second autocorrelation. The abbreviations of the filters are the same as Table 8.1.

Table 8.2 displays statistics on the first order autocorrelation coefficient of filtered Thai macroeconomic time series. The first difference and the moving average filters cannot account for the persistence of the output, consumption and investment series. The first difference and the moving average produce dramatically smaller measures of persistence compared with the other filters. The reason is that the first difference filter eliminates more of highly persistent, low-frequency components, and intensifies the much less persistent, high-frequency components of the series in Thailand during the observation period. The Hodrick-Prescott filter and the two band pass filters provide a very similar degree of persistency of output, consumption and investment. The Hodrick-Prescott filter generates the persistency that is slight lower than those of the two band pass filters for output, consumption and investment. The two band pass filters are also very much similar in term of persistence. The Hodrick-Prescott filter provides a high persistency for output, consumption, and investment, but not for wage, interest rate and labour.

8.3.1.3 Correlation

Table 8.3 exhibits the results of the correlation between various macro variables and GDP of Thailand. The Hodrick-Prescott filters again produce statistics that are approximately similar to those computed using the two band pass filters. The Hodrick-Prescott and the two band pass filters produce a strong correlation with output for consumption and investment, but weak correlation with output for labour, wages and interest rate. The first difference filter and the moving average generate a significantly

smaller correlation in all cases. The first difference and the moving average filters give a different view to explain the degree of co-movement of consumption and investment. Co-movement of consumption and investment to output by these two filters have lower volatility than those of the others.

Table 8.3 The effect of filtering on moments from the Thai data; Co-movement with respect to output

	FD	MV	HP	BP(BK)	BP(CF)
Output	1.000	1.000	1.000	1.000	1.000
Consumption	0.714	0.341	0.954	0.989	0.983
Investment	0.474	0.139	0.895	0.946	0.917
Labour	0.096	0.229	-0.069	-0.087	-0.521
Wages	-0.075	-0.111	-0.275	-0.722	-0.602
Interest rate	-0.036	-0.019	0.017	-0.039	-0.113

Note: Co-movement with respect to output is measured by the correlation coefficient of the series of output and another series. The abbreviations of the filters are the same as Table 8.1.

8.3.1.4 Conclusions

The Hodrick-Prescott and the two band pass filters yield similar outcomes for volatility, persistency and co-movement. The two band pass filters give the smoother time series than the Hodrick-Prescott does. Even though, the first difference and the moving average filters can remove a stochastic trend, there are some problems regarding these two filters. The first difference filter yields a phase shift and also it cannot separate the cyclical variation within a particular band frequency. The moving average filter cannot separate the cyclical variation within a particular band frequency. These factors could be the reasons why we find difficulties in extracting the cyclical component using the first difference and the moving average filters. The Hodrick-Prescott filter works as a high pass filter passing through only a high frequency component and removing the lower ones. The two band pass filters can separate the cyclical component variation within 6 and 32 quarters. Therefore, these two filters suggest a similar result. The volatility from the band pass by Christiano and Fitzgerald is just lightly lower than that from the band pass by Baxter and King. For the persistence and co-movement, these two filters generally perform the same.

Baxter and King (1999) claim that overall the researchers should use the band pass filter, the Hodrick-Prescott filter on the U.S. time series give a similar impression of the nature of business cycles. But using the first difference filter will give different view of the

central business cycle facts. They suggest that the first difference filter downweights the lower frequencies relative to the alternative filters. Due to this fact, the first difference filter produces filtered series with lower volatility than those generated by the other filters. For the same reason, it produces time series with much lower persistence and correlation with output than those produced by other filters, including the Christiano and Fitzgerald filter.

8.3.2 Model moments

The model economy is usually evaluated by comparing the model moments with the moments computed from the data. This section shows how application of the filters in section 8.3.1 affects the model moments. The real business cycle model from chapter 5 with the calibration parameters according to table 5.7 is used to calculate the model moments for this section¹⁰. Chapter 5 shows that the shock to productivity can generate the business cycle, not the shock to government spending¹¹. Therefore, this section follows that result. The closed model economy simulated for Thailand experiences a 1% increase in technology and is filtered by differently kinds of filters. The ideal band pass filter is represented both by the band pass by Baxter and King and the band pass by Christiano & Fitzgerald. Once again, this section focuses on three main features of time series, volatility, persistence and co-movement with respect to output of each variable.

Table 8.4 The effect of filtering on moments from the model; Volatility

	FD		MV		HP		Ideal BP	
	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>
Output	4.04	1.000	11.56	1.000	5.73	1.000	4.69	1.000
Consumption	1.66	0.411	7.30	0.631	2.47	0.431	1.96	0.418
Investment	10.96	2.713	26.86	2.324	15.34	2.678	12.64	2.700
Labour	4.23	1.047	7.51	0.650	6.10	1.065	4.97	1.059
Wages	2.56	0.634	9.83	0.850	3.88	0.677	3.12	0.665
Interest rate	6.94	1.718	18.41	1.593	9.97	1.740	8.14	1.735

Note: Volatility is measured by the percentage of standard deviation of series. FD stands for the first difference filter. MV is the moving average. HP denotes the Hodrick-Prescott. Ideal BP is the ideal band pass filter.

Table 8.4 displays the amplitude of fluctuations measured by the percentage of standard deviation. For all the filters in the table, the model economy shows the business cycle

¹⁰ The discussion of the model construction can be seen in chapter 5.

¹¹ The government spending shock produces a negative wealth effect. The agents are likely to cut their consumption and increase their labours. Therefore, the consumption is negatively correlated to output.

properties. Consumption is less volatile than output and investment is the most volatile series. In fact, consumption is less volatile than all the other cyclical series. Investment is about three times more volatile than output, except for the investment filtered by the moving average filter. The model economy filtered by the moving average produces a considerably high degree of volatility for all series. The model filtered by the Hodrick-Prescott filter displays a high volatility compare to those filtered by the ideal band pass filter. However, the first difference filter generates slightly less fluctuated time series than the ideal band pass filters do.

Table 8.5 The effect of filtering on moments from the model; Persistence

	FD		MV		HP		Ideal BP	
	$(x+1)$	$(x+2)$	$(x+1)$	$(x+2)$	$(x+1)$	$(x+2)$	$(x+1)$	$(x+2)$
Output	0.897	-0.032	0.981	0.938	0.763	0.558	0.897	0.632
Consumption	0.900	-0.004	0.992	0.973	0.788	0.601	0.900	0.644
Investment	0.896	-0.037	0.974	0.917	0.757	0.548	0.896	0.63
Labour	0.898	-0.028	0.980	0.935	0.769	0.568	0.898	0.636
Wages	0.899	-0.017	0.988	0.962	0.778	0.583	0.899	0.639
Interest rate	0.898	-0.030	0.978	0.928	0.767	0.565	0.898	0.635

Note: Persistence is measured by the first and the second autocorrelation. The abbreviations of the filters are the same as Table 8.1 A.

Table 8.5 shows that models filtered by different kind of filters clearly show a high degree of persistency for all variables. The first order autocorrelation coefficient for all variable is approximately 0.9 for the first difference, the moving average and the ideal band pass filter. However, the Hodrick-Prescott filter produces a slightly lower degree of persistence which is approximately 0.8. The second order autocorrelation drops dramatically for the first difference filter. The second order autocorrelation produced by the moving average is slightly lower than the first order autocorrelation.

Table 8.6 The effect of filtering on moments from the model; Co-movement

	FD	MV	HP	Ideal BP
Output	1.000	1.000	1.000	1.000
Consumption	0.957	0.913	0.949	0.957
Investment	0.991	0.961	0.993	0.994
Labour	0.897	0.872	0.897	0.898
Wages	0.941	0.949	0.942	0.943
Interest rate	0.916	0.769	0.908	0.914

Note: Co-movement with respect to output is measured by the correlation coefficient of the series of output and another series. The abbreviations of the filters are the same as Table 8.1 A.

Table 8.6 shows the results for the contemporaneous correlation of various aggregates with GDP. The table suggests that all variables are strongly pro-cyclical for all filters. The investment series shows the strongest contemporaneous correlation features of all filters.

8.3.3 Moments comparison

Understanding the role of measurement errors is crucial to proper data interpretation. This section creates three additional tables, table 8.7, 8.8 and 8.9 to facilitate the moment comparison. These tables provide the deviation of the model's prediction from the fact for volatility, persistence and co-movement respectively.

Table 8.7 The deviation of the model's predictions from the fact; Volatility

	FD		MV		HP		BP(BK)		BP(CF)	
	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>	<i>Std</i>	<i>Std/y</i>
Output	2.08	0.00	10.82	0.00	1.96	0.00	1.56	0.00	1.63	0.00
Consumption	-0.19	-0.54	6.66	-0.24	-1.51	-0.63	-1.56	-0.71	-1.53	-0.72
Investment	4.29	-0.69	23.92	-1.67	3.02	-0.60	2.19	-0.64	3.27	-0.36
Labour	0.18	-1.02	5.38	-2.25	3.33	0.33	4.94	1.05	4.91	1.04
Wages	-1.92	-1.65	7.13	-2.82	0.89	-0.12	1.90	0.28	1.58	0.16
Interest rate	3.92	0.17	16.76	-0.64	7.28	1.03	5.95	1.04	5.92	1.01

Note: The deviation is derived from the model's prediction subtracting the fact. The abbreviations of the filters are the same as Table 8.1 A.

As in the tables 8.1 and 8.4, the model economy with the first difference filter and the moving average shows significantly high volatility compared to the facts. The deviations of the model's prediction from the fact by the moving average are obviously high as in table 8.7. For consumption, labour and interest rate, the deviations of the model's prediction from the fact by the first difference are lower than the Hodrick-Prescott and the two band pass filters. The first difference and the moving average filters also predict strong persistence in the model economy approximately 0.9 where as the facts show approximately negative figures as shown table 8.2 and 8.5. Table 8.8 also suggests that the deviation of the model's persistence is high comparing to other filters. The model economy with these two filters shows a strongly contemporaneous correlation with respect to output while the facts show the weak correlation as in table 8.3, 8.6 and 8.9. Thus the model economies filtered by the first difference and the moving average do not provide the good match for the variables in the Thai economy.

The model economy filtered by the Hodrick-Prescott filter and the two band pass filters captures the facts better than the first difference and the moving average filters for volatility, persistence and co-movement. Although the model economy filtered by the Hodrick-Prescott filter and the ideal band pass do not produce a perfect explanation for all variables in Thai economy. The model economy does result in business cycle with reasonable characteristics with respect to output, consumption and investment.

Table 8.8 The deviation of the model's predictions from the fact; Persistence

	FD	MV	HP	BP(BK)	BP(CF)
	$(x+1)$	$(x+1)$	$(x+1)$	$(x+1)$	$(x+1)$
Output	0.513	1.384	-0.096	-0.011	-0.020
Consumption	0.419	1.353	-0.109	-0.021	-0.023
Investment	0.743	1.525	-0.109	-0.041	-0.042
Labour	1.077	1.249	0.837	0.237	0.060
Wages	1.485	1.717	0.893	0.020	0.109
Interest rate	1.185	1.520	0.299	-0.019	-0.001

Note: The deviation is derived from the model's prediction subtracting the fact. The abbreviations of the filters are the same as Table 8.1 A.

Table 8.9 The deviation of the model's predictions from the fact; Co-movement

	FD	MV	HP	BP(BK)	BP(CF)
Output	0.000	0.000	0.000	0.000	0.000
Consumption	0.243	0.572	-0.005	-0.032	-0.026
Investment	0.517	0.822	0.098	0.048	0.077
Labour	0.801	0.643	0.966	0.985	1.419
Wages	1.016	1.060	1.217	1.665	1.545
Interest rate	0.952	0.788	0.891	0.953	1.027

Note: The deviation is derived from the model's prediction subtracting the fact. The abbreviations of the filters are the same as Table 8.1 A.

The volatility of the model economy filtered by the Hodrick-Prescott is higher than that of the model economy filtered by the band pass filters. The volatilities of output and investment of the model economy filtered by the Hodrick-Prescott and the ideal band pass filter are slightly higher than those generated by Thai facts. Persistency is strong for the model filtered by the band pass filters which is consistent with the facts. However, persistence is also strong for the model filtered by the Hodrick-Prescott filter for output, consumption and investment. Persistence is considerably weaker for the Hodrick-Prescott filter for labour, wages and rental rate. The model filtered by the Hodrick-Prescott and the ideal band pass predicts a strong co-movement with output for all variables. However, for the facts, the Hodrick-Prescott filter and the two band pass

filters provide a strong contemporaneous correlation for output, consumption and investment while the co-movement of labour, wage and interest rate shows a weak and even negative co-movement. Overall, the model filtered by the Hodrick-Prescott and the ideal band pass filters can mimic the facts better than the model filtered by the first difference and the moving average filters.

8.4 Conclusion

By using different kind of filters, the purpose of this chapter is to document some features of aggregate economic fluctuations which referred to as business cycles. The Thai data and the real business cycle model are used to study the effect of different filters on the business cycle properties. The investigation uses quarterly data from the Thai economy during 1993q1 to 2006q3. The real business cycle model investigated in this chapter follows the closed economy model in chapter 5. Specifically, this chapter explores how the five different filters, the first difference filter, the equally weighted moving average, the Hodrick-Prescott filter, the band pass filter by Baxter and King and the band pass filter by Christiano and Fitzgerald could yield different results in terms of business cycle properties. For the purpose of business cycle analysis, the two band pass filters are better than the others. The band pass filter by Baxter and King and the band pass filter by Christiano and Fitzgerald are similar in measuring business cycles. The majority of research uses the band pass filter by Baxter and King. It is reasonable to use the Baxter and King band pass filter in our analysis. For this reason we felt justified in using the standard band pass Baxter and King in this thesis.

The aim is to evaluate the filters in terms of their ability to retain the business cycle variation in the data and in the model economy. Following the study of Burns and Mitchell (1946), the business cycle has a periodicity no less than 6 quarters and no more than 32 quarters. The first difference, the moving average and the Hodrick-Prescott filter cannot retain the cyclical variation within a particular band frequency.¹² One problem which arises from the first difference filter is that there is a phase shift in the filtered series. The Hodrick-Prescott filter is approximately a high pass filter, which remove the low frequency components but retain the high ones. The two band pass filters can

¹² The first difference is the difference of x between t and $t-1$. The moving average takes a weighted average of successive value of an interesting series. The Hodrick-Prescott is similar to the high pass filter.

remove both a low frequency components and a high frequency component of a time series while keeping the frequency in between 6 and 32 quarters. This suggests that, the first difference and the moving average filters are not desirable filters for the purpose of business cycle analysis. The Hodrick-Prescott filter in some cases can approach the ideal band pass. Nonetheless, the two band pass filters produce a better approximation to the ideal filter.

Comparing the models to the facts, the first difference filter and the moving average show a sizable fluctuation, strong persistence and high contemporaneous correlation with respect to output. The model economy filtered by the Hodrick-Prescott filter and the ideal band pass better capture the facts than the either first difference and the moving average filters. The Hodrick-Prescott filter produces slightly more volatile than the band pass filters do for both the model and the facts. The Hodrick-Prescott filter produces strong persistency from the model and the facts only for output, consumption and investment. Persistency is strong for the model and the Thai fact filtered by the band pass filters. The model filtered by the band pass filter and the Hodrick-Prescott filter provides a strong contemporaneous correlation with output for all variables. However, the facts filtered by the Hodrick-Prescott filter and the two band pass filters produce a strong co-movement with respect to output for consumption and investment.

9.1 Introduction

The explanation of cyclical fluctuation that arises in real business cycle theory, a theoretical model economy based on the neoclassical theory of capital accumulation, is focused on the role of productivity shocks originating from technological progress. The innovation in technology optimally reproduces the behaviour of people to intertemporally substitute their consumption and labour, ultimately inducing the economic fluctuation. The real business cycle research in developed countries has been done in both theoretical and empirical studies. But this is less the case in developing countries with respect to Thailand, there is still lack of a comprehensive study of the real business cycle. This is a gap this thesis has sought to fill. The thesis has explored the characteristics of the Thai business cycle and presented a dynamic macroeconomic model that can simulate its fluctuations. It adopted the basic framework of real business cycle theory to test and study the ability to explain the salient business cycle features of the Thai economy. The decision rules used to generate the predicted paths for the endogenous variables reported in this thesis were computed in a manner consistent with the method of finding a linear approximation to the first order conditions by King *et al.*(1988).

This thesis also extends the theory of real business cycle to a framework of a small open economy, using alternative versions of a standard one good two country international real business cycle model based on Baxter and Crucini (1995), in which each country has its own set of preference and technology parameters. The country spillovers in technology as well as trade are the transmission mechanism of the fluctuations across countries. By studying the open economy model, the cross correlation between the variables of two countries can be analyzed. The thesis has firstly summarized the facts of the business cycle in Thailand and other countries then compares those facts with and the results of the model prediction.

9.2 The main findings

The main findings are that there are many striking business cycle fluctuations in the developing countries studied during 1965-2004. The volatility of investment and government spending are higher than that of aggregate output. Consumption is more volatile than output. These variables in general are pro-cyclical and are high persistent. However, net exports are highly volatile and countercyclical. There are some crucial similarities between the cyclical fluctuation in developing countries and those of the developed country. Co-movement and persistence for consumption, investment and government spending are quite similar. Nonetheless, the business cycle features of the developing countries fluctuate more than those of the developed countries. The business cycle regularities in the Asian countries exhibit a high degree of co-movement. It also finds that the business cycle fluctuations of output among developing countries are positively correlated.

To evaluate the dynamics of a closed economy model calibrated for Thai economy, the model and the real economy are filtered by the band pass and the HP filters. For the annual observed data during 1976-2005, the model economies were filtered by both the band pass and the Hodrick-Prescott filters to obtain crucial business cycle properties. The results indicate consumption and output fluctuate less than investment does. Consumption in the model economy is less volatile than that of output. The models suggest a high degree of persistence and co-movement with output. Nonetheless, the HP filtered model exhibits lower persistence than that of the band pass filtered model. The striking fact of the Thai business cycle is that consumption fluctuates more than output does. This outstanding fact cannot be explained by the model economy. For quarterly data during 1993q1-2006q3, the model economies exhibit business cycle regularities. Even though the models cannot explain the high volatility of consumption relative to output, they solve the persistent problem of the model with annual data. The model economy can mimic the facts better with the band pass filter for the quarterly data for volatility, persistence and cross correlation.

The closed economy model is simulated by three different kinds of driving parameters, capital, productivity and government spending. Other shocks cannot generate the real business cycle properties except for the productivity shock. The model is also simulated

by multiple shocks and also by shocks off the steady state. The idea of the shocks off the steady state is introduced to simulate the impact of one shock after another shock has already occurred. By doing the first shock, the model economy is able to generate the situation which is off the steady state such as the periods of rapid expansion and contraction. By using another shock, the policymaker is able to stabilize the economy. In chapter 5, this thesis replicates the 1997 economic crisis in Thailand and tries to minimize the cycle by using the government spending shock. Compared to the technology shock, the government spending shock has a minimal effect in the aggregation fluctuations. In this respect taxation is more efficient. However, although it is possible to use fiscal policy in the short-term to effect macro-economic stability, there are long-term problems. This is because of the different dynamics of the fiscal and productivity shocks.

The calibrated values have an important influence on the model performance. The persistence parameters of the technology shock of the Thai economy during the observation periods are significantly high. The idea that variations in output can be accounted for by large and highly persistent shocks to productivity is confirmed for Thai economy. In this model economy, the volatility of output is higher than in the Thai economy, suggesting that the productivity shock can be one reason for the output fluctuations, but not the only reason. Similar to the Thai facts, investment in the model economy fluctuates much more than output. However, the model fails to explain the volatility of consumption relative to output for Thai economy. The model predicts that consumption fluctuates less than output does while the Thai facts show the opposite. The volatility and the contemporaneous correlation with real output of labour, wages and interest rate are substantially lower in the data than in the models. This discrepancy implies a crucial missing component in this model for capturing the business cycle of developing countries, in this case Thailand. The model economy generates a high persistence and strong co-movement, just as it is in the Thai economy for output, consumption and investment.

The first part of investigating the open economy model for Thailand is to explore empirically the properties of business cycle for 7 OECD countries (the USA, the UK, EU, Canada, Australia, Japan and Korea) over the period 1986q1-2006q4. It finds that the output volatilities of Japan and Korea are larger than that of other countries.

Consumption is less volatile than output for all countries, except for UK and Korea. Relative to output, investment for all countries have been two to four times more volatile. The cyclical components of all the series are highly persistent. The trade variables exhibit a negative correlation with output for all countries, apart from Canada. Employment, consumption and investment have been pro-cyclical respect to with output. Consumption and investment show a strongly positive correlation with output. The cross country correlations of the US output are positive in most cases, except for Japan and Korea. It is noticeable from this empirical analysis that the cross-country correlations are stronger among neighbours and weaker with regard to remote countries. The cross-country correlations of variables in the USA and Canada are higher than that of variables between the USA and Japan or the USA and Korea.

The stylized facts of the Thai economy are explored using quarterly data only over 1993Q1-2006Q4, due to the lack of Thai data. The characteristics of the business cycle of Thailand, the USA and Japan are also investigated during this period because the USA and Japan are the main trading partners of Thailand. The volatility of all Thai variables is greater than that of the US and Japan variables. Output and trade in Thailand fluctuate about three times more than that of the USA and Japan. All variables for all three countries exhibit a high persistence. Investment has been shown to be strongly pro-cyclical with consumption and employment in the USA and Japan. However, consumption in Thailand is more strongly correlated with output than investment is. The result shows that the Thai cycle negatively fluctuated with the US cycle during 1993-2006.

For the open economy model, special attention is given to the implication of the calibrated parameters for Thailand and the USA by using the one good two country model with complete markets in line with Baxter and Crucini (1995). The capital stock of the USA is derived from the US investment series. The calibrated parameters imply that the movements in productivity are highly persistent in both Thailand and the USA. There is evidence of transmission of shocks across countries. The technology spillovers significantly transfers from the USA to Thailand at 26.1%, but only 2.2% from Thailand to the USA. The eigenvalues of the coefficient matrix of technology shock are nearly 1. For this reason, it will take a long time before the steady state is obtained following a shock. The contemporaneous correlation of technology innovation for Thailand and the

USA is -0.186. The total factor productivity of each country shows evidence of a random walk. Therefore it is possible to assume and examine the effects on Thai business cycle of a permanent shock.

The starting point of the open economy model investigation is the impulse responses. The impulse response is done for both a permanent and a realistic shock.¹ Three forcing processes, technology, government spending and taxes, are examined. The technology shock in Thailand increases investment, consumption and output in Thailand. The investment and output in the USA declines on the impact because capital is shifted to Thailand. The increase in consumption is negligible in both countries because of the small country specific shock location. The responses in Thailand are significant if the shock originates in the USA. Consumption noticeably increases in both countries in this case. The realistic shock based on the calibration of the USA and Thailand is investigated as a temporary shock with technology spillovers. The realistic shock in the US and Thailand generally results in a trade surplus for home country. The technology shock must have a very high persistence to induce a trade deficit in the home country. Because of the effect of technology spillovers, investment in the USA falls while the labour input rises on the impact of the realistic shock originating in the USA. The effects of the persistence of government spending and taxation on Thai business cycle are also investigated. Government spending creates a pure wealth effects while tax shocks generates both wealth and substitution effect. The higher the persistence is, the higher the wealth effect is. The shocks off the steady state are also explored in the context of the open economy model. Comparing to the technology and tax shocks, the government spending shock has a small impact to the variables in the model economy. The USA does not respond to a shock generated in Thailand. Nonetheless, Thailand is dominated by a shock originating in the USA. The multiple shocks are once again the combination of several shocks.

There is strong evidence that there is a significant impact of a technology shock from a large open economy on other countries, particularly, a small open economy. Small open economies are very susceptible to large country shocks. However, the large countries do not respond significantly to small country shocks. The economic events in the large

¹ The realistic shocks are derived from the data from the USA and Thailand as in equation (7.1) in chapter 7.

economies such as the USA, EU and Japan could have a strong impact on other countries. They, more importantly, hardly respond to the economic activities in small countries such as Thailand. The economic crisis in Thailand in 1997 for instance did not have any great impact in the US economy. But the economic crisis generating in the US and the EU in 2008 has had substantial effects on small economies, including Thailand.

Comparing the predictions of the model with the data, the model is evaluated for the permanent technology shock, realistic technology shock with and without technology spillovers, realistic technology shock with technology spillovers with changes in the variance of the technology innovation. The realistic technology shock with technology spillover with high variance (1%) captures the Thai stylized facts better than other specifications. The model requires a high variance of technology innovation (1%) to explain the Thai facts. Even though, the model cannot exactly match the data, the model can capture the business cycle pattern of volatility. It performs quite well in replicating the persistence. The shock in the USA can explain the co-movement in Thailand better than the shock originating in Thailand itself. However, the model fails to explain the counter-cyclical behavior of trade in Thailand and in the USA. In term of international co-movement, the model performs poorly in exactly matching the data. It can only match the sign of some variables. It fails to mimic the negative cross country correlation of consumption of the USA and Thailand. The shock in the USA can capture the negative sign of investment cross correlation, but not the shock originating in Thailand. The model predicts that the cross country correlation of consumption is higher than that of output. However, the facts show the opposite.

There are some key differences in terms of results between the closed economy and the open economy model in this thesis. One is that the closed economy model cannot analyze the cross country correlation. The open economy model is particularly extended from the closed economy model to focus on the cross country correlation. The open economy model exhibits that the cross country correlation of consumption is higher than the cross country correlation of output. However, the facts show the opposite.

This thesis also evaluates the filters in terms of their ability to capture the characteristics of business cycles in both data and the model economy. Five different filters, which are the first difference, the equally weighted moving average, the Hodrick-Prescott filter, the

band pass filter by Baxter and King, the band pass filter by Christiano and Fitzgerald, are used to filter the Thai data and the model economy. The model follows the closed economy model in chapter 5. The two band pass filters can remove both a low and high frequency components but retain the frequency between 6 and 32 quarters which is the frequency range for the purpose of business cycle analysis. The two band pass filters are similar in volatility, persistence and co-movement. The first difference and the moving average filters are not desirable filters. The Hodrick-Prescott filter is approximately a high pass filter, which removes only the low frequency. For this reason we felt justified in using the standard band pass Baxter and King in our analysis.

While macroeconomic theories try to explain the business cycle, economic policy tries to control it. The model can contribute to a policy decisions by offering measures of economic impact and predictions of economic behaviour. The model can predict how much the deviation of output is in response to a shock. Therefore, the model economy can replicate the pattern of the deviation of output in periods such as that of economic decline during 1979-1985, expansion during 1986-1996 and even the 1997 crisis period by using the concept of multiples shocks and the shocks off the steady state. The fiscal policy such as a government spending can be analyzed by this model economy to minimize the cycle.

9.3 Limitations

The model economy presented in this thesis has some limitations. Firstly, the real business cycle theory suggests that a supply side of the economy determines the fluctuation in the economy. Therefore, there is a distinct difference regarding how a change in government spending affects consumption in the real business cycle model and Keynesian model. An increase in government spending in the real business cycle theory decreases after tax income and in turn reduces consumption. In contrast, an increase in government spending based on Keynesian theory increases after tax income inducing an increase in consumption. The real business cycle theory considers the supply side effect of government spending whereas the Keynesian model focuses on the demand side effect. The technology and tax shock, in real business cycle theory, directly changes productivity. In addition, the government variable, in this thesis, is not integrated in the agent's utility function. Thus, no impact of government spending on utility which in part

explains the small impact of government spending shocks. In the presence of supply side shock, the demand side policies seem to have a limited applicability.

For the open economy model, the complete market assumption is introduced to link between the two countries. Under this particular assumption of market structures, individuals are able to diversify country specific risk across states of nature. Individuals equate their marginal utilities of consumption across countries. Therefore, a technology shock in one country results in an increase in consumption in both countries. The cross country consumption correlations are expected to be high.

Another limitation is that money and financial sectors are not included in the model economy. The model is therefore not useful for issues where financial intermediation is of first-order importance. The present crisis in the US and the EU is, unfortunately, one of those issues.

Finally, the world is made up of more than two countries. A multi country real business cycle model covering say 20 countries is ambitious, but would particularly from small open economy be an interesting exercise.

9.4 Implication

If one is to build an economic model of a small open or even medium sized open economy then it is important to use a two country model. They are arguably as affected as much by US shock as its own. But the US is relatively unaffected by shocks elsewhere. The economic shocks in a large economy dominate the economic activities in a small economy. However, a large economy does not respond to the shocks in a small country.

Fiscal policy whilst stabilizing in the home country can add to initial productivity shocks impact on other countries. The negative technology shock in the home country causes the movement of capital to the foreign country. Therefore, investment and output in foreign countries increase. The fiscal policy, for example an increase in government spending shock, induces people in the home country to work more. The output in home country increases. The government spending shock in home country drives up the world

interest rate, inducing people in the foreign country work harder. The output in the foreign country also increases. In the case of a tax shock, the substitution effects are stronger than the wealth effect, the labour input in home country increases. The output in the home country increases. The marginal product of labour decreases in the foreign country. Therefore, the output in the foreign country decreases because labour input in the foreign country decreases. Whilst fiscal policy can be used to neutralize the impact of a technology shock, it too has long run impacts which will make the economy away from the steady state in the long-run.

Even though governments might also attempt to influence the level of total output and income in an economy by changing their levels of spending on output, transfers, and taxes, there are some problems with fiscal policies such as timing. It takes time for the government to recognize the need for intervention, to agree on a policy, to implement the policy, and for the policy to start affecting the behavior of consumers and firms.

The model suggests that the shocks both permanent and temporary can have a long lived impact. A shock impact can last a long time. Thus the current crisis, if we assume it as a negative technology shock, may have impacts for a decade or even more.

9.5 Next research in real business cycle for Thailand

Real business cycle theory is still an incomplete theory of the business cycle. In particular, simple real business cycle models have demonstrated that equilibrium models are not necessarily inconsistent with many characteristics attributed to the business cycle facts. None is perfectly satisfied. The model economy only gives us a guideline of what we can learn using the neoclassical model as the explanation of how the Thai economy behaves. It is possible to outline more sophisticated models for the Thai economy. The appealing results obtained with the model suggest other use of topics for future research for Thailand such as the indivisible labour model and the fiscal policy in general equilibrium in Thailand. Even though the technology shocks have dominated the central focus in the literature, other shocks such as, money could be included. The combination of the Keynesian theory such as the sticky prices and wages could be interesting to introduce to the model. Regarding to the open economy model with complete markets, the main limitation is that the agents are able to smooth lifetime consumption by sharing

exposure to technology shocks. Therefore, the cross-country output correlations are theoretically near zero or even negative but the cross-country consumption correlations are close to one. One way to solve this problem is to limit the ability of agents to share risk for example in the incomplete market model by Baxter and Crucini (1995). The two-goods two-country model along the lines of Backus, Kehoe and Kydland (1982) are also challenging for future research. The extension of the international real business cycle model for exploring saving and investment correlations across countries such as in Baxter and Crucini (1996) is also interesting. It is a challenging open question that should stimulate a much needed and exciting research effort for the Thai economy as well as other developing countries.

Section A: Technical appendix

1. The first order necessary conditions of HP filter.

An observable time series, y_t , is composed of two main components, the growth component, g_t , and the cyclical component, c_t . Let $y_t = g_t + c_t$,

$$\text{Min}_{\{g_t\}_{t=1}^T} \left\{ \sum_{t=1}^T (y_t - g_t)^2 + \lambda \sum_{t=2}^{T-1} [(g_{t+1} - g_t) - (g_t - g_{t-1})]^2 \right\}$$

FOCs

$$\begin{aligned} g_1 &: -2(y_1 - g_1) + \lambda 2[(g_3 - g_2) - (g_2 - g_1)] = 0 \\ g_2 &: -2\lambda 2[(g_3 - g_2) - (g_2 - g_1)] - 2(y_2 - g_2) + \lambda 2[(g_4 - g_3) - (g_3 - g_2)] = 0 \\ g_3 &: 2\lambda[g_3 - 2g_2 + g_1] - 2\lambda[g_4 - 2g_3 + g_2] - 2(y_3 - g_3) + 2\lambda[g_5 - 2g_4 + g_3] = 0 \\ &\vdots \\ g_{T-1} &: -2(y_{T-1} - g_{T-1}) + 2\lambda[g_{T-3} - 2g_{T-2} + g_{T-1}] - 2\lambda[g_{T-2} - 2g_{T-1} + g_T] = 0 \\ g_T &: -2(y_T - g_T) + 2\lambda[g_{T-2} - 2g_{T-1} + g_T] = 0 \end{aligned}$$

The FOC can be rewritten as the matrix from $c_{Tx1} = \lambda P_{TxT} g_{Tx1}$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ \vdots \\ \vdots \\ c_{T-1} \\ c_T \end{bmatrix}_{Tx1} = \begin{bmatrix} 1 & -2 & 1 & 0 & \dots & \dots & 0 \\ -2 & 5 & -4 & 1 & 0 & \dots & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & \dots & \dots & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & \dots & \dots & \dots & 0 & 1 & -4 & 5 & -2 \\ 0 & \dots & \dots & \dots & \dots & 0 & 1 & -2 & 1 \end{bmatrix}_{TxT} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ \vdots \\ \vdots \\ g_{T-1} \\ g_T \end{bmatrix}_{Tx1}$$

2. The spectral representation

Consider the deterministic time series $y_t = a \cos(\omega t)$. Let realization be λ , and form the time series $y_t = a \cos(\omega t + \lambda)$ where $\tilde{\lambda} \sim U(o, \pi)$. We can write,

$$y_t = a[\cos(\omega t) \cos(\lambda) - \sin(\omega t) \sin(\lambda)]$$

Let $y_t = y_{1t} + y_{2t}$ where $y_{1t} = \cos(w_1 t) a_1 \cos(\lambda_1) - \sin(w_1 t) a_1 \sin(\lambda_1)$
 $y_{2t} = \cos(w_2 t) a_2 \cos(\lambda_2) - \sin(w_2 t) a_2 \sin(\lambda_2)$

Or $y_{jt} = \cos(w_j t) a_j \cos(\lambda_j) - \sin(w_j t) a_j \sin(\lambda_j)$

$$\begin{aligned} y_{jt} &= \sum_{j=1}^N y_{jt} = \sum_{j=1}^N [\cos(w_j t) a_j \cos(\lambda_j) - \sin(w_j t) a_j \sin(\lambda_j)] \\ &= \sum_{j=1}^N [\cos(w_j t) u_j - \sin(w_j t) v_j] \end{aligned}$$

The limit of sums of the areas in these intervals is the integral

$$y_t = \int_0^\pi [\cos(wt) du(w) - \sin(wt) dv(w)] = \int_0^\pi \frac{e^{iwt} + e^{-iwt}}{2} du(w) - \int_0^\pi \frac{e^{iwt} - e^{-iwt}}{2i} dv(w)$$

Let $dz(w) = \frac{1}{2}(du(w) + idv(w))$ and $dv(w) = \frac{2dz(w) - du(w)}{i}$

$$\begin{aligned} y_t &= \int_0^\pi \frac{e^{iwt} + e^{-iwt}}{2} du(w) - \int_0^\pi \frac{e^{iwt} - e^{-iwt}}{2i} \left(\frac{2dz(w) - du(w)}{i} \right) \\ y_t &= \int_0^\pi e^{-iwt} du(w) + \int_0^\pi e^{iwt} dz(w) - \int_0^\pi e^{-iwt} dz(w) \\ &= \int_{-\pi}^\pi e^{iwt} dz(w) \text{ and it is called Cramer representation of } y_t \end{aligned}$$

So, $y_t = \lim_{N \rightarrow \infty} \sum_{j=1}^N a_j \cos(w_j t + \lambda_j) = \int_{-\pi}^\pi e^{iwt} dz(w)$. Since the process $dz(w)$ is built up from independent increment, we can also write

$$dz(w) = \frac{1}{2} \{ du(w) + idv(w) \} \text{ and } dz^*(w) = \frac{1}{2} \{ du(w) - idv(w) \}$$

For coincident increments, we can define the function of $S_y(w)dw$ to be

$$E[dz(w)dz^*(w)] = \begin{cases} S_y(w) & \text{If } \lambda=w \\ 0 & \text{otherwise} \end{cases}$$

That is $S_y(w)dw$ is the variance of the w frequency component of y_t and is called the spectral density function of y_t

3. The auto-covariance and the population spectrum

The population spectrum $S_y(w)$ and the sequence of auto-covariance $\{\gamma_j\}_{j=-\infty}^{\infty}$ contain exactly the same information. If we know $\{\gamma_j\}_{j=-\infty}^{\infty}$, we can calculate the value of $S_y(w)$ for any w . The converse is also true.

Let define $S_y(w)$ as $\int_{-\pi}^{\pi} S_y(w) e^{i w k} dw = \gamma_k$

We can show that

$$\begin{aligned} \int_{-\pi}^{\pi} S_y(w) e^{i w k} dw &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{j=-\infty}^{\infty} \gamma_j e^{-i w j} e^{i w k} dw = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \int_{-\pi}^{\pi} e^{i w (k-j)} dw \\ &= \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \int_{-\pi}^{\pi} [\cos w(k-j) + i \sin w(k-j)] dw \end{aligned}$$

$$\begin{aligned} \text{For } k = j, \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \int_{-\pi}^{\pi} [\cos w(k-j) + i \sin w(k-j)] dw &= \frac{1}{2\pi} \gamma_k \int_{-\pi}^{\pi} [\cos(0) + i \sin(0)] dw \\ &= \frac{1}{2\pi} \gamma_k \int_{-\pi}^{\pi} dw = \gamma_k \end{aligned}$$

$$\begin{aligned} \text{For } k \neq j, \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \int_{-\pi}^{\pi} [\cos w(k-j) + i \sin w(k-j)] dw \\ = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j [(k-j)^{-1} \{ \sin \pi(k-j) - \sin(-\pi(k-j)) \} - (k-j)^{-1} \{ \cos \pi(k-j) - \cos(-\pi(k-j)) \}] \\ = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j [(k-j)^{-1} \{ 2 \sin \pi k \cos(0) - 2 \sin \pi k \sin \pi k \}] \end{aligned}$$

$$\text{Therefore, } \int_{-\pi}^{\pi} S_y(w) e^{i w k} dw = \gamma_k$$

4. Steady state growth (The details of KPR (2002))

According to Swan (1963) and Phepls (1966), it must be restrictions on the form of technical progress if a steady state feasible

$$\text{Let } \gamma_q = \frac{q_t}{q_{t-1}} = 1 + \text{growth rate of } q_t = 1 + \frac{q_t - q_{t-1}}{q_{t-1}}$$

Suppose $Y_t = AF(X_{kt}K_t, X_{nt}N_t)$, where is the capital augmenting technical progress X_{kt}
where is labor augmenting technical progress X_{nt}

$$\begin{aligned}\gamma_Y &= \frac{Y_{t+1}}{Y_t} = F(X_{k,t+1}K_{t+1}, X_{nt}N_t) \\ &= \frac{X_{k,t+1}K_{t+1}}{X_{k,t}K_t} \frac{F(1, \frac{X_{n,t+1}N_{t+1}}{X_{k,t+1}K_{t+1}})}{F(1, \frac{X_{nt}N_t}{X_{k,t}K_t})} \\ &= \gamma_{xk}\gamma_k \frac{F(1, Z_{t+1})}{F(1, Z_t)}, \text{ where } Z_t = \frac{X_{n,t}N_t}{X_{k,t}K_t}\end{aligned}$$

If the ratio Z_t is constant overtime, we would have $\gamma_Y = \gamma_{xk}\gamma_k$. Consider the economy without distorting taxes, $C_t + K_{t+1} - (1 - \delta)K_t \leq Y_t$.

$$\begin{aligned}\gamma_k &= \frac{K_{t+1}}{K_t} = \frac{Y_t - C_t}{K_t} + (1 - \delta_k) \\ &= \frac{Y_t}{K_t} - \frac{C_t}{K_t} + (1 - \delta_k) \quad \text{if } Y > C, \text{ the investment is strictly positive.}\end{aligned}$$

In order for γ_k to be constant, $\frac{Y}{K}, \frac{C}{K}$ must constant as well. The constancy of $\frac{Y}{K}$ implies that $\gamma_k = \gamma_Y$. Since $\gamma_Y = \gamma_{xk}\gamma_k$, $\gamma_{xk} = 1$. Thus, the feasibility of steady state growth requires that there be no capital augmenting technical progress.

If the ratio $\frac{F(1, Z_{t+1})}{F(1, Z_t)}$ is constant irrespective of the constancy of the ratio $Z_t = \frac{X_{n,t}N_t}{X_{k,t}K_t}$, it is possible to write the technical progress as labor augmenting for Cobb-Douglas production function

$$\begin{aligned}Y_t &= A(X_{kt}K_t)^{1-\alpha} (X_{nt}N_t)^\alpha \\ Y_t &= AK_t^{1-\alpha} X_{kt}^{1-\alpha} X_{nt}^\alpha N_t^\alpha = AK_t^{1-\alpha} \widetilde{X}_{nt}^\alpha N_t^\alpha \quad \text{where } \widetilde{X}_{nt} = X_{kt}^{(1-\alpha)/\alpha} X_{nt}\end{aligned}$$

Therefore, technical change must be expressible in labor augmenting form. We utilize the Cobb-Douglas form in our analysis and we can write that production function as

$$Y_t = A_t K_t^{1-\alpha} (N_t X_t)^\alpha$$

where A_t is a temporary displacement to total factor productivity. X_t is a smoothly varying trend growth in labor augmenting technical progress.

5. Feasible Steady State (The details of KPR (2002))

Under the assumption that $A_t = A$ or A_t is constant overtime. Since time devoted to work is bounded by the endowment, it cannot grow in the steady state. Thus, the only admissible constant growth rate of N is zero, i.e; $N_{t+1} = N_t$ or $\gamma_N = 1$.

Given the commodity resource constraint, the growth rate of output can be written

$$\begin{aligned}\frac{Y_{t+1}}{Y_t} &= \frac{C_{t+1} + I_{t+1}}{Y_t} = \frac{C_{t+1}}{Y_t} + \frac{I_{t+1}}{Y_t} = \frac{C_{t+1}}{C_t} \frac{C_t}{Y_t} + \frac{I_{t+1}}{I_t} \frac{I_t}{Y_t} = \gamma_{y,t+1} \\ \gamma_{y,t+1} &= \gamma_{c,t+1} \frac{C_t}{Y_t} + \gamma_{I,t+1} \frac{I_t}{Y_t} = \gamma_{c,t+1} S_{ct} + \gamma_{I,t+1} S_{it}\end{aligned}$$

Recall $C_t + I_t = Y_t$ we have $1 = S_{ct} + S_{it}$

Therefore, $\gamma_{y,t+1} = \gamma_{c,t+1} S_{ct} + \gamma_{I,t+1} (1 - S_{ct})$

$$\gamma_{y,t+1} - \gamma_{I,t+1} = (\gamma_{c,t+1} - \gamma_{I,t+1}) S_{ct}$$

$$S_{ct} = \frac{\gamma_{y,t+1} - \gamma_{I,t+1}}{\gamma_{c,t+1} - \gamma_{I,t+1}}$$

The above condition can be fulfilled only by one of the two following situation, if there is a steady state growth. One is that S_c is constant if it is not true that $\gamma_c - \gamma_I = \gamma_Y - \gamma_I = 0$. Another is that $\gamma_c = \gamma_Y = \gamma_I$. However, if S_c is constant, then it turns out that $\gamma_c = \gamma_Y$ and in turn $\gamma_c = \gamma_Y = \gamma_I$ which violate the first argument. Consequently, $\gamma_c = \gamma_Y = \gamma_I$ must hold in any steady state.

From accumulation equation $K_{t+1} = (1 - \delta)K_t + I_t$

We can write $\frac{K_{t+1}}{K_t} = (1 - \delta) + \frac{I_t}{K_t}$.

$$\gamma_K = (1 - \delta) + \frac{I_t}{K_t} \quad \text{if } \gamma_K \text{ is constant, it follows that } \gamma_K = \gamma_I$$

Consider the production function with constant return to scale $Y_t = A_t F(K_t, X_t)$

We can write $Y_t = X_t A_t F(\frac{K_t}{X_t}, 1)$. If γ_Y grows at constant rate, $\frac{K_t}{X_t}$ must grows at constant rate. Another word, $\gamma_K = \gamma_X$.

There is another way to illustrate this concept with the Cobb-Douglas production function, $Y_t = A_t K_t^{1-\alpha} (N_t X_t)^\alpha$.

$$\frac{Y_{t+1}}{Y_t} = \frac{A_{t+1}}{A_t} \frac{K_{t+1}^{1-\alpha}}{K_t^{1-\alpha}} \frac{(N_{t+1} X_{t+1})^\alpha}{(N_t X_t)^\alpha}, \text{ where } A_t = A \text{ and } \gamma_N = 1$$

$$\log \gamma_Y = \log(\gamma_K^{1-\alpha} \gamma_X^\alpha)$$

$$\log \gamma_Y = (1-\alpha) \log \gamma_K + \alpha \log \gamma_X$$

$$\alpha \log \gamma_X = \alpha \log \gamma_K, \text{ since } \gamma_K = \gamma_Y$$

Therefore, $\gamma_K = \gamma_X$

Collecting these results, we conclude that $\gamma_Y = \gamma_C = \gamma_K = \gamma_I = \gamma_X$. Thus, this model has a unique steady state growth rate.

6. Steady state marginal product (The details of KPR (2002))

The marginal product of capital $AD_1 F(K_t, NX_t)$ is constant overtime.

Under Cobb-Douglas,

$$Y_t = A_t K_t^{1-\alpha} (N_t X_t)^\alpha, \text{ where } A_t = A$$

$$\frac{\partial Y_t}{\partial K_t} = A(1-\alpha) K_t^{-\alpha} (NX_t)^\alpha = (1-\alpha) A N^\alpha \left(\frac{X_t}{K_t}\right)^\alpha$$

Since $\gamma_K = \gamma_X$, $\frac{\partial Y_t}{\partial K_t}$ is constant at the steady state.

The marginal product of a unit of time $X_t AD_2 F(K_t, NX_t)$ grows at γ_X .

Under Cobb-Douglas, $\frac{\partial Y_t}{\partial N} = A_t \alpha K_t^{1-\alpha} N^{\alpha-1} X_t^\alpha$.

Since $\gamma_K = \gamma_X$, $\frac{\partial Y_t}{\partial N}$ grows at γ_X .

7. Local elasticities (The details of KPR (2002))

Define $k_t = \frac{K_t}{X_t}$, $y_t = \frac{Y_t}{X_t}$, so that $y_t = A_t F(k_t, N_t)$

Define S_K is the shares of capital and S_N is the share of labor, so that $S_K + S_N = 1$

Let $Y = F(K, N)$, then $\frac{Y}{N} = F\left(\frac{K}{N}, 1\right) = f(k)$, where $k = \frac{K}{N}$, so that $Y = NF\left(\frac{K}{N}, 1\right)$.

$$\begin{aligned} \frac{\partial Y}{\partial N} &= N(-KN^{-2})F_N\left(\frac{K}{N}, 1\right)(-KN^{-2}) + F\left(\frac{K}{N}, 1\right) \\ &= -\frac{K}{N}F_N\left(\frac{K}{N}, 1\right) + F\left(\frac{K}{N}, 1\right) \end{aligned}$$

$$= f(k) - kDf(k)$$

$$\frac{\partial Y}{\partial K} = F_K\left(\frac{K}{N}, 1\right) = Df(k)$$

The marginal rate of technical substitution of labor for capital $mrs_{k,N}$.

$$mrs_{k,N} = \frac{\frac{\partial Y}{\partial N}}{\frac{\partial Y}{\partial K}} = \frac{f(k) - kDf(k)}{Df(k)}$$

$$\frac{\partial mrs_{k,N}}{\partial k} = \frac{Df(k)[Df(k) - (kD^2 f(k) + Df(k))] - [f(k) - kDf(k)]D^2 f(k)}{[Df(k)]^2}$$

Therefore,
$$\frac{\partial mrs_{k,N}}{\partial k} = -\frac{f(k)D^2 f(k)}{[Df(k)]^2}$$

Denote ζ_{kN} is the local elasticity of substitution of labor for capital.

$$\zeta_{kN} = \frac{TRS}{k/N} \frac{d(k/N)}{dTRS} = \frac{d \ln(k/N)}{d \ln|TRS|}$$

TRS is the technical rate of substitution, which measures the slope of an isoquant line. The elasticity of substitution measures the percentage change in the factor ratio divided by the percentage change in TRS, with output being held fixed.

Under the Cobb-Douglas, calculate TRS

$$Y = f(K, N) = K^{1-\alpha} N^\alpha$$

$$\frac{\partial f(K, N)}{\partial K} = (1-\alpha)K^{-\alpha} N^\alpha = (1-\alpha)\left(\frac{N}{K}\right)^\alpha, \quad \frac{\partial f(K, N)}{\partial N} = \alpha K^{1-\alpha} N^{\alpha-1} = \alpha\left(\frac{K}{N}\right)^{1-\alpha}$$

Let $f(K, N(K^*)) \equiv Y$

$$Y^* = f(K^*, N^*)$$

Total differentiate

$$\frac{\partial f(K, N(K^*))}{\partial K} + \frac{\partial f(K, N(K^*))}{\partial N} \frac{\partial N(K^*)}{\partial K} = 0$$

$$\frac{\partial N(K^*)}{\partial K} = -\frac{\frac{\partial f(K, N(K^*))}{\partial K}}{\frac{\partial f(K, N(K^*))}{\partial N}} = TRS$$

Another way to solve for TRS

$$dy = \frac{\partial f}{\partial K} dK = \frac{\partial f}{\partial N} dN, \quad 0 = \frac{\partial f}{\partial K} dK = \frac{\partial f}{\partial N} dN, \quad \text{and we can write} \quad \frac{dN}{dK} = -\frac{\frac{\partial f}{\partial K}}{\frac{\partial f}{\partial N}} = TRS$$

$$\text{So that } TRS = -\frac{(1-\alpha) \left(\frac{N}{K}\right)^\alpha}{\alpha \left(\frac{K}{N}\right)^{1-\alpha}} = -\frac{(1-\alpha) K}{\alpha N}$$

$$\frac{K}{N} = -\frac{\alpha TRS}{(1-\alpha)}, \quad \ln\left(\frac{K}{N}\right) = \ln\left[\frac{\alpha}{(1-\alpha)}\right] + \ln|TRS|$$

$$\text{Therefore, } d\ln\left(\frac{K}{N}\right) = d\ln|TRS|, \quad \zeta_{kN} = \frac{d\ln\left(\frac{K}{N}\right)}{d\ln|TRS|} = 1 \text{ for the Cobb-Douglas}$$

$$\zeta_{kN} = \frac{d\log(k/N)}{d[\log(\frac{D_1 F(k_t, N_t)}{D_2 F(k_t, N_t)})]}, \quad \text{where } \log x = -\log \frac{1}{x}$$

$$\zeta_{kN} = \frac{d\log(k/N)}{d[\log(\frac{D_2 F(k_t, N_t)}{D_1 F(k_t, N_t)})]}, \quad \text{where } k_t = \frac{K_t}{X_t} \text{ and } y_t = A_t F(k_t, N_t)$$

$$= \frac{d\log\left(\frac{K}{N}\right)}{d\log(mrs_{k,N})}$$

$$\begin{aligned} \xi_{NN} &= \frac{F_{NN} N}{F_N} = \frac{\frac{\partial}{\partial N} \left(\frac{\partial Y}{\partial N} \right) N}{\frac{\partial Y}{\partial N}} = \frac{N \frac{\partial}{\partial N} \left[-\frac{K}{N} F_N \left(\frac{K}{N}, 1 \right) + F \left(\frac{K}{N}, 1 \right) \right]}{f(k) - kDf(k)} \\ &= \frac{N \left\{ \left(-\frac{K}{N} F_{NN} \left(\frac{K}{N}, 1 \right) K (-N^{-2}) + F_N \left(\frac{K}{N}, 1 \right) K N^{-2} \right) + F_N \left(\frac{K}{N}, 1 \right) K (-N^{-2}) \right\}}{f(k) - kDf(k)} = \frac{k^2 D^2 f(k)}{f(k) - kDf(k)} \end{aligned}$$

$$\begin{aligned} \xi_{NK} &= \frac{F_{NK} K}{F_N} = \frac{\frac{\partial}{\partial K} \left(\frac{\partial Y}{\partial N} \right) K}{\frac{\partial Y}{\partial N}} \\ &= \frac{\left(K - \frac{K}{N} F_{NK} \left(\frac{K}{N}, 1 \right) \frac{1}{N} - \frac{1}{N} F \left(\frac{K}{N}, 1 \right) + \frac{1}{N} F_K \left(\frac{K}{N}, 1 \right) \right)}{f(k) - kDf(k)} = -\frac{k^2 D^2 f(k)}{f(k) - kDf(k)} \end{aligned}$$

$$\xi_{KK} = \frac{F_{KK} K}{F_K} = \frac{K \frac{\partial}{\partial K} \left(F_K \left(\frac{K}{N}, 1 \right) \right)}{Df(k)} = \frac{kD^2 f(k)}{Df(k)}$$

$$\xi_{KN} = \frac{F_{NN}N}{F_K} = \frac{N \frac{\partial}{\partial K} (F_K(\frac{K}{N}, 1))}{Df(k)} = \frac{NF_{KN}(\frac{K}{N}, 1)(-KN^{-2})}{Df(k)} = -\frac{kD^2 f(k)}{Df(k)}$$

Therefore, $\xi_{NN} = -\xi_{NK}$ and $\xi_{KK} = -\xi_{KN}$

Define factor share

$$S_N = \frac{\frac{\partial Y}{\partial N} N}{Y} = \frac{N[f(k) - kDf(k)]}{F(K, N)} = \frac{f(k) - kDf(k)}{f(k)}$$

$$S_K = \frac{\frac{\partial Y}{\partial K} K}{Y} = \frac{K[Df(k)]}{F(K, N)} = \frac{kDf(k)}{f(k)}$$

$$\text{Since } \zeta_{KN} = \frac{d \log(\frac{K}{N})}{d \log(mrs_{k,N})} = -\frac{Df(k)[f(k) - kDf(k)]}{kf(k)D^2 f(k)}, \text{ we have}$$

$$\frac{S_K}{\xi_{NN}} = \frac{kDf(k)}{f(k)} \frac{[f(k) - kDf(k)]}{k^2 D^2 f(k)} = \zeta_{KN}$$

$$\frac{S_N}{\xi_{KK}} = \frac{[f(k) - kDf(k)]}{f(k)} \frac{Df(k)}{kD^2 f(k)} = \zeta_{KN}$$

$$\text{Thus } \xi_{KK} = -\xi_{KN} = -\frac{S_N}{\zeta_{KN}}, \quad \xi_{NN} = -\xi_{NK} = -\frac{S_K}{\zeta_{KN}}$$

For Cobb-Douglas,

$$\zeta_{KN} = 1 \text{ and } S_N = \alpha, \text{ so that } \xi_{KK} = -\frac{S_N}{\zeta_{KN}} = -\alpha \text{ and } \xi_{NN} = -\frac{S_K}{\zeta_{KN}} = -\frac{(1-S_N)}{\zeta_{KN}} = -(1-\alpha)$$

8. Restriction on Preferences (The details of KPR (2002))

(i) The optimal steady state restricts the utility function so that there is an invariant intertemporal substitution in consumption.

According to efficiency conditions,

$$\begin{aligned} \beta^{t+1} D_1 u(C_{t+1}, 1 - N_{t+1}) &= \Lambda_{t+1} \\ \beta^t D_1 u(C_t, 1 - N_t) &= \Lambda_t \end{aligned}$$

$$\text{So we can write } \frac{\beta^t D_1 u(C_{t+1}, L)}{D_1 u(C_t, L)} = \gamma_\Lambda$$

$$\frac{D_1 u(C_t, L)}{D_1 u(C_{t+1}, L)} = \frac{\beta}{\gamma_\Lambda} = \beta[AD_1 F(k, N) + (1 - \delta_K)]$$

where $\frac{D_1 u(C_t, L)}{D_1 u(C_{t+1}, L)}$ is the marginal utility of consumption grows at constant rate.

If marginal utility is to grow at constant rate for all potential values of x_0 , it must be the case

that
$$\frac{D_{11} u(C_{t+1}, L) C_{t+1}}{D_1 u(C_t, L)} \frac{dx_0}{x_0} - \frac{D_{11} u(C_t, L) C_t}{D_1 u(C_t, L)} \frac{dx_0}{x_0} = 0$$

and $\frac{D_{11} u(C_t, L) C_t}{D_1 u(C_t, L)}$, defined as σ_C : the elasticity of marginal utility with respect to consumption, must be constant overtime.

(ii) Invariance of efficient steady state labor

According to efficiency conditions, $\Lambda_t = \beta^t D_1 u(C_t, L_t)$

$$\Lambda_t AD_2 F(K_t, NX_t) = \beta^t D_2 u(C_t, L_t)$$

Therefore, $D_1 u(C_t, L_t) AD_2 F(K_t, NX_t) = D_2 u(C_t, L_t)$

$$\log[D_1 u(C_t, L_t)] + \log[AD_2 F(K_t, NX_t)] = \log[D_2 u(C_t, L_t)]$$

Therefore, the steady state implies

1) The marginal product of effective labor is constant at steady state.

$$AD_2 F(K_t, NX_t) = AD_2 F(k, N)$$

2) X_t , technology, grows at rate γ_x

3) The marginal utility of consumption $D_1 u(C_t, L_t)$ grows at $\gamma_C^{-\sigma_C}$, where σ_C is the elasticity and $\gamma_x = \gamma_C$.

9. Level and Ratios at Steady State (The details of KPR (2002))

Use the efficiency conditions together with the requirement that all variables grow at a constant rate to get the steady state value of several variables and ratios.

Recall $\frac{D_1 u(C_t, L)}{D_1 u(C_{t+1}, L)} = \frac{\beta}{\gamma_\Lambda}$, then. $\frac{1}{\gamma_C^{-\sigma_C}} = \frac{\beta}{\gamma_\Lambda}$

$$\gamma_\Lambda = \beta \gamma_C^{-\sigma_C}, \text{ so that } -\sigma_C \log \gamma_C = \log\left(\frac{\gamma_\Lambda}{\beta}\right) = -\sigma_C \log \gamma_x$$

Recall efficiency conditions,

$$\begin{aligned}\frac{\Lambda_{t+1}}{\Lambda_t}[(1-\tau)AD_1F(k, N) + (1-\delta_K)] &= 1, \quad k = \frac{K}{X} \\ [(1-\tau)AD_1F(k, N) + (1-\delta_K)] &= \frac{1}{\gamma_\Lambda}, \quad \gamma_\Lambda = \beta\gamma_C^{-\sigma_C} \\ \beta[(1-\tau)AD_1F(k, N) + (1-\delta_K)] &= \gamma_C^{\sigma_C}, \quad \gamma_C = \gamma_X \\ \frac{\gamma_X}{\gamma_X^{\sigma_C}} \beta[(1-\tau)AD_1F(k, N) + (1-\delta_K)] &= \gamma_X^{\sigma_C} \frac{\gamma_X}{\gamma_X^{\sigma_C}}\end{aligned}$$

Consequently, $\beta\gamma_X^{1-\sigma_C}[(1-\tau)AD_1F(k, N) + (1-\delta_K)] = \gamma_X$

Let $\beta^* = \beta\gamma_X^{1-\sigma_C}$, we write $\gamma_X = \beta^*[(1-\tau)AD_1F(k, N) + (1-\delta_K)]$

Since $S_K = \frac{\partial Y}{\partial K} \frac{K}{Y}$, $\frac{\gamma_X - \beta^*(1-\delta_K)}{\beta^* S_K \Omega} = \frac{Y}{K}$

For the Cobb-Douglas case, $\frac{\gamma_X - \beta^*(1-\delta_K)}{\beta^* \frac{\partial Y}{\partial K} \frac{K}{Y} \Omega} = \frac{\beta^* [\Omega A_t (1-\alpha) (\frac{X_t N_t}{K_t})^\alpha] - \beta^*(1-\delta_K)}{\beta^* \frac{K}{Y} \Omega A_t (1-\alpha) (\frac{X_t N_t}{K_t})^\alpha} = \frac{Y}{K}$

Therefore, the output capital ratio $\frac{Y}{K} = \frac{\gamma_X - \beta^*(1-\delta_K)}{\beta^* S_K \Omega}$, if $\gamma_X \geq 1, \beta^* < 1$, then $\frac{Y}{K} > 0$

Consider $S_N = \frac{\partial Y}{\partial N} \frac{N}{Y} = \alpha A_t X_t^\alpha (\frac{K_t}{N_t})^{1-\alpha} \frac{N}{Y} = \alpha$

$$\left[\frac{\gamma_X - \beta^*(1-\delta_K)}{\beta^* S_K \Omega A} \right]^{\frac{1}{S_N}} = \left(\frac{Y}{KA} \right)^{\frac{1}{\alpha}} = \left(\frac{AK_t^{1-\alpha} (NX)^\alpha}{AK} \right)^{\frac{1}{\alpha}} = \frac{NX}{K}$$

Therefore, the effective labor-capital ratio $\frac{NX}{K} = \left[\frac{\gamma_X - \beta^*(1-\delta_K)}{\beta^* S_K \Omega A} \right]^{\frac{1}{S_N}}$

We can write $\frac{\frac{Y}{K}}{\frac{NX}{K}} = \frac{Y}{NX} = \frac{\frac{\gamma_X - \beta^*(1-\delta_K)}{\beta^* S_K \Omega}}{\left(\frac{\gamma_X - \beta^*(1-\delta_K)}{\beta^* S_K \Omega A} \right)^{\frac{1}{S_N}}} = A^{\frac{1}{\alpha}} \left(\frac{\gamma_X - \beta^*(1-\delta_K)}{\beta^* S_K \Omega} \right)^{1-\frac{1}{\alpha}}$

$$\begin{aligned} &= A^{\frac{1}{\alpha}} \left(\frac{\beta^* S_K \Omega}{\gamma_X - \beta^*(1-\delta_K)} \right)^{\frac{1-\alpha}{\alpha}} \\ S_K &= \frac{\partial Y}{\partial K} \frac{K}{Y} = \frac{A_t (1-\alpha) K_t^{-\alpha} (N_t X_t)^\alpha K}{A_t K_t^{1-\alpha} (N_t X_t)^\alpha} = 1-\alpha\end{aligned}$$

Therefore, the output-labor ratio $\frac{Y}{NX} = A^{\frac{1}{\alpha}} \left(\frac{\gamma_X - \beta^*(1-\delta_K)}{\beta^* S_K \Omega} \right)^{\frac{S_K}{S_N}}$

$$\begin{aligned}\text{Shares of output devoted to gross investment } S_i &= [\gamma_x - (1 - \delta_K)] \frac{K}{Y} \\ &= \frac{[\gamma_x - (1 - \delta_K)][\beta^* S_K \Omega]}{\gamma_x - \beta^* (1 - \delta_K)}\end{aligned}$$

Given that the share of government is governed by $S_C = 1 - S_i - S_g$
According to the efficiency condition,

$$D_1 u(C_t, L) \Omega X_t A D_2 F(K_t, N X_t) = D_2 u(C_t, L)$$

with $\sigma_c = 1$, we have $\frac{1}{S_C} \Omega \frac{\alpha}{N} = D v(1 - N)$

if $u(C, L) = \log C + \theta_l \log(L)$, the steady state level of labor $N = \frac{\alpha \Omega}{\alpha \Omega + S_C \theta_l}$

10. The Transformed Economy (The details of KPR (2002))

Transform variables, expressing preferences and technology in term of variables that will be constant in the steady state. Capital accumulation equation is alter as follow

Denote $c = C/X, k = K/X, g = G/X$,

$$\begin{aligned}\text{Recall, } K_{t+1} &= (1 - \delta_K) K_t + I_t \\ &= (1 - \delta_K) K_t + A_t F(K_t, N_t X_t) - C_t\end{aligned}$$

$$\begin{aligned}\text{Since } \frac{K_{t+1}}{X_t} &= \frac{K_{t+1}}{X_{t+1}} \frac{X_{t+1}}{X_t} = \gamma_x k_{t+1}, \text{ we write } \gamma_x k_{t+1} = (1 - \delta_K) k_t + i_t \\ &= (1 - \delta_K) k_t + A_t F(k_t, N_t) - c_t\end{aligned}$$

The process of transforming consumption (by dividing by X_t) alters the effective rate of time preference.

$$U = \sum_{t=0}^{\infty} \beta^t [u(C_t, L_t)] = \sum_{t=0}^{\infty} \beta^t [u(c_t X_t, L_t)]$$

If $\sigma_c \neq 1$

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t [u(c_t X_t, L_t)] &= \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1 - \sigma_c} (c_t X_t)^{1 - \sigma_c} v(L) \right] \\ &= \sum_{t=0}^{\infty} \beta^t X_t^{1 - \sigma_c} \left[\frac{1}{1 - \sigma_c} c_t^{1 - \sigma_c} v(L) \right] \\ &= X_0^{1 - \sigma_c} \sum_{t=0}^{\infty} \beta^t \frac{X_t^{1 - \sigma_c}}{X_0^{1 - \sigma_c}} \left[\frac{1}{1 - \sigma_c} c_t^{1 - \sigma_c} v(L) \right]\end{aligned}$$

$$\begin{aligned}
&= X_0^{1-\sigma_c} \sum_{t=0}^{\infty} \beta^t \gamma_x^{1-\sigma_c} \left[\frac{1}{1-\sigma_c} c_t^{1-\sigma_c} v(L) \right] \\
&= \sum_{t=0}^{\infty} \beta^{*t} \left[\frac{1}{1-\sigma_c} c_t^{1-\sigma_c} v(L) \right], \text{ where } \beta^* = \beta(\gamma_x)^{1-\sigma}
\end{aligned}$$

If $\sigma_c = 1, \beta^{*t} = \beta^t$

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t [u(c_t X_t, L_t)] &= \sum_{t=0}^{\infty} \beta^t [\log c_t X_t + v(L)] \\
&= \sum_{t=0}^{\infty} \beta^t [\log c_t + v(L) + \log X_t] \\
&= \sum_{t=0}^{\infty} \beta^{*t} [\log c_t + v(L)]
\end{aligned}$$

The term $X_0^{1-\sigma_c}$ and $\sum_{t=0}^{\infty} \beta^{*t} \log X_t$ can be ignored since they do not affect the preference ordering on the transformed variables. We may set $X_0 = 1$ or $\sum_{t=0}^{\infty} \beta^{*t} \log X_t = 0$.

11. The discounted sum (The details of KPR (2002))

Hansen-Sargent (1980,1981)

Computation of $\tilde{\lambda}(I_t)$

Recall the transformed shadow price

$$\tilde{\lambda}_t = \mu_2^{-1} \tilde{\lambda}_{t+1} - \mu_2^{-1} [p_{21}^* R_{ke} + p_{22}^* R_{\lambda e}] \hat{e}_{t+1} - \mu_2^{-1} [p_{21}^* Q_{ke} + p_{22}^* Q_{\lambda e}] \hat{e}_t$$

we can write in the form

$$\tilde{\lambda}_t = \mu_2^{-1} \tilde{\lambda}_{t+1} + z_1 \hat{e}_{t+1} + z_0 \hat{e}_t$$

$\tilde{\lambda}_t$ has $ns \times 1$, μ_2^{-1} has $ns \times ns$, z_1, z_0 have $ns \times ne$, and \hat{e}_t has $ne \times 1$ dimension where ns is the number of state variables. Under the certainty equivalence assumption, this difference equation is assumed to hold in expectation form,

$$\tilde{\lambda}_t = \mu_2^{-1} E_t \tilde{\lambda}_{t+1} + z_1 E_t \hat{e}_{t+1} + z_0 E_t \hat{e}_t$$

Hansen-Sargent formulas can be employed line-by-line basis.

First, partition z_0 and z_1 into ns row vectors, denoted z_{0i} and z_{1i} , $i = 1, 2, \dots, ns$.

It follows that $\tilde{\lambda}_{it} = \mu_{2i}^{-1} E_t \tilde{\lambda}_{i,t+1} + z_{1i} E_t \hat{e}_{t+1} + z_{0i} \hat{e}_t$ solve it by forward recursive

$$\tilde{\lambda}_{i,t+1} = \mu_{2i}^{-1} E_{t+1} \tilde{\lambda}_{i,t+2} + z_{1i} E_{t+1} \hat{e}_{t+2} + z_{0i} E_{t+1} \hat{e}_{t+1}$$

$$\begin{aligned}
\tilde{\lambda}_{it} &= \mu_{2i}^{-1} E_t \left\{ \mu_{2i}^{-1} E_{t+1} \tilde{\lambda}_{i,t+2} + z_{1i} E_{t+1} \hat{e}_{t+2} + z_{0i} E_{t+1} \hat{e}_{t+1} \right\} + z_{1i} E_t \hat{e}_{t+1} + z_{0i} \hat{e}_t \\
&= \mu_{2i}^{-2} E_t \tilde{\lambda}_{i,t+2} + \mu_{2i}^{-1} z_{1i} E_t \hat{e}_{t+2} + \mu_{2i}^{-1} z_{0i} E_t \hat{e}_{t+1} + z_{1i} E_t \hat{e}_{t+1} + z_{0i} E_t \hat{e}_t \\
&= \mu_{2i}^{-k} E_t \tilde{\lambda}_{i,t+k} + \sum_{j=0}^{k-1} \mu_{2i}^{-j} z_{1i} E_t \hat{e}_{t+1+j} + \sum_{j=0}^{k-1} \mu_{2i}^{-j} z_{0i} E_t \hat{e}_{t+j} \\
&= \mu_{2i}^{-k} E_t \tilde{\lambda}_{i,t+k} + z_{1i} \sum_{j=0}^{k-1} \mu_{2i}^{-j} E_t \hat{e}_{t+1+j} + z_{0i} \sum_{j=0}^{k-1} \mu_{2i}^{-j} E_t \hat{e}_{t+j}
\end{aligned}$$

The vector of exogenous variables is an $AR(1)$

$$\begin{aligned}
\hat{e}_t &= \rho \hat{e}_{t+1} + \varepsilon_t \\
\hat{e}_{t+j} &= \rho \hat{e}_{t+j-1} + \varepsilon_{t+j} \\
\hat{e}_{t+j+1} &= \rho \hat{e}_{t+j} + \varepsilon_{t+j+1}
\end{aligned}$$

$$\begin{aligned}
\text{Consequently, } \tilde{\lambda}_{it} &= \mu_{2i}^{-k} E_t \tilde{\lambda}_{i,t+k} + z_{1i} \sum_{j=0}^{k-1} \mu_{2i}^{-j} E_t (\rho \hat{e}_{t+j} + \varepsilon_{t+j-1}) + z_{0i} \sum_{j=0}^{k-1} \mu_{2i}^{-j} E_t (\rho \hat{e}_{t+j-1} + \varepsilon_{t+j}) \\
&= \mu_{2i}^{-k} E_t \tilde{\lambda}_{i,t+k} + z_{1i} \sum_{j=0}^{k-1} \mu_{2i}^{-j} \rho E_t \hat{e}_{t+j} + z_{0i} \sum_{j=0}^{k-1} \mu_{2i}^{-j} \rho E_t \hat{e}_{t+j-1} \\
&= \mu_{2i}^{-k} E_t \tilde{\lambda}_{i,t+k} + z_{1i} \frac{1}{(1 - \rho \mu_{2i}^{-1})} E_t \hat{e}_{t+j} + z_{0i} \frac{1}{(1 - \rho \mu_{2i}^{-1})} E_t \hat{e}_{t+j-1} \\
&= \mu_{2i}^{-k} E_t \tilde{\lambda}_{i,t+k} + (z_{1i} E_t \hat{e}_{t+j} + z_{0i} E_t \hat{e}_{t+j-1}) \frac{1}{(1 - \rho \mu_{2i}^{-1})} \\
&= \frac{(z_{1i} \rho + z_{0i}) \hat{e}_t}{(1 - \rho \mu_{2i}^{-1})} \quad \text{since } \mu_{2i} > 1
\end{aligned}$$

12. Tobin's q

Consider an investment dynamic with adjustment costs. A firm wants to minimize discounted cost of producing a given level.

$$\text{Firm maximize } -\sum_{t=0}^{\infty} \beta^t (w_t n_t - p_t^i i_t)$$

$$\text{subject to } y_t = f(k_t, n_t) \text{ and } k_{t+1} = k_t - \delta k_t + h\left(\frac{i_t}{k_t}\right) k_t$$

$$\text{Or, } V(k_t) = \text{Max}_{i_t, n_t, k_{t+1}} - \left\{ (w_t n_t - p_t^i i_t) + \beta E_t V(k_{t+1}) \right\}$$

$$\text{subject to } y_t = f(k_t, n_t) \text{ and } k_{t+1} = k_t - \delta k_t + h\left(\frac{i_t}{k_t}\right) k_t$$

$$\text{Therefore, } V(k_t) = - \left\{ (w_t n_t - p_t^i i_t) + \beta E_t V(k_{t+1}) \right\} + \lambda_t \{ y_t - f(k_t, n_t) \}$$

$$+\psi_t \left\{ h\left(\frac{i_t}{k_t}\right)k_t - k_{t+1} + k_t - \delta k_t \right\}$$

FOCs

$$i_t: \quad p_t^i = \psi_t h'\left(\frac{i_t}{k_t}\right)$$

$$n_t: \quad -\psi_t = \lambda_t f_n(k_t, n_t)$$

$$k_{t+1}: \quad \beta E_t V_{k_{t+1}}(k_{t+1}) = \psi_t$$

$$\lambda_t: \quad y_t = f(k_t, n_t)$$

$$\psi_t: \quad k_{t+1} = (1-\delta)k_t + h\left(\frac{i_t}{k_t}\right)k_t$$

The Envelope theorem

$$V_{k_t}(k) = -\lambda_t f_n(k_t, n_t) + \psi_t \{h(z) - zh(z) - \delta - 1\}$$

$$V_{k_{t+1}}(k_{t+1}) = -\lambda_{t+1} f_{k_{t+1}}(k_{t+1}, n_{t+1}) + \psi_{t+1} \{h(z_{t+1}) - z_{t+1}h(z_{t+1}) - \delta - 1\}$$

$$\begin{aligned} \text{Tobin's } q &= \frac{\text{marginal value of unit of capital}}{\text{its cost of capital}} = \frac{\beta E_t V_{k_{t+1}}(k_{t+1})}{p_t^i} \\ &= \frac{\psi_t}{p_t^i} = \frac{1}{h'\left(\frac{i_t}{k_t}\right)} \end{aligned}$$

Section B: Programming code

```
%*****
%**
%**                                     Thai RBC Model
%**
%*****

% Observation period for calibration 1976-2005 quaterly
clc;
disp('THE THAI BUSINESS CYCLE 1976-2005');
disp('*****');
disp(' ');
clear all;

%Variable Description
%*****
nc=2; %controls c n
nbs=1; %endogenous backward looking state k
nfs=1; %endogenous forward looking state ld
ns=nbs+nfs; %total number of states
ne=3; %exogenous variables a g om
nf=4; %flows variables are y w r i
```



```

%Technology Specification
%*****
alpha=0.64; %share of labour
        % (1-alpha) is share of capital
%the elasticity of substitution for Cobb-Douglas is 1
cikk=-alpha; %the elasticity of MP of k with respect to k
cikn=alpha; % the elasticity of MP of k with respect to n
cinn=-(1-alpha); % the elasticity of MP of n with respect to k
cink=(1-alpha); % the elasticity of MP of n with respect to k
%disp('The elasticities of substitution for the Cobb-Douglas');
%disp(['cikk is: ' num2str(cikk)]);
%disp(['cikn is: ' num2str(cikn)]);
%disp(['cinn is: ' num2str(cinn)]);
%disp(['cink is: ' num2str(cink)]);
%disp(' ');

%Capital accumulation
delta=0.0099; %capital depreciation rate

%Government
%*****
sg=0.0998; % SS share of government spending
omega=1; %wedge function

%Steady State
%*****
gamma=1.01173; %growth of labour augmenting tech
r=0.0173;
Nbar=0.192; %a time devoted to market work at steady state
betas=gamma/(1+r); %steady state discount factor
kyratio=(betas*(1-alpha)*omega)/(gamma-betas*(1-delta)); % Capital-output
ratio
si=(gamma-(1-delta))*kyratio; %investment-output ratio
sc=1-si-sg; %consumption-output ratio
ikratio=(gamma-1+delta);
phi=gamma/(gamma-(1-delta));
%disp('Steady State Calculation');
%disp('*****');
%disp(['gamma = ' num2str(gamma)]);
%disp(['r = ' num2str(r)]);
%disp(['nbar = ' num2str(nbar)]);
%disp(['bstar = ' num2str(bstar)]);
%disp(['kyratio = ' num2str(kyratio)]);
%disp(['si = ' num2str(si)]);
%disp(['sc = ' num2str(sc)]);
%disp(['ikratio = ' num2str(ikratio)]);
%disp(' ');

%The steady state elasticities of the aftertax marginal product of capital
etaa=(gamma-betas*(1-delta))*(1)/gamma;
etak=(gamma-betas*(1-delta))*(cikk)/gamma;
etan=(gamma-betas*(1-delta))*(cikn)/gamma;
etag=(gamma-betas*(1-delta))*(0)/gamma;
%disp('The elasticities of the aftertax marginal product of capital');
%disp(['etaa is: ' num2str(etaa)]);
%disp(['etak is: ' num2str(etak)]);

```

```

%disp(['etan is: ' num2str(etan)]);
%disp(['etag is: ' num2str(etag)]);
%disp(' ');

%Preference Specification
%*****
% denote l is leisure
sigma=2; % the degree of intertemporal substitution
theta=sc/(alpha*omega*((1-Nbar)/Nbar)+sc);
%disp(['theta = ' num2str(theta)]);
disp(' ');
cicc=theta*(1-sigma)-1; % the elasticity of MU of c with respect to c
cicl=(1-theta)*(1-sigma); % the elasticity of MU of c with respect to l
cilc=theta*(1-sigma); % the elasticity of MU of l with respect to c
cill=(1-theta)*(1-sigma)-1; % the elasticity of MU of l with respect to l

%disp('The elasticities of the marginal utility');
%disp(['cicc is: ' num2str(cicc)]);
%disp(['cicl is: ' num2str(cicl)]);
%disp(['cilc is: ' num2str(cilc)]);
%disp(['cill is: ' num2str(cill)]);
%disp(' ');

%System Matrices
%*****

%Matrices in Control Equations
%*****

%Define Mcc Matrix
%
Mcc=[cicc          -cicl*(Nbar/(1-Nbar))
     cilc          -cill*(Nbar/(1-Nbar))-(cinn)];

%Define Mcs Matrix
%
Mcs=[ 0          1
     cink          1];

%Define Mce Matrix
%
Mce=[ 0          0          0
     1          0          1];

%Matrices in State Equations
%*****

%Define Mss(B) Matrices
%
Mss0=[ etak          1
     si*phi          0];

%
Mss1=[          0          -1
     -(1-alpha)-si*(phi-1)    0];

```

```

%Define Msc(B) Matrices
%
Msc0=[0 etan
      0 0];

%
Msc1=[ 0 0
      -sc alpha];

%Define Mse(B) Matrices
%
Mse0=[-etaa -etag -etaa
      0 0 0];

%
Mse1=[ 0 0 0
      1 -sg 0];
% Matrices in Flow Equations
%*****

%Define FVc
%
FVc=[ 0 alpha
      0 alpha-1
      0 alpha
      -sc/si alpha/si];

%Define FVke
%
FVke=[ 1-alpha 1 0 0
      1-alpha 1 0 1
      -alpha 1 0 1
      (1-alpha)/si 1/si -sg/si 0];

FVl=zeros(nf,1);

%Fundamental State-Costate Difference eq
%*****
MSss0=Mss0-Msc0*(Mcc\Mcs);
MSss1=Mss1-Msc1*(Mcc\Mcs);
MSse0=Mse0+Msc0*(Mcc\Mce);
MSse1=Mse1+Msc1*(Mcc\Mce);

%Convert to a normal difference eq
W=-(MSss0\MSss1);
R=MSss0\MSse0;
Q=MSss0\MSse1;

%Compute the eigenvalues and eigenvectors of W
[P,MU]=eig(W);
%disp('Eigenvectors(P) of the state transition matrix');
%disp(P);
%disp('Eigenvalues(MU) of the state transition matrix');
%disp(MU);

```

```

%Order the eigenvalues, in the order of ascending absolute value
AMU=abs(MU);

%Verifying the counting rule
disp('Blanchard and Khan condition')
disp('*****')
disp('Check Blanchard and Khan condition')
Deigv=diag(abs(MU));
Vcr=sum(Deigv>1);
if Vcr==nfs;
    disp('Number of non-predetermined variable = Number of unstable root');
    disp('Counting Rule is hold');
else
    disp('Counting Rule does not hold');
end;
disp(' ');

%Sort by the absolute value of the eigenvectors, then reorder the
%eigenvector by this sort
[MU,j]=sort(diag(AMU));
P=P(:,j);
PS=inv(P);
MU=PS*W*P;
%disp('Eigenvalues in order of ascending absolute value');
%disp(MU);

%Partitoning matrix of eigenvalue
MU1=MU(1:nbs,1:nbs);
MU2=MU(nbs+1:2*nbs,nbs+1:2*nbs);

%Partitioning matrix of eigenvector
P11=P(1:nbs,1:nbs);
P12=P(1:nbs,nbs+1:2*nbs);
P21=P(nbs+1:2*nbs,1:nbs);
P22=P(nbs+1:2*nbs,nbs+1:2*nbs);

%Partitioing matrix of inverst eigenvector
PS11=PS(1:nbs,1:nbs);
PS12=PS(1:nbs,nbs+1:2*nbs);
PS21=PS(nbs+1:2*nbs,1:nbs);
PS22=PS(nbs+1:2*nbs,nbs+1:2*nbs);

%Partitioning matrix R
Rke=R(1:nbs,1:ne);
Rle=R(nbs+1:2*nbs,1:ne);

%Partitioning matrix Q
Qke=Q(1:nbs,1:ne);
Qle=Q(nbs+1:2*nbs,1:ne);

%lambda
SP1=-MU2\ (PS21*Rke+PS22*Rle);
SP2=-MU2\ (PS21*Qke+PS22*Qle);

```

```

%capital
KLK=P11*MU1/P11;
KTL=(P11*MU1*PS12+P12*MU2*PS22)/PS22;

%Driving Process
%*****
rhoa=0.930; %Persistence of the technology shock
rhog=0.944; %Persistence of the government spending shock

%Persistence Matrix
%
RHO=[rhoa    0    0
      0    rhog    0
      0    0    0];

%System Dynamics
%*****
FL=SP1*RHO+SP2;
IRHO=eye(size(RHO));
for i=1:nbs;
    q=FL(i,1:ne);
    mu2i=1/MU2(i,i);
    disu=inv((IRHO-mu2i*RHO));
    LE(i,1:ne)=q*disu;
end

KEC=Rke*RHO+Qke+KTL*LE;

%Vaughan's approach
ULE=PS22\LE;
ULK=-PS22\PS21;

%State Transition Matrix
Mke=[KLK    KEC
      zeros(ne,nbs)    RHO];

%Incorporation of lambda, controls and other flows
Lke=[ULK ULE];
T=Mcc\Mcs;
MOck=T(1:nc,1:nbs);
MOcl=T(1:nc,nbs+1:2*nbs);
MOce=Mcc\Mce;
clear T;

MOcke=[MOck+MOcl*ULK    MOce+MOcl*ULE];
Fke=[FVc*MOcke+FVke+FV1*Lke];

PI=[Lke
     MOcke
     Fke];
PI'; %Illustrate PI transpose

%Impluse Response
%*****
disp('Driving process');

```

```

disp('*****');
[H,nMke]=size(Mke); %Store dimensions of Mke
S=zeros(nMke,1); %Create a shock column vector
disp('The shock variables are denoted as the number below');
disp('1 = Capital(k)');
disp('2 = Total factor productivity(A)');
disp('3 = Governemt spending(g)');

disp(' ');
IVS=input('Select a number 1 to 4 for a shock variable = ');
S(IVS)=input('Enter size of shock in percent of variance = ');
disp(' ');
add=input('Enter 1 to add the second shock, Enter 0 to shock now= ');
    if add == 0,
        S=S;
    elseif add == 1,
        IVS=input('Select a number 1 to 4 for a shock variable = ');
        S(IVS)=input('Enter size of shock in percent of variance = ');
        disp(' ');
        add=input('Enter 1 to add the third shock, Enter 0 to shock now= ');
            if add == 0,
                S=S;
            elseif add == 1,
                IVS=input('Select a number 1 to 4 for a shock variable = ');
                S(IVS)=input('Enter size of shock in percent of variance = ');
                disp(' ');
                add=input('Enter 1 to add the fourth shock, Enter 0 to shock now= ');
                    if add == 0,
                        S=S;
                    elseif add == 1,
                        IVS=input('Select a number 1 to 4 for a shock variable = ');
                        S(IVS)=input('Enter size of shock in percent of variance = ');
                        S=S;
                    end
                end
            end
        end
    end

np=12; %number of periods for which the impluse response is to estimate
[npi,K]=size(PI);

IRM=zeros(np,np+nMke+1);
for i=1:2;
    IRM(i,1)=i;
    IRM(i,2:nMke+1)=S';
    IRM(i,nMke+2:npi+nMke+1)=(PI*S)';
    S=Mke*S;
end
SP=S;

disp(' ');
disp('Shock again');
S=zeros(nMke,1); %Create a shock column vector
disp('The shock variables are denoted as the number below');
disp('1 = Capital(k)');
disp('2 = Total factor productivity(A)');

```

```

disp('3 = Governemt spending(g)');
disp(' ');
IVS=input('Select a number 1 to 4 for a shock variable = ');
S(IVS)=input('Enter size of shock in percent of variance = ');
disp(' ');
SN=S;
SM=SP+SN;

for i=3:4;
    IRM(i,1)=i;
    IRM(i,2:nMke+1)=SM';
    IRM(i,nMke+2:npi+nMke+1)=(PI*SM)';
    SM=Mke*SM;
end

SP=SM;

disp(' ');
disp('Shock again');
S=zeros(nMke,1); %Create a shock column vector
disp('The shock variables are denoted as the number below');
disp('1 = Capital(k)');
disp('2 = Total factor productivity(A)');
disp('3 = Governemt spending(g)');
disp(' ');
IVS=input('Select a number 1 to 4 for a shock variable = ');
S(IVS)=input('Enter size of shock in percent of variance = ');
disp(' ');

SN=S;
SM=SP+SN;

for i=5:6;
    IRM(i,1)=i;
    IRM(i,2:nMke+1)=SM';
    IRM(i,nMke+2:npi+nMke+1)=(PI*SM)';
    SM=Mke*SM;
end

SP=SM;

disp(' ');
disp('Shock again');
S=zeros(nMke,1); %Create a shock column vector
disp('The shock variables are denoted as the number below');
disp('1 = Capital(k)');
disp('2 = Total factor productivity(A)');
disp('3 = Governemt spending(g)');
disp(' ');
IVS=input('Select a number 1 to 4 for a shock variable = ');
S(IVS)=input('Enter size of shock in percent of variance = ');
disp(' ');

SN=S;
SM=SP+SN;

```

```

for i=7:12;
    IRM(i,1)=i;
    IRM(i,2:nMke+1)=SM';
    IRM(i,nMke+2:npi+nMke+1)=(PI*SM)';
    SM=Mke*SM;
end

IRM(:,7)=sc*IRM(:,7); %Consumption response in good units
IRM(:,12)=si*IRM(:,12); %Investment response in good units

%Plot simulation
%*****
quarter=IRM(:,1);
disp(' ');
disp('Plot Impluse Response');
disp('    1 = Plot all variables');
disp('    2 = plot an individual variable');
disp('    3 = Plot three variables in the same graph');
disp(' ');
xx=input('Enter the number = ');
if xx==1,
    figure(1)
    subplot(4,1,1); plot(quarter,IRM(:,7),'r');
    title('consumption');
    subplot(4,1,2); plot(quarter,IRM(:,12),'b');
    title('investment');
    subplot(4,1,3); plot(quarter,IRM(:,9),'r');
    title('output');
    subplot(4,1,4); plot(quarter,IRM(:,8),'b');
    title('labour');
    figure(2)
    subplot(4,1,1); plot(quarter,IRM(:,2),'r');
    title('capital');
    subplot(4,1,2); plot(quarter,IRM(:,3),'b');
    title('total factor productivity');
    subplot(4,1,3); plot(quarter,IRM(:,10),'r');
    title('real wage');
    subplot(4,1,4); plot(quarter,IRM(:,11),'b');
    title('rental price of capital');
elseif xx==2
    vl=[
        '
        '          capital          '
        ' total factor productivity '
        '
        '
        '          consumption        '
        '          labour            '
        '          output             '
        '          real wage          '
        ' rental price of capital    '
        '          investment         '];
    disp(' ');
    disp('List of a variable and its number');

```



```

disp('2 = capital(k)');
disp('3 = total factor productivity(A)');
disp('7 = consumption(c)');
disp('8 = labour(N)');
disp('9 = output(y)');
disp('10 = real wage(w)');
disp('11 = rental price of capital(R)');
disp('12 = investment(i)');
yy=input('Enter the number = ');
plot(quarter,IRM(:,yy)),
title(vl(yy,:),:)),
elseif xx==3,
disp('List of a variable and its number');
disp('2 = capital(k)');
disp('3 = total factor productivity(A)');
disp('7 = consumption(c)');
disp('8 = labour(N)');
disp('9 = output(y)');
disp('10 = real wage(w)');
disp('11 = rental price of capital(R)');
disp('12 = investment(i)');
lg=[
'      ',
' k ',
' A ',
'      ',
'      ',
'      ',
' c ',
' N ',
' Y ',
' w ',
' R ',
' i '];
j = zeros(3,1);
j(1) = input('Enter number of the first variable to plot = ');
add = input('Enter 1 to add variables, Enter 0 to plot now = ');
if add == 0,
plot(quarter,IRM(:,j(1)),'-'),
title('Thai RBC Impluse Response'),
legend(lg(j(1),:)),
elseif add == 1,
j(2) = input('Enter number of the second variable to plot = ');
');
end

if j(2) > 0,
add = input('Enter 1 to add variables, Enter 0 to plot now = ');
if add == 0,
plot(quarter,IRM(:,j(1)),'-',quarter,IRM(:,j(2)),':'),
title('Thai RBC Impluse Response'),
legend(lg(j(1),:),lg(j(2),:),2),
elseif add == 1,
j(3) = input('Enter number of the third variable to plot = ');
end
end
end

```

```

        if j(3) > 0,
            plot(quarter,IRM(:,j(1)),'-
',quarter,IRM(:,j(2)),':',quarter,IRM(:,j(3)),'--'),
                title('Thai RBC Impluse Response'),
                legend(lg(j(1),:),lg(j(2),:),lg(j(3),:),3),
                    end
end

% Frequency domain Filtering
disp(' ');
disp('Want to filter the model economy?');
disp('*****');
disp('If yes, input fme=1');
disp('If no, input fme=0');
fme=input('fme=')

if (fme==1)
disp(' ');
disp('Frequency Domain Filtering');

% Set the points of frequency
j=100;
w=-1+(1/j):2/j:1-(1/j);
w=w*pi;
nw=max(size(w));

% Define imaginary number
i=sqrt(-1);
% Define complex scalar or  $z=\cos(w)-i\sin(w)$ 
z=exp(-i*w);

% Apply alternative filters
disp('Enter the filters for the frequency domain:')
disp(' 1 = First Difference')
disp(' 2 = Moving Average 3 points')
disp(' 3 = Ideal Band pass')
disp(' 4 = Hodrick Prescott')
disp(' ')
enter=input('The filter = ')

if (enter==1)
    % First Difference Filter
    fgf_fd=ones(size(w))-z;           %filter generating function
    tff_fd=abs(fgf_fd).^2;           %transfer function
    plot(w/pi,tff_fd),title('Transfer Function of First Difference Filter'),
    xlabel('Frequency * Pi')
    tff=tff_fd;

elseif (enter==2)
    % Moving Average 3 points
    fgf_mv=(1/3)*(1+2*cos(w)); %filter generating function is
    (1/3)*(inv(L)+1+L)
    tff_mv=abs(fgf_mv).^2;

```

```

    plot(w/pi,tff_mv),title('Transfer Function of Moving Average Filter at
point 3')
    xlabel('Frequency * Pi')
    tff=tff_mv;

elseif (enter==3)
    % Ideal Bandpass Filter
    %low periodicity
    ucf=6;
    %high periodicity
    lcf=32;
    % Calculate frequencies
    hw=2*pi/ucf;
    lw=2*pi/lcf;
    % Speciy the transfer function
    onet=abs(w)>hw; % high frequencies
    oneb=abs(w)<lw; % low frequencies
    tff_ibp=ones(size(w))-onet-oneb;
    plot(w/pi,tff_ibp),title('Transfer Function of Ideal Bandpass')
    xlabel('Frequency * Pi')
    tff=tff_ibp;

elseif (enter==4)
    % Hodrick Prescott Filter
    % Identify the value of lamda
    disp(' ')
    disp('Enter the value of lamda:')
    disp(' 100 = Annual data')
    disp(' 1600 = Quarterly data')
    disp(' 14400 = Monthly data')
    disp(' ')
    lamda=input('The value of lamda = ')
    % Calculate the transfer function
    tff_hp=(4*lamda*(1-cos(w)).^2)./(1+4*lamda*(1-cos(w)).^2);

    %To plot
    VL=[
        'Lamda = 100 for annual data      '
        'Lamda = 1600 for quarterly data'
        'Lamda = 14400 for monthly data '];
    if (lamda==100)
        ll=1
    elseif (lamda==1600)
        ll=2
    else ll=3
    end

    plot(w/pi,tff_hp),title({'Transfer Function of Hodrick-Prescott
Filter';[VL{(ll),:}]}),
    xlabel('Frequency * Pi')
    tff=tff_hp;

end
end

```

```

% Calculate the filtered spectrum density of state variables
%*****
disp(' ');
%specify the matrix of variance of the shocks
sigma_A=0.0156;
var_A=sigma_A^2;
sigma_g=0.0184;
var_g=sigma_g^2

MVE=[0      0      0      0
      0  var_A      0      0
      0      0  var_g      0
      0      0      0      0]; %Matrix of variance of epsilon

%identify lags for the filter
lagv=[0 1 2 3 4 5]; % identify lag vector
nlags=max(size(lagv)); %
leadv=[0 -1 -2 -3 -4 -5]; %identify lead vector
nleads=max(size(leadv)); %
IMke=eye(size(Mke));
IMAC_lag=zeros(nMke,nMke*nlags);%prepare the filterd state matrix with
lags

%calculate the spectrum density of state variables
for q=1:nw;
    SSD=inv(exp(i*w(q))*IMke-Mke);
    SSD=(SSD*MVE*SSD')/(2*pi); % The spectrum density of state variables
    FSSD=tff(q)*SSD; %The spectrum density of the filtered state variables
    FSSD=kron(exp(i*w(q)*lagv),FSSD); %The spectrum density of FSSD with
lags
    IMAC_lag=IMAC_lag+FSSD;
end

IMAC_lead=zeros(nMke,nMke*nleads);%prepare the filterd state matrix with
leads
for q=1:nw;
    SSD=inv(exp(i*w(q))*IMke-Mke);
    SSD=(SSD*MVE*SSD')/(2*pi); % The spectrum density of state variables
    FSSD=tff(q)*SSD; %The spectrum density of the filtered state variables
    FSSD=kron(exp(i*w(q)*leadv),FSSD); %The spectrum density of FSSD with
leads
    IMAC_lead=IMAC_lead+FSSD;
end

IMAC_lag=real(IMAC_lag)*(pi/50);
IMAC_lead=real(IMAC_lead)*(pi/50);
size(IMAC_lag);
size(IMAC_lead);

IMPI=[IMke
      PI]; %prepare matrix for calculating variace-covariace matrix

sIMPI=size(IMPI); %size of IMPI

```

```

rIMPI=sIMPI(1);    %number of row of IMPI

%Variance and Correlation of the contemporaneous control and flow
variables
VCF=IMAC_lag(1:nMke,1:nMke)*IMPI';
VCF=IMPI*VCF;    %calculate the variance of the controls and flows in 2
steps
sdvf=sqrt(diag(VCF));    %calculate the standard deviation of VCF
tsdvf=kron(sdvf,sdvf');
COR=VCF./tsdvf; %calculate the correlation of VCF

%Calculate the relative standard deviation
disp('Select a reference variable by entering the number allocated below')
disp('List of a variable and its number');
    disp('6 = consumption(c)');
    disp('7 = labour(N)');
    disp('8 = output(y)');
    disp('9 = real wage(w)');
    disp('10 = rental price of capital(R)');
    disp('11 = investment(i)');
    disp(' ');
rv=input('The reference variable = ')

rvsdvf=sdvf/sdvf(rv);

%Prepare the comovement, persistence and volatility table
lagv=[1 2 3 4 5];    %identify lag vector
leadv=[1 2 3 4 5];    %identify lead vector
acv=[1 2 3];    %identify autocorrelation vector
nlags=max(size(lagv));    % number of lags
nleads=max(size(leadv));
nautos=max(size(acv));    % number of autocorrelation

%the blank table
CPVT=zeros(rIMPI,nlags+1+nleads+nautos+2);
%sdvf in the table
CPVT(1:rIMPI,nlags+1+nleads+nautos+1)=sdvf;
%rvsdvf in the table
CPVT(1:rIMPI,nlags+1+nleads+nautos+2)=rvsdvf;

%the position of selected reference variable
rvpos=nlags+1;

%the contemporaneous correlation of rv in the table
CPVT(1:rIMPI,rvpos)=COR(1:rIMPI,rv);

%calculate the cross correlations with selected rv
for q=1:nlags;
    HOLD_lag=(IMPI*IMAC_lag(:,q*nMke+1:(q+1)*nMke)*IMPI')./tsdvf;
    HOLD_lead=(IMPI*IMAC_lead(:,q*nMke+1:(q+1)*nMke)*IMPI')./tsdvf;
    CPVT(1:rIMPI,rvpos-q)=HOLD_lag(rv,1:rIMPI);
    CPVT(1:rIMPI,rvpos+q)=HOLD_lead(rv,1:rIMPI);
end

%calculate the autocorrelation

```

```

for q=1:nautos;
    HOLD_lag=(IMPI*IMAC_lag(:,q*nMke+1:(q+1)*nMke)*IMPI')./tsdvf;
    CPVT(1:rIMPI,nlags+1+nleads+q)=diag(HOLD_lag);
end
TABLE=CPVT;

disp('*****')
disp('The instruction for the Thai business cycle table')
disp('*****')
disp('For rows,')
disp('Row 6-11 represent consumption, labour, output, wage')
disp('          rental price of capital and investment respectively')
disp(' ')
disp('For columns,')
disp('Column 1-5 are the lags corr with selected reference variable')
disp('Column 6 is lag 1 period,...,Column 11 is lag 5 period')
disp('Column 12 is the correlation with the selected reference variable')
disp('Column 13-17 are the leads corr with the selected reference variable')
disp('Column 18 is lead 1 period,...,Column 22 is lead 5 period')
disp('Column 23-26 are the autocorrelation for each variable')
disp('Column 27 is auto corr 1 period,...,Column 30 is auto corr 4 period')
disp('Column 31 is the standard deviation for each variable')
disp('Column 32 is the standard deviation relative to the selected variable')
pause(5)
TABLE

disp('Want to look at a particular variable?')
disp('*****');
disp('If yes, input lpv=1');
disp('If no, input lpv=0');
lpv=input('lpv=');
disp(' ')
if (lpv==1)
disp('Enter the business cycle features as the number below')
disp(' 1 = Volatility')
disp(' 2 = Comovement')
disp(' 3 = Persistence')
disp(' ')
enter=input('The feature = ')
    if (enter==1)
        % Volatility
        disp(' ')
        disp('For volatility,')
        disp('***** ')
        sdvf
        disp('Select a variable by entering the number allocated below')
        disp('List of a variable and its number');
        disp('6 = consumption(c)');
        disp('7 = labour(N)');
        disp('8 = output(y)');
        disp('9 = real wage(w)');
        disp('10 = rental price of capital(R)');
        disp('11 = investment(i)');
    end
end

```

```

disp(' ')
disp('Enter the number for its volatility')
iv=input('the variable = ')
volatility=sdvf(iv)

elseif (enter==2)
% Comovement
disp('For comovement,')
disp('***** ')
disp('Enter the business cycle features as the number below')
disp(' 1 = contemporaneous comovement')
disp(' 2 = lead or lag comovement')
cm=input('The feature = ')
if (cm==1)
    q=0;
    HOLD=(IMPI*IMAC_lag(:,q*nMke+1:(q+1)*nMke)*IMPI')./tsdvf;
    disp('Select a variable by entering the number allocated below')
    disp('List of a variable and its number');
    disp('6 = consumption(c)');
    disp('7 = labour(N)');
    disp('8 = output(y)');
    disp('9 = real wage(w)');
    disp('10 = rental price of capital(R)');
    disp('11 = investment(i)');
    disp(' ')
    pv=input('Enter the pair of variables in the form of [1 2] = ')
    result=HOLD(pv(1),pv(2))
elseif (cm==2)
    disp('Enter 1 for lag and 0 for lead')
    lorl=input('Enter = ')
    if (lorl==1)
        disp('Lags are the following')
        lagv
        disp('Enter the lag')
        iplag=input('lag = ')
        q=iplag;
        HOLD=(IMPI*IMAC_lag(:,q*nMke+1:(q+1)*nMke)*IMPI')./tsdvf;
        disp('Select a variable by entering the number allocated
below')

        disp('List of a variable and its number');
        disp('6 = consumption(c)');
        disp('7 = labour(N)');
        disp('8 = output(y)');
        disp('9 = real wage(w)');
        disp('10 = rental price of capital(R)');
        disp('11 = investment(i)');
        pv=input('Enter the pair of variables in the form of [1 2]
= ')

        result=HOLD(pv(1),pv(2))

    elseif (lorl==0)
        disp('Leads are the following')
        leadv
        disp('Enter the lead')
        iplead=input('lead = ')
        q=iplead;

```

```

        HOLD=(IMPI*IMAC_lead(:,q*nMke+1:(q+1)*nMke)*IMPI')./tsdvf;
        disp('Select a variable by entering the number allocated
below')

        disp('List of a variable and its number');
        disp('6 = consumption(c)');
        disp('7 = labour(N)');
        disp('8 = output(y)');
        disp('9 = real wage(w)');
        disp('10 = rental price of capital(R)');
        disp('11 = investment(i)');
        pv=input('Enter the pair of variables in the form of [1 2]
= ');
        result=HOLD(pv(1),pv(2))
    end
end
elseif (enter==3)
% Persistence
    disp(' ')
    disp('For persistence,')
    disp('*****')
    disp('Autocorrelations are the following')
    acv
    disp('Enter the autocorrelation')
    ipautoc=input('the autocorrelation = ')
    q=ipautoc;
    HOLD_lag=(IMPI*IMAC_lag(:,q*nMke+1:(q+1)*nMke)*IMPI')./tsdvf;
    CPVT(1:rIMPI,nlags+1+nleads+q)=diag(HOLD_lag);
    autov=diag(HOLD_lag)
    disp('Select a variable by entering the number allocated below')
    disp('List of a variable and its number');
    disp('6 = consumption(c)');
    disp('7 = labour(N)');
    disp('8 = output(y)');
    disp('9 = real wage(w)');
    disp('10 = rental price of capital(R)');
    disp('11 = investment(i)');
    disp(' ')
    disp('Enter the number for its persistence at the chosen autocorr')
    iv=input('the variable = ')
    Autocorrelation=autov(iv)

elseif (fme==0)
    disp(' ');
    disp('End of the program');
elseif (lpv==0)
    disp(' ');
    disp('End of the program');
end
end
end

```


Bibliography:

- ABADIR, K. & TALMAIN, G. (2001) Depreciation Rates and Capital Stocks. *Manchester School* (14636786), 69, 42.
- ABADIR, K. & TALMAIN, G. (2002) Aggregation, Persistence and Volatility in a Macro Model. *Review of Economic Studies*. Blackwell Publishing Limited.
- AHMED, S., ICKES, B. W., WANG, P. & YOO, B. S. (1993) International Business Cycles. *The American Economic Review*, 83, 335-359.
- AHN, H. & KIM, S. H. (2003) Dynamics of Open Economy Business Cycle Models: The Case of Korea. *Macroeconomics Working Papers, East Asian Bureau of Economic Research*, 88.
- AKIN & KOSE, M. A. (2008) Changing nature of North and South linkages: Stylized facts and explanations. *Journal of Asian Economics*, 19, 1-28.
- ANGELETOS, G.-M. & PANOUSI, V. (2007) Revisiting the Supply Side Effects of Government Spending Under Incomplete Markets. *NBER, Working paper 13136*.
- ARVANITIS, A. V. & MIKKOLA, A. (1996) Asset-Market Structure and International Trade Dynamics. *American Economic Review*, 86, 67.
- BACKUS, D. K. & CRUCINI, M. J. (2000) Oil Prices and the Terms of Trade. *Journal of International Economics*, 50, 185.
- BACKUS, D. K. & KEHOE, P. J. (1992) International Evidence on the Historical Properties of Business Cycles. *American Economic Review*, 82, 864-388.
- BACKUS, D. K. & KEHOE, P. J. (1993) International Business Cycles: Theory vs. Evidence. *Quarterly Review* (02715287), 17, 14.
- BACKUS, D. K., KEHOE, P. J. & KYDLAND, F. E. (1992) International Real Business Cycles. *Journal of Political Economy*, 100, 745.
- BACKUS, D. K., KEHOE, P. J. & KYDLAND, F. E. (1994) Dynamics of the Trade Balance and the Terms of Trade: The J-Curve? (cover story). *American Economic Review*, 84, 84-103.
- BACKUS, D. K., KEHOE, P. J. & KYDLAND, F. E. (1995) International Business Cycles: Theory and Evidence. IN COOLEY, T. F. (Ed.) *Frontiers of Business Cycle Research*. . Princeton, Princeton University Press.

- BARRO, R. J. (1991) Economic Growth in a Cross Section of Countries. *The Quarterly Journal of Economics*, 106, 407-443.
- BARRO, R. J. & KING, R. G. (1984) Time-Separable Preferences and Intertemporal-Substitution Models of Business Cycles. *The Quarterly Journal of Economics*, 99, 817-839.
- BARRO, R. J., MANKIW, N. G. & SALA-I-MARTIN, X. (1995) Capital Mobility in Neoclassical Models of Growth. *The American Economic Review*, 85, 103-115.
- BARRO, R. J. & SALA-I-MARTIN, X. (1995) *Economic Growth*, Cambridge, MA, The MIT Press.
- BAXTER, M. (1991) Approximating Suboptimal Dynamic Equilibria: An Euler Equation Approach. *Journal of Monetary Economics*, 28, 173-200.
- BAXTER, M. & CRUCINI, M. J. (1993) Explaining Saving-Investment Correlations. *American Economic Review*, 83, 416.
- BAXTER, M. & CRUCINI, M. J. (1995) Business Cycles and the Asset Structure of Foreign Trade. *International Economic Review*, 36, 821.
- BAXTER, M., CRUCINI, M. J. & ROUWENHORST, K. G. (1990) Solving the Stochastic Growth Model by a Discrete-State-Space, Euler-Equation Approach. *Journal of Business & Economic Statistics*, 8, 19-21.
- BAXTER, M. & KING, R. G. (1993) Fiscal Policy in General Equilibrium. *American Economic Review*, 83, 315.
- BAXTER, M. & KING, R. O. (1999) Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series. *Review of Economics & Statistics*, 81, 575-593.
- BAXTER, M. & KOUPARITSAS, M. A. (2005) Determinants of Business Cycle Comovement: A Robust Analysis. *Journal of Monetary Economics*, 52, 113-157.
- BERGOEING, R., KEHOE, P. J., KEHOE, T. J. & SOTO, R. (2002) A Decade Lost and Found: Mexico and Chile in the 1980s. *Review of Economic Dynamics*, 5, 166.
- BERGOEING, R. & SOTO, R. (2005) Testing Real Business Cycle Models in an Emerging Economy. *Central Bank of Chile, Santiago*, 221-259.
- BERNANKE, B. S. & GURKAYNAK, R. S. (2001) Is Growth Exogenous? Taking Mankiw, Romer, and Weil Seriously. *NBER Macroeconomics Annual*, 16, 11-57.

- BLACKBURN, K. & RAVN, M. O. (1992) Business Cycles in the United Kingdom: Facts and Fictions. *Economica*, 59, 583-401.
- BLANCHARD, O. J. & FISHER, S. (1989) *Lectures on Macroeconomics*, Cambridge, MA, The MIT Press.
- BLANCHARD, O. J. & KAHN, C. M. (1980) The Solution of Linear Difference Models Under Rational Expectations. *Econometrica*, 48, 1305-1311.
- BLANKENAU, W., KOSE, M. A. & KEI-MU, Y. (2001) Can world real interest rates explain business cycles in a small open economy? *Journal of Economic Dynamics & Control*, 25, 867.
- BOILEAU, M. (1996) Growth and the international transmission of business cycles. *International Economic Review*, 37, 737.
- BOILEAU, M. (1999) Trade in capital goods and the volatility of net exports and the terms of trade. *Journal of International Economics*, 48, 347-365.
- BOLDRIN, M., CHRISTIANO, L. J. & FISHER, J. D. M. (2001) Habit Persistence, Asset Returns, and the Business Cycle. *The American Economic Review*, 91, 149-166.
- BOLDRIN, M., CHRISTIANO, L. J. & FISHER, J. D. M. (2001) Habit Persistence, Asset Returns, and the Business Cycle. *The American Economic Review*, 91, 149-166.
- BOSWORTH, B. P. & COLLINS, S. M. (2003) The Empirics of Growth: An Update. *Brookings Papers on Economic Activity*, 113-206.
- BRAUN, R. A. (1994) Tax Disturbances and Real Economic Activity in the Postwar United States. *Journal of Monetary Economics* 33, 441-462.
- BROCK, W. A. & MIRMAN, L. J. (1972) Optimal Economic Growth and Uncertainty: The Discounted Case. *Journal of Economic Theory*, 4, 479.
- BROCK, W. A. & MIRMAN, L. J. (1973) Optimal Economic Growth and Uncertainty: The No Discounting Case. *International Economic Review*, 14, 560.
- BURDETT, K. & WRIGHT, R. (1989) Optimal Firm Size, Taxes, and Unemployment. *Journal of Public Economics*, 39, 275.
- BURN, A. M. & MITCHELL, W. C. (1946) Measuring Business Cycles. *National Bureau of Economic Research*.
- BURNSIDE, C. (1999) Real Business Cycle Models: Linear Approximation and GMM Estimation. *The World Bank*, Revision 7 (preliminary).

- BURNSIDE, C. & EICHENBAUM, M. (1996) Factor-Hoarding and the Propagation of Business-Cycle Shocks. *American Economic Review*, 86, 1154-1174.
- BURNSIDE, C., EICHENBAUM, M. & REBELO, S. (1993) Labor Hoarding and the Business Cycle. *Journal of Political Economy*, 101, 245.
- CALDERON, C. & FUENTES, R. (2006) Characterizing the Business Cycles of Emerging Economies. *The Central Bank of Chile*, Mimeo.
- CAMPBELL, J. Y. (1994) Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model. *Journal of Monetary Economics*, 33, 463-506.
- CAMPBELL, J. Y. & MANKIW, N. G. (1989) Consumption, Income, and Interest rates: Reinterpreting the Time Series Evidence. IN BLANCHARD, O. J. & FISCHER, S. (Eds.) *NBER Macroeconomics Annual*.
- CANOVA, F. & GIANNI, D. N. (2003) On the Sources of Business Cycles in the G-7. *Journal of International Economics*, 59, 77-100.
- CANOVA, F. & RAVN, M. O. (1996) International Consumption Risk Sharing. *International Economic Review*, 37, 573.
- CARDIA, E. (1991) The Dynamics of a Small Open Economy in Response to Monetary, Fiscal, and Productivity Shocks. *Journal of Monetary Economics*, 28, 411.
- CASS, D. (1965) Optimum Growth in an Aggregative Model of Capital Accumulation. *Review of Economic Studies*, 32, 233.
- CHARI, V. V., CHRISTIANO, L. J. & KEHOE, P. J. (1994) Optimal Fiscal Policy in a Business Cycle Model. *Journal of Political Economy*, 102, 617.
- CHARTFIELD, C. (2004) *The Analysis of Time Series An Introduction*, Chapman & Hall/CRC.
- CHRISTIANO, L. (2002) Solving Dynamic Equilibrium Models by a Method of Undertermined Coefficients. *Computational Economics*, 20, 21-55.
- CHRISTIANO, L. & FITZGERALD, T. (2003) The Band Pass Filter. *International Economic Review*, 44, 435-465.
- CHRISTIANO, L. J. (1990) Linear-Quadratic Approximation and Value-Function Iteration: A Comparison. *Journal of Business & Economic Statistics*, 8, 99-113.
- CHRISTIANO, L. J. & EICHENBAUM, M. (1992) Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations. *American Economic Review*, 82, 430-450.

- CHRISTIANO, L. J. & EICHENBAUM, M. (1995) Liquidity Effects, Monetary Policy, and the Business Cycle. *Journal of Money, Credit & Banking*, 27, 1113-1136.
- CHRISTIANO, L. J., EICHENBAUM, M. & VIGFUSS, R. (2003) What Happens After A Technology Shock? *Working Papers -- US Federal Reserve Board's International Finance Discussion Papers*.
- CICCO, J. G., PANCRAZI, R. & URIBE, M. (2006) Real Business Cycles in Emerging Countries? *NBER, Working paper 12629*.
- CLARIDA, R., GALA, J. & GERTLER, M. (1999) The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature*, 37, 1661.
- COLE, H. L. & KEHOE, T. J. (1996) A Self-Fulfilling Model of Mexico's 1994-1995 Debt Crisis. *Journal of International Economics*, 41, 309-330.
- COOLEY, T. F. (1997) Calibrated Models. *Oxford Review of Economic Policy*, 13, 55.
- COOLEY, T. F. & HANSEN, G. D. (1989) The Inflation Tax in a Real Business Cycle Model. *American Economic Review*, 79, 733.
- COOLEY, T. F. & HANSEN, G. D. (1995) Money and the Business Cycle. IN COOLEY, T. F. (Ed.) *Frontier of Business Cycle Research*. Princeton University Press.
- COOLEY, T. F. & HANSEN, G. D. (1997) Unanticipated Money Growth and the Business Cycle Reconsidered. *Journal of Money, Credit & Banking*, 29, 624-648.
- COOLEY, T. F. & HANSEN, G. D. (1998) The Role of Monetary Shocks in Equilibrium Business Cycle Theory: Three Examples. *European Economic Review*, 42, 605-617.
- COOLEY, T. F. & OHANIAN, L. E. (1997) Postwar British Economic Growth and the Legacy of Keynes. *The Journal of Political Economy*, 105, 439-472.
- COOLEY, T. F. & PRESCOTT, E. C. (1995) Economic Growth and Business Cycles, in T.F. Cooley, ed. *Frontiers of Business Cycle Research*. Princeton: Princeton University Press.
- COOPER, R. & ADDA, J. (2003) *Dynamic Economics*, Cambridge, MA and London, England, The MIT Press.
- CORREIA, I., NEVES, J. C. & REBELO, S. (1995) Business Cycles in a Small Open Economy. *European Economic Review*, 39, 1089-1113.
- CORSETTI, G. & MULLER, G. J. (2006) Twin Deficits: Squaring Theory, Evidence and Common Sense. *Economic Policy*, 21, 597-638.

- CRUCINI, M. J. (1997) Country Size and Economic Fluctuations. *Review of International Economics*, 5, 204.
- CRUCINI, M. J. (1999) On International and National Dimensions of Risk Sharing. *Review of Economics & Statistics*, 81, 73-84.
- DANTHINE, J.-P. & DONALDSON, J. B. (1981) Stochastic Properties of Fast VS. Slow Growing Economies. *Econometrica*, 49, 1007-1033.
- ELLEY, R. D. G., GOMES, V. & SACHISDA, A. (2002) Business Cycle Fluctuations in Brazil. *Revista Brasileira de Economia, Graduate School of Economics, Getulio Vargas Foundation (Brazil)*, 52.
- ENDERS, W. (2004) *Applied Econometric Time Series*, Wiley.
- GAIL, M. (1998) Stylized Facts and International Business Cycles-The German Case. *Volkswirtschaftliche Diskussionsbeitraege* 69-98, *Univesitaet Siegen*.
- GALLANT, A. R. (1997) *An Introduction to Econometric Theory*, Princeton, New Jersey, Princeton University Press.
- GHEZ, G. & BECKER, G. (1975) The Allocation of Time and Goods over the Life Cycle. *Columbia University Press, New York*.
- GLICK, R. & ROGOFF, K. (1995) Global Versus Country-Specific Productivity Shocks and the Current Account. *Journal of Monetary Economics*, 35, 159-192.
- GOODFRIEND, M. & KING, R. G. (1997) The New Neoclassical Synthesis and the Role of Monetary Policy. *NBER/Macroeconomics Annual*, 12, 9-9.
- GRANGER & HATANAKA (1964) *Spectral Analysis of Economic Time Series*, Princeton, Princeton University Press.
- GRAVELLE, H. & REES, R. (2004) *Microeconomics*, Prentice Hall.
- GREENWOOD, J., HERCOWITZ, Z. & HUFFMAN, G. W. (1988) Investment, Capacity Utilization, and the Real Business Cycle. *American Economic Review*, 78, 402.
- GREENWOOD, J., HERCOWITZ, Z. & KRUSELL, P. (2000) The Role of Investment-Specific Technological Change in the Business Cycle. *European Economic Review*, 44, 91-115.
- GREENWOOD, J., ROGERSON, R. & WRIGHT, R. (1995) Household Production in Real Business Cycle Theory, in T.F. Cooley, ed. *Frontiers of Business Cycle Research*. Princeton, Princeton University Press.

- GRILLI, V. & ROUBINI, N. (1996) Liquidity Models in Open Economies: Theory and Empirical evidence. *European Economic Review*, 40, 847-859.
- HAMILTON, J. D. (1994) *Time Series Analysis*, Princeton, Princeton University Press.
- HANSEN, G. D. (1985) Indivisible Labor and The Business Cycle. *Journal of Monetary Economics*, 16, 309-327.
- HANSEN, G. D. & PRESCOTT, E. C. (1993) Did Technology Shocks Cause the 1990-1991 Recession? *The American Economic Review*, 83, 280-286.
- HANSEN, G. D. & WRIGHT, R. (1992) The Labor Market in Real Business Cycle Theory. *Quarterly Review* (02715287), 16, 2.
- HANSEN, L. P. & SARGENT, T. J. (1980) Formulating and Estimating Dynamic Linear Rational Expectations Models. *Journal of Economic Dynamics & Control*, 2, 7.
- HANSEN, L. P. & SARGENT, T. J. (1981) A note on Wiener-Kolmogorov prediction formulas for rational expectations models. *Economics Letters, Elsevier*, 8, 255-260.
- HARBERGER, A. C. (1978) On the Use of Distributional Weights in Social Cost-Benefit Analysis. *Journal of Political Economy*, 86, S87.
- HAYASHI, F. (2002) *Econometrics*, Princeton and Oxford, Princeton University Press.
- HEIJDRA, B. J. & VAN DER PLOG, F. (2002) *Foundations of Modern Macroeconomics*, Oxford, Oxford University Press.
- HODRICK, R. J. & PRESCOTT, E. C. (1997) Postwar U.S. Business Cycles: An Empirical Investigation. *Journal of Money, Credit & Banking*, 29, 1-16.
- INADA, K. (1963) On the Two Sector-Model of Economic Growth: Comments and a Generalization. *Review of Economic Studies*, 30, 119-127.
- JUDD, K. L. (1992) Projections Methods for Solving Aggregate Growth Models. *Journal of Economic Theory*, 58, 410-452.
- JUSTER, F. T. & STAFFORD, F. P. (1991) The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement. *Journal of Economic Literature*, 29, 471-522.
- KAMINSKY, G., LIZONDO, S., REINHART, C. M., DI TATA, J. C. & HERNANDEZ-CATA, E. (1998) Leading indicators of currency crises. *IMF Staff Papers*, 45, 1.

- KAMINSKY, G. L. & REINHART, C. M. (1996) The Twin Crises: The Causes of Banking and Balance-of-Payments Problems. *Working Papers -- US Federal Reserve Board's International Finance Discussion Papers*.
- KEHOE, P. J. & PERRI, F. (2002) International Business Cycles with Endogenous Incomplete Markets. *Econometrica*, 70, 907-928.
- KIM, S. H., KOSE, M. A. & PLUMMER, M. G. (2003) Dynamics of Business Cycles in Asia: Differences and Similarities. *Review of Development Economics*, 7, 462-477.
- KING, R. G., PLOSSER, C. I. & REBELO, S. T. (1988) Production, Growth and Business Cycles I. The Basic Neoclassical Model. *Journal of Monetary Economics*, 21, 195-232.
- KING, R. G., PLOSSER, C. I. & REBELO, S. T. (1988) Production, Growth and Business Cycles II. New Directions. *Journal of Monetary Economics*, 21, 309-341.
- KING, R. G., PLOSSER, C. I. & REBELO, S. T. (2002) Production, Growth and Business Cycles: Technical Appendix. *Computational Economics*, 20, 87-116.
- KING, R. G., PLOSSER, C. I., STOCK, J. H. & WATSON, M. W. (1991) Stochastic Trends and Economic Fluctuations. *The American Economic Review*, 81, 819-840.
- KING, R. G. & REBELO, S. T. (1993) Low Frequency Filtering and Real Business Cycles. *Journal of Economic Dynamics & Control*, 17, 207.
- KING, R. G. & REBELO, S. T. (1994) Resuscitating Real Business Cycles. *Journal of Monetary Economics*, 33, 405-438.
- KING, R. G. & WATSON, M. W. (1998) The Solution of Singular Linear Difference Systems Under Rational Expectations. *International Economic Review*, 39, 1015-1026.
- KING, R. G. & WATSON, M. W. (2002) System Reduction and Solution Algorithms for Singular Linear Difference Systems under Rational Expectations. *Computational Economics*, 20.
- KLEIN, M. W. (2002) *Mathematical Methods for Economics*, Addison Wesley.
- KLEIN, P. (2000) Using the Generalized Schur form to Solve a Multivariate Linear Rational Expectations Model. *Journal of Economic Dynamics & Control*, 24, 1405.

- KOLLMANN, R. (1996) Incomplete Asset Markets and the Cross-Country Consumption. *Journal of Economic Dynamics & Control*, 20, 945.
- KOOPMANS, T. C. (1965) On the Concept of Optimal Economic Growth. *The econometric approach to development planning*. Chicago, IL, Rand-McNally.
- KOOPMANS, T. C. (1967) Objectives, Constraints, and Outcomes in Optimal Growth Models. *Econometrica*, 35, 1-15.
- KOSE, M. A. (2002) Explaining Business Cycles in Small Open Economies: 'How Much Do World Prices Matter?'. *Journal of International Economics*, 56, 299-327.
- KOSE, M. A., PRASAD, E. S. & TERRONES, M. E. (2007) How Does Financial Globalization Affect Risk Sharing? Patterns and Channels. *IMF Working Papers 07/238, International Monetary Fund*.
- KOSE, M. A. & YI, K.-M. (2001) International Trade and Business Cycles: Is Vertical Specialization the Missing Link? *American Economic Review*, 91, 371-375.
- KOSE, M. A. & YI, K.-M. (2006) Can the Standard International Business Cycle Model Explain the Relation Between Trade and Comovement? *Journal of International Economics*, 68, 267-295.
- KOUPARITSAS, M. A. (2001) Evidence of the North-South business cycle. *Economic Perspectives*, 25, 46.
- KOUPARITSAS, M. A. (2002) Understanding U.S. regional cyclical comovement: How important are spillovers and common shocks? *Economic Perspectives*, 26, 30.
- KYDLAND, F. E. (1992) On the Econometrics of World Business Cycles. *European Economic Review*, 36, 476-482.
- KYDLAND, F. E. & PRESCOTT, E. C. (1982) Time to Build and Aggregate Fluctuations. *Econometrica*, 50, 1345-1370.
- KYDLAND, F. E. & PRESCOTT, E. C. (1990) Business cycles: Real facts and a monetary myth. *Quarterly Review (02715287)*, 14, 3.
- KYDLAND, F. E. & PRESCOTT, E. C. (1991) Hours and Employment Variation in Business Cycle Theory. *Economic Theory*, 1, 63-81.
- LAXTON, D., N'DIAYE, P. & PESENTI, P. (2006) Deflationary Shocks and Monetary Rules: An Open-Economy Scenario Analysis. *Journal of the Japanese & International Economies*, 20, 665-698.

- LJUNQVIST, L. & SARGENT, T. (2001) *Recursive Macroeconomic Theory*, Cambridge, MA, The MIT Press, .
- LONG, J. B. & PLOSSER, C. I. (1983) Real Business Cycles. *Journal of Political Economy*, 91, 39-69.
- LUCAS JR, R. E. (1972) Expectations and the Neutrality of Money. *Journal of Economic Theory*, 4, 103.
- MANKIW, N. G. (1985) Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly. *Quarterly Journal of Economics*, 100, 529-538.
- MANKIW, N. G. (1989) Real Business Cycles: A New Keynesian Perspective. *Journal of Economic Perspectives*, 3, 79-90.
- MANKIW, N. G. (2006) The Macroeconomist as Scientist and Engineer. *Journal of Economic Perspectives*, 20, 29-46.
- MAS-COLELL, A., WHINSTON, D. & GREEN, R. J. (1995) *Microeconomics Theory*. Oxford University Press, 308.
- McCallum, B. T. & Nelson, E. (1999) An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis. *Journal of Money, Credit and Banking*, 31, 296-316.
- MCGRATTAN, E. R. (1994) The Macroeconomic Effects of Distortionary Taxation. *Journal of Monetary Economics*, 33, 573-601.
- MCGRATTAN, E. R. (1994) A Progress Report on Business Cycle Models. *Quarterly Review (02715287)*, 18, 2.
- MCGRATTAN, E. R., ROGERSON, R. & WRIGHT, R. (1997) An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy. *International Economic Review*, 38, 267-290.
- MENDOZA, E. G. (1991) Real Business Cycles in a Small Open Economy. *American Economic Review*, 81, 797-818.
- MENDOZA, E. G. (1995) The Terms of Trade, The Real Exchange Rate, and Economic Fluctuations. *International Economic Review*, 36, 101.
- NEHRU, V. & DHARESHWAR (1993) A New Database on Physical Capital Stock: Sources, Methodology, and Results. *Revista de Analisis Economico* 8, 37-59.

- NELSON, C. R. & PLOSSER, C. I. (1982) Trends and Random Walks in Macroeconomic Time Series. *Journal of Monetary Economics*, 10, 139-162.
- NEUMEYER, P. A. & PERRI, F. (2005) Business Cycles in Emerging Economies: The Role of Interest Rates. *Journal of Monetary Economics*, 52, 345-380.
- OBSTFELD, M. (1995) International Currency Experience: New Lessons and Lessons Relearned. *Brookings Papers on Economic Activity*, 119-220.
- OBSTFELD, M. (1996) Models of Currency Crises with Self-Fulfilling Features. *European Economic Review*, 40, 1037-1047.
- OBSTFELD, M. & ROGOFF, K. (1995) The Mirage of Fixed Exchange Rates. *Journal of Economic Perspectives*, 9, 73-96.
- ORTEGA, E., ESTUDIOS, S. D. & ESPANA, B. D. (1998) The Spanish Business Cycle and Its Relationship to Europe. *Working Paper No.9819, The Bank of Spain*.
- PAKKO, M. R. (1997) International Risk Sharing and Low Cross-Country Consumption Correlations: Are They Really Inconsistent? *Review of International Economics*, 5, 386.
- PAKKO, M. R. (1997) Trade, Investment, and International Borrowing in Two-Country Business Cycle Models. *Working Paper 1997-023A. Federal Reserve Bank of St. Louis*.
- PAKKO, M. R. (2002) Substitution Elasticities and Investment Dynamics in Two-Country Business Cycle Models. *Working Paper 2002-030B. Federal Reserve Bank of St. Louis*.
- PHELPS, E. S. (1966) *Golden Rules of Economic Growth* (Norton New York).
- PLOSSER, C. I. (1989) Understanding Real Business Cycles. *The Journal of Economic Perspectives*, 3, 51-77.
- POVOLEDO, L. (2004) Business Cycle in Italy: The Facts and Some Models. *Royal Economic Society Annual Conference 2004*, 139.
- PRESCOTT, E. C. (1986) The Theory Ahead of Business-Cycle Measurement. *Federal Reserve Bank of Minneapolis Quarterly Review*, 10, 9-22.
- RADE, L. & WESTERGREN, B. (2004) *Mathematics Handbook for Science and Engineering*, Springer.
- RAMSEY, F. P. (1928) A Mathematical Theory of Saving. *Economic Journal*, 38, 543-559.

- RAVN, M. O. (1997) International Business Cycles in Theory and in Practice. *Journal of International Money & Finance*, 16, 255.
- RAVN, M. O. (1997) Permanent and Transitory Shocks, and the UK Business Cycle. *Journal of Applied Econometrics*, 12, 27-48.
- REBELO, S. (2005) Real Business Cycle Models: Past, Present and Future. *Scandinavian Journal of Economics*, 107, 217-238.
- ROBERTO, D. G. E. J., VICTOR, G. & ADOLFO, S. (2002) Business Cycle Fluctuations in Brazil. *Revista Brasileira de Economia, Graduate School of Economics, Getulio Vargas Foundation (Brazil)*, 56.
- ROCHE, M. J. (1996) Government Spending and the International Business Cycle. *The Canadian Journal of Economics / Revue canadienne d'Economie*, 29, 865-884.
- ROGERSON, R. (1988) Indivisible Labor, Lotteries and Equilibrium. *Journal of Monetary Economics*, 21, 3-16.
- ROMER, D. (2001) *Advanced Macroeconomics*, McGraw-Hill
- SARGENT, T. J. (1987) *Dynamic Macroeconomic Theory*, Cambridge, MA and London, England, Harvard University Press.
- SCHLAGENHAUF, D. E. & WRASE, J. M. (1995) Liquidity and Real Activity in A Simple Open Economy Model. *Journal of Monetary Economics*, 35, 431-461.
- SCHLITZER, G. (1995) Business Cycles in Italy: A Statistical Investigation. *European Journal of Political Economy*, 11, 683-689.
- SCHMITT-GROHE, S. (1998) The international Transmission of Economic Fluctuations: Effects of U.S. Business Cycles on the Canadian Economy. *Journal of International Economics*, 44, 257-287.
- SCHMITT-GROHE, S. & URIBE, M. (2004) Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function. *Journal of Economic Dynamics & Control*, 28, 755.
- SIMS, C. A. (2002) Solving Linear Rational Expectations models. *Computational Economics*, 20, 1-20.
- SOLOW, R. M. (1956) A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70, 65-94.

- SOLOW, R. M. (1957) Technical Change and the Aggregate Production Function. *The Review of Economics and Statistics*, 39, 312-320.
- SOLOW, R. M. (1994) Perspectives on Growth Theory. *The Journal of Economic Perspectives*, 8, 45-54.
- STOCK, J. H. & WATSON, M. W. (1999) Business Cycle Fluctuations in US Macroeconomic Time Series, in J.B. Taylor and M. Woodford, eds. *Handbook of Macroeconomics*, 1A.
- STOCK, J. H. & WATSON, M. W. (2002) Has the Business Cycle Changed and Why? *NBER/Macroeconomics Annual*, 17, 159-218.
- STOCK, J. H. & WATSON, M. W. (2003) *Introduction to Econometrics*, Addison-Wesley.
- STOCK, J. H. & WATSON, M. W. (2005) Understanding Changes in International Business Cycle Dynamics. *Journal of the European Economic Association*, 3, 968-1006.
- STOCKMAN, A. C. & TESAR, L. L. (1995) Tastes and technology in a two-country model of the business cycle: Explaining international. *American Economic Review*, 85, 168.
- STOKEY, N., LUCAS, R. & PRESCOTT, E. C. (1989) *Recursive Methods in Economic Dynamics*, Harvard University Press.
- STOKEY, N. L. & REBELO, S. (1995) Growth effects of flat-rate taxes. *Journal of Political Economy*, 103, 519.
- SUNGHYUN HENRY, K. & KOSE, M. A. (2003) Dynamics of Open-Economy Business-Cycle Models: Role of the Discount Factor. *Macroeconomic Dynamics*, 7, 263-290.
- SWAN, T. (1964) Growth Models of Golden Ages and Production Functions. IN BERRIL, K. (Ed.) *Economic Development with Special Reference to East Asia*. Macmillan, London.
- TALMAIN, G. (2003) Growth and Business Cycles. IN S. ALTUG, J. C., AND C.NOLAN, (Ed.) *Elements of Dynamic Macroeconomic Analysis*. Cambridge, Cambridge University Press.
- TALVI, E. & VEGH, C. A. (2005) Tax Base Variability and Procyclical Fiscal Policy in Developing Countries. *Journal of Development Economics*, 78, 156-190.
- TAYLOR, J. B. & UHLIG, H. (1990) Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution Methods. *Journal of Business & Economic Statistics*, 8, 1-17.

- TOBIN, J. (1968) Notes on Optimal Monetary Growth. *The Journal of Political Economy*, 76, 833-859.
- VAUGHAN, D. R. (1970) A Non Recursive Algorithm Solution for The Discret Ricatti Equation. *IEEE Transaction on Automatic Control*, AC, 597-599.
- WICKENS, R. M. (1996) Interpreting Cointegrating Vectors and Common Stochastic Trends. *Journal of Econometrics*, 74, 255-271.
- ZADROZNY, P. A. (1998) An Eigenvalue Method of Undetermined Coefficients for Solving Linear Rational Expectations Models. *Journal of Economic Dynamics & Control*, 22, 1353.